Auto-Regressive Latent Variable Modeling: A General Framework for Bayesian Spatial Structural Equation Models

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Abstract

Spatial analytic approaches are classical models in econometric literature (LeSage & Pace, 2009). Recently, the behavioral sciences have seen an increase in their application, but spatial effects are generally still ignored (Stakhovych et al., 2012; Musafer et al., 2017; Oud & Folmer, 2008; Hogan & Tchernis, 2004). Spatial analysis models are synonymous with social network auto-regressive models which are also gaining popularity in the behavioral sciences. Structural Equation Models (SEM) are widely used in psychological research for measuring and testing multi-faceted constructs (Bollen, 1989). While SEM models are widely used, limitations remain, in particular latent interaction/polynomial effects are troublesome (Brandt et al., 2014). Recent work has produced methods to account for these issues (Brandt et al., 2018). Further, recent work has established methods to account for spatial and network effects in SEM (Oud & Folmer, 2008). However, a cohesive framework which can simultaneously estimate latent interaction/polynomial effects and account for spatial effects, has not been established. To accommodate this I provide a novel model, the Bayesian Spatial Auto-Regressive Structural Equation Model (SASEM). In the first chapter of this dissertation I review existing literature relevant to spatial analysis and latent interaction effects in SEM. In the next chapter I present a new modeling framework which can accommodate these effects. In the next chapter I investigate model performance with a series of Monte-Carlo studies. Results are promising particularly for one sub-model of the SASEM. I provide an empirical example using the spatially dependent extended US southern homicide data (Messner et al., 1999; Land et al., 1990) to show the rich interpretations made possible by the SASEM. Finally, I discuss results, implications, limitations, and recommendations.

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Chapter 1

Introduction

Waldo Tobler was a geographer and statistician at Columbia University. He is credited with establishing what is known as the first law of geography, which states "Everything is related to everything else, but near things are more related than distant things." In the economic, political, and geostatistical sciences spatial relationships are often hypothesized and tested under the spatial analysis framework (LeSage & Pace, 2009). In the behavioral sciences, these models are more commonly known as *Network Auto-Correlation Models*, which are used to model the reciprocal effects agents in a network have on one another (Valente, 2005).

Latent variable techniques are a hallmark of the behavioral sciences. They boast the ability to make predictions with complex features comprised of multiple observed items. The establishment of techniques for estimating latent interaction/polynomial effects further enhances latent variable method applications in behavioral sciences. These effects can be used to model hypotheses about moderators, as shown by Kernis et al. (1991) establishing the stability of self-esteem as having a multiplicative relationship with self-esteem when predicting depression. They can also model curvilinear relationships, as in work by Shi (1998), which shows that when age is used to predict healthy behaviors in humans rates are highest early and late in life and lowest around midlife. While work has been done with both latent spatial techniques and latent interaction/polynomial effects, a cohesive framework which unites the two has not been developed.

In the first chapter of this dissertation I introduce spatial auto-regressive techniques, latent variable techniques, extensions to latent interaction effects, and prior work with latent spatial effects, and then discuss the benefits of the combination of these approaches. Then in the second half, I present two new nonlinear latent variable models which accommodate spatial and interaction/

polynomial effects. Next, I investigate model performance under empirically relevant conditions with Monte-Carlo methods. I then provide two empirical applications of these models which exemplify their uses. Finally, I discuss the limitations of this study and the models and provide guidelines and recommendations to their application in future research.

1.1 Spatial Models

1.1.1 Regional differences in psychological research

The study of personality and cultural differences of geographic regions is a focal point of multiple fields in behavioral sciences, such as Personality Psychology, Sociology, and Anthropology to name a few. In large scale studies of regional differences, it is plausible to believe that nearer regions are more similar to one another than further regions in some domains. For example, in the United States, Nisbett (2018) showed *culture of honor* to be higher in the south than the north. Statistical inference of variables which exhibit spatial effects but are not accounted for is problematic. When a spatial relationship is present in data but not modelled estimates are biased (Anselin, 1988a).

Spatial analytics account for these types of violations and thus decrease erroneous statistical decisions. Spatial methodology also allows for the exploration of unstudied hypotheses. For example, McMillen et al. (2007) used spatial analysis approaches to show that tuition costs are more strongly effected by competing neighboring school tuition rates than schools further away. There are many behaviorally relevant applications for spatial models. However, metaphorical space may provide even more direct applications in the behavioral sciences. The next section briefly describes the application of spatial models to metaphorical distance via social network auto-correlation modeling.

1.1.2 Network auto-correlation models

Fields which employ network analysis like sociology and cognitive sciences use techniques described as *network auto-correlation models*. From a statistical standpoint network auto-correlation models and spatial models are the same. However, the representation of what constitutes the spatial relationship differs between them. In spatial models, the relationships between cases are denoted by their locations relevant to one another in space. In network auto-correlation models the physical representation of the spatial relationship is traded for theoretically relevant alternatives like influence or social comparison of agents (Leenders, 1995).

Network Auto-Correlation Models are utilized primarily to explicitly model *spillover effects* between cases. *Spillover effects* describe the process of one case affecting another case which then affects even more cases (LeSage & Pace, 2014b). Consider a spatial example in which we measure the property value of three neighboring houses, *A*, *B*, and *C*. Homeowner *A* adds a pool to their backyard which increases their property value. This increase raises the property value of their neighbor, homeowner *B*. The property value increase to homeowner *B* also spills over to homeowner *C*. The increase that homeowner *B* experienced also spills back to homeowner *A* increasing that property's value even more. In a social network context, spillover effects typically describe individual agent's effects on other agents. Consider the following network example from Valente et al. (1997). The authors measure contraceptive use opinions in a large social network of women in South Africa and use a network auto-correlation model to investigate spillover effects. They operationalize the spatial representation as the influence the women have on one another. Their results suggest that if participant *A* has high influence on participant *B*, when participant *A* adopts a positive opinion of contraceptive use *B* is more likely to develop positive opinions as well. This effect then spreads to other participants to which *B* has influence, as well as reinforcing the use of contraceptives by participant *A* (Kincaid, 2000).

Spatial or network auto-correlation models capable of providing such information are limited in other ways. A latent variable model that simultaneously controls for spatial dependence, provides spillover estimates, while also allowing for latent interaction effects does not exist. This is a major drawback. In psychology, higher order cognitive and behavioral traits, like facets of intelligence, depression, and implicit racism are typically measured by many observed variables. Current methods do not allow researchers to include these directly estimated latent traits into auto-correlation or spatial models. The next section outlines in more detail the motivations for using this class of models.

1.2 Motivations for Spatial and Network Auto-Correlation Models

1.2.1 Independence of observation

The *independence of observation* assumption in traditional statistical models is ubiquitous. Observations are assumed either completely independent or conditionally independent and accounted for. In other words, no observation is influenced by any other observation that the model does not consider. This is violated if $E[(x_i - \mu) * (x_j - \mu)] \neq 0$, where μ is a grand mean, and x_i and x_j are scores for case i and j (Kenny & Judd, 1986).

An intuitive way to conceptualize independence is to consider two scores (x_1 and x_2) are drawn from a population. If these scores are truly independent, then the score of x_1 will provide no information regarding the score of x_2. If these cases are related, they will provide information regarding the other. For example, if the scores we sample are from a married couple and x_1 is above the mean, it is more likely that x_2 is also above the mean.

Generally speaking, violations of independence occur in part from a lack of random sampling. This can be unintended, such as convenience sampling from an introductory psychology course resulting in a deceptively homogeneous sample. However, it can also be intentional, as is the case with repeated measures research. When random sampling directly inhibits exploration of the phenomena and is thus discarded, we must use modeling techniques to account for the violation and obtain accurate results. When observations are repeated, researchers use statistical techniques that account for the violation and produce a conditionally independent sample. When cases are geographic units like countries, states, or counties, or exist in space like houses or schools, it is likely that those closer to one another will be more related than those further away, thus violating independence. Spatial or network auto-correlation models take this into account resulting in *conditional independence*. In a network context, independence is also violated. The network of agents interact with one another, which is in fact the phenomena intended to be studied. However, this interaction itself violates independence, meaning a network auto-correlation model would be appropriate to account for the reciprocal interaction and provide a conditionally independent sample.

1.3 Omitted Variables

When studying phenomena which inherently prohibit random sampling, *omitted variables* are a common concern. Traditional regression models assume that the unexplained variance (error term) does not correlate with predictors in the model. When the variables in the model are spatially dependent, it is plausible that spatially dependent omitted variables are also present. That is, variables which are not included in the model but are related to both the dependent variable and predictor(s). The result is an error term in the model that has a non-zero correlation with predictors. When the error term is correlated with predictors or covariates in the model the resulting estimates are inaccurate (Clarke, 2005), and thus referred to as *omitted variable bias*.

Consider the following example of omitted variable bias. A researcher aims to predict crime rates in a set of neighboring counties. They collect some relevant covariates: average income; average education level; and the availability of narcotics. Then, they use traditional multiple regression analysis to make predictions. It is likely that some spatially dependent omitted variables will be correlated with the dependent variable and covariates. For example, the amount of police funding a county has is likely related to the availability of narcotics as well as the crime rate. This omitted variables effect is embedded in the error term affecting the model estimates.

When omitted variables arise from dependent data, accounting for the dependence using spatial or network auto-regressive models makes estimates more accurate and reliable (LeSage & Pace, 2009). Omitted variable bias has other potential implications which are not discussed here as they are not relevant to the topics in this dissertation, for more details see Clarke (2005, 2009).

1.3.1 Spatial auto-regressive linear models

Depending on prior theoretical understandings of the spatial effect, the spatial model is specified differently. When we aim to hypothesize about the magnitude of the spatial auto-correlation we specify a *dependent lag*. If we believe the spatial dependence is solely in the error term, we specify a spatial term in the disturbance sub-model.[1]

To provide a baseline of comparison, the first model considered is the standard OLS regression model without spatial components. Equation 1.1 provides a standard regression model with an intercept (β_0), a slope (β_1) estimate for a single predictor (x), and an error term (ε) which is normally distributed with a mean of 0 and non zero variance (σ).

$$y = \beta_0 + \beta_1 x + \varepsilon, \quad \varepsilon \sim N(0, \sigma I_N) \tag{1.1}$$

As mentioned earlier, spatial dependence can be specified in the model equation, the disturbance sub-model, or both. Eq. 1.2 provides the spatial model equation with both lags present; $\rho W y$ is the dependent spatial lag and $\lambda W \mu$ is the disturbance lag. ρ and λ are scalar summaries of the magnitude of the spatial auto-correlation of the dependent variable and disturbance term respectively. W is an n by n matrix which summarizes the spatial relationship between cases. In the network context, W represents the theoretically relevant metaphorical distance chosen, which in this example is influence each participant has on one another. μ provides the conditionally independent error term after spatial dependence in the disturbance has been accounted for. It is worth noting that if the magnitude of the spatial effects estimated by ρ and λ are 0, this model provides equivalent estimates to that of OLS regression.

[1]While not discussed in this paper, less common spatial specifications exist; for example, a predictor lag. This dissertation does not aim to cover all potential lags, as this would detract from the aim of the paper. For more information on alternative spatial model specifications, see LeSage & Pace (2009).

$$
\begin{aligned}
y &= \underbrace{\rho W y}_{\text{Spatial dependent lag}} + \beta_0 + \beta_1 x + \mu \\
\mu &= \underbrace{\lambda W \mu}_{\text{Spatial error lag}} + \varepsilon \qquad (1.2) \\
\mu &\sim N(0, \sigma_\mu)
\end{aligned}
$$

The decision of where to include spatial lags will depend on prior theoretical information regarding the variables of the model. If the researcher anticipates that the dependent variable is not spatially dependent and does not aim at evaluating that dependence, but is fearful that spatially relevant omitted variables may be present, just the disturbance lag is specified. Inversely, if the researcher aims at estimating the magnitude of the spatial dependence in the dependent variable, but has reason to believe there are no spatially dependent omitted variables then a dependent lag is specified. These are uncommon circumstances, as more often both are specified (Plümper & Neumayer, 2010).

1.3.2 Estimation

Traditionally the spatial models outlined above have been estimated with Maximum Likelihood (ML) estimation; however, Bayesian Monte-Carlo Markov-Chain (MCMC) estimation is gaining popularity. When estimated with MCMC techniques, prior distributions need to be specified. Commonly accepted priors and associated citations are provided in Table 1.1; where c_i ($i = 1,2$) are the hyper parameter means for normally distributed parameters, t_i provide the hyper parameter variances, $U(a,b)$ provides a uniform prior distributions with lower and upper bounds a,b for the auto-correlation summaries. Typically, a and b are set to -1 and 1 respectively to reflect the bounds of the spatial auto-correlation. However, if strong prior information suggests negative auto-correlation is not plausible 0 and 1 are used. Finally, $\gamma^-1(d_i, v_i)$ is the inverse gamma distribution for variances, with hyperparameters v_i and d_i. Alternative priors for standard deviations are half

Cauchy distributions (Gelman et al., 2013).

Table 1.1: Common priors for spatial regression parameters.

Parameter	Description	Prior	Citation
ε	Error term	$\sim N(0,\sigma^2)$	(LeSage, 1997; Gelman et al., 2013)
μ	Disturbance term	$\sim N(0,\sigma_\mu^2)$	(LeSage, 1997; Gelman et al., 2013)
β_0	Intercept	$\sim N(c_1,t_1)$	(LeSage, 1997; Gelman et al., 2013)
β_1	Slope	$\sim N(c_2,t_2)$	(LeSage, 1997; Gelman et al., 2013)
ρ	Dependent lag auto-correlation	$\sim U(0,1)$	(LeSage, 1997)
ϕ	Disturbance lag auto-correlation	$\sim U(0,1)$	(LeSage, 1997)
σ^2	Hyper standard deviation of σ	$\sim \gamma^-1(d_1,v_1)$	(LeSage, 1997)
σ_μ^2	Hyper standard deviation of μ	$\sim \gamma^-1(d_2,v_2)$	(LeSage, 1997)

1.3.3 Assumptions

The typical linear modelling assumptions apply to spatial models. The disturbance term is assumed *IID*, with mean 0, and finite variance σ^2. The elements of the X matrix has full column rank (lack of multi-collinearity). However, additional assumptions involving the spatial component/s are made. The elements of W are known constants and rank $(I-\rho W)=n$ for all $|\rho|<1$. The row and column sums of W and $(I-\rho W)^{-1}(I-\rho W\prime)$ are uniformly bounded in absolute value. Per the *No Islands* assumption, there are no cases present without neighbors. Finally, the *isotropy assumption*, states that the spatial effect is uniformly effective in every direction. It is worth noting that other spatial approaches measuring the changing magnitude of spatial effects across a location have been developed; however, those models assume the spatial effect is uniform across the sample.

1.4 Defining the Dependence Structure

The first stage in performing an auto-regressive analysis is establishing W, which defines the dependent relationships between cases. Two general options exist, contiguity or distance approaches (LeSage & Pace, 2009). Contiguity treats each case as having a dichotomous relationship to each other, meaning it is either a neighbor or it is not. The inverse distance based representation treats each case as being a certain distance from each other case. Common distance representations are Euclidean distance and Manhattan distances. Theory should guide these decisions, although the availability of information (precision of geographic location, for example) also plays a role.

Under contiguity representations, what constitutes a "neighbor" will vary based on the phenomena being studied. By virtue of the auto-regressive effects of these models, each case will impact all other cases to some degree. This is the process of indirect impacts. Case A impacts its neighbor, case B, then case B thus impacts its neighbor, case C; therefore, case A has indirectly impacted case C. However, in some settings it may be reasonable to assume case A also directly impacts case C. In this scenario, case C may be specified as a "neighbor" to case A. In econometric literature this is referred to as establishing *second order neighbors*. In addition, econometricians use the terms *Rook Contiguity* and *Queen Contiguity* to describe the process of determining first order neighbors. *Rook Contiguity* establishes only vertical and horizontal adjoining cases as neighbors, while *Queen Contiguity* also includes diagonally positioned neighbors.

An example of theoretically derived weighting schemes is given by McMillen et al. (2007), who implements higher order neighbor representations in an analysis measuring the spatial relationships of college tuition pricing. McMillen et al. (2007) justify specifying higher order neighbors because they argue that nearby colleges directly compete for student enrollment, regardless of being first or second order neighbors. Colleges which are near one another but not first order neighbors directly impact tuition prices, as well as indirectly through first order neighbors. Therefore, they use a *Second Order Queen Contiguity* representation of W.

After the neighbor or distance structure of W is established, common practice is to normalize W prior to analysis. Normalization constrains the range of auto-correlation estimates. Row normalization is a popular choice as it constrains auto-correlation estimates to be between -1 and 1 and is given by

$$W^* = \frac{W_k}{\Sigma W_k} \quad \forall\, k \tag{1.3}$$

where k represents the rows of the weight matrix W and W^* is the normalized matrix. Other choices of normalization appear in literature (LeSage & Pace, 2014a; Lee, 2013), but are not discussed here. For a more complete discussion of defining spatial units including normalization techniques see LeSage & Pace (2009).

In network fields, the spatial dependence is traded for social dependence and is constructed to represent the influence each case has on one another. For example, Kincaid (2000) investigated contraceptive use in a dependent network of women. The dependence structure was operationalized by measuring the self-reported influence of each participant with one another and constructing weighted values. For a more complete discussion of network auto-correlation representations of dependence see Ponds et al. (2009).

1.4.1 Contiguity

Contiguity represents spatial relationships as dichotomous, meaning you are either a neighbor to another case or not. The matrix which represents these relationships is referred to as W and is n by n. Each row and column of the W matrix represents a case. The elements contain ones with regions that share borders, and zeros with those who do not. The diagonal is always 0, as a case can not be a neighbor to itself.

1.4.2 Distance

An alternative to using contiguity specifications for W is an inverse distance matrix. In a pragmatic sense, distance from a central location can be more realistic information to obtain. For example, boundary data may be unavailable or unrealistic to define. Determining the boundaries of a large set of regions can be difficult, time consuming, and potentially subjective. In network settings, distance measures are used. For example, in the aforementioned work by Valente et al. (1997) elements of the W matrix represent the influence a participant has with other participants.

Inverse distance ($\frac{1}{\text{distance}}$) is used (as opposed to distance) due to the magnitude of the spatial units implied in an analysis. This weighs the spatial relationship so that nearer regions have higher values in W and further regions have lower values.

Simulation results indicate a high convergence between parameter estimates when comparing distance to contiguity specifications of W in econometric applications with physical space (LeSage & Pace, 2014a). In a simulation study comparing the differences between inverse distance and contiguity W specifications in a spatial regression model, similar parameter estimates and spatial parameters were observed. The correlation between β parameter estimates from both specifications varied from .74 to .86. The spatial parameter estimates ρ were very close with the correlation between specifications ranging .92 to .98 (LeSage & Pace, 2014a).

1.5 The Latent Variable Framework

I now take an aside from spatial models to provide a brief background on Structural Equation Modeling (SEM), adaptations to include structural interaction/ polynomial effects, and extensions to spatial effects.

The latent variable framework is a hallmark of behavioral statistics by measuring unobserved constructs via sets of related proxies. Latent variable models provide a framework for making predictions with and comparing multifaceted features. Some work has been done to extend spatial techniques to the latent variable framework (Christensen & Amemiya, 2001, 2002, 2003; Oud & Folmer, 2008; Stakhovych et al., 2012; Liu et al., 2005) but limitations remain. This section will provide a brief overview of the model equations for Structural Equation Modeling (SEM). For a more in depth understanding of latent variable techniques and SEM, see (Bollen, 1989).

Generally speaking, latent variable techniques provide a framework for producing and testing latent factors from observed sets of variables. In classical settings, the approach first started with an Exploratory Factor Analysis (EFA) to determine the number of factors and which observed items were included within factors,and then utilized Confirmatory Factor Analysis (CFA) to test the factor structure. However, in contemporary work the typical procedure is to use substantive theory to group items together and produce latent constructs, skipping the EFA stage. We then assess model fit and determine whether the model implied covariances match the observed covariances between observed variables. Once good model fit is established, interpretation of the model parameters is permissible. This process is referred to as establishing a measurement model and informs researchers as to the relationships between observed items and their associated latent constructs. Once a proper measurement model is established, researchers may extend the model to make predictions with latent constructs. This extension is referred to as SEM. SEM allows for predictions (and comparisons) to be made at the latent level. That is, the prediction of latent features by or on other latent factors or observed variables (Bollen, 1989). In SEM the dependent latent variables are referred to as *endogenous*, while latent predictor variables are referred to as *exogenous*.

SEM are comprised of two sub-models, the measurement model and the structural model. The measurement model establishes the relationships between observed indicators and the generated latent factors. With SEM, there is a separate measurement model for endogenous and exogenous latent variables, see Eq. (1.4) and Eq. (1.5).

$$y = \Lambda_y \eta + \varepsilon \tag{1.4}$$

$$x = \Lambda_x \xi + \delta \tag{1.5}$$

Where $y\,(p \times 1)$ and $x\,(q \times 1)$ are vectors of observed variables, q representing the number of observed exogenous variables, and p representing the number of observed endogenous variables. $\Lambda_y(p \times m)$ and $\Lambda_x(q \times n)$ are matrices of factor loadings summarizing the relationships between latent (η, ξ) and observed constructs (y, x). Measurement error is assumed orthogonal to ξ and η.

When establishing the measurement model, latent variable techniques require a method for identifying the mean and variance of the latent variables. This is called the latent variable scaling technique. This can be achieved in two ways, the *marker variable* approach or standardizing the latent variable. The marker variable approach constrains the factor loading to one, for one variable of each factor. The factor then takes up the mean and variance of that item. However, in some situations, you may want to have all of the factor loadings estimated, in which case, standardizing the latent construct to have mean of 0 and variance of 1 is preferred.

The structural equation for SEM is provided in Eq. (1.6), and represents the latent predictions of the model.

$$\eta = \alpha + B\eta + \Gamma\xi + \zeta, \quad \zeta \sim N(0, \sigma I_N) \tag{1.6}$$

Where η is an $m \times 1$ vector of endogenous latent variables. α provides the intercept, ξ is an $n \times 1$ (n = number of exogenous latent variables) vector of exogenous (predictor) latent variables. ζ is an $m \times 1$ matrix of latent errors. B is an $m \times m$ coefficient matrix of the latent endogenous variables. Γ is the coefficient matrix for the latent exogenous variables, commonly referred to as factor loadings, both Γ and B are linear slopes. It is worth noting that if a single endogenous variable is supplied there is no matrix of linear slopes defining their relationships. The result is the $B\eta$ term being dropped from the structural level of the model. Covariances can be specified between the exogenous predictors. These estimates are the off diagonal elements of ϕ an $n \times n$ variance covariance matrix. ψ is the $m \times m$ variance covariance matrix of the error terms. If no covariances are specified, ϕ and ψ are diagonal matrices containing variances.

1.5.1 Estimation

Traditionally SEMs with continuous observed variables are estimated using Maximum Likelihood (ML) estimation (Bollen, 1989). ML for SEM, just as with other models, aims at maximizing the F_{ML} function. In the case of SEM, maximizing the F_{ML} minimizes the discrepancy between the estimated population covariance matrix Σ and the model implied covariance matrix $\Sigma(\theta)$. Eq. (1.7) provides the F_{ML} function (Suhr, 2006). The population covariance is estimated as a function of the observed sample covariance matrix S. In other words, the F_{ML} function provides a means to optimize the model based on optimizing model fit.

$$F_{ML} = \log|\Sigma(\theta)| + \mathrm{Trace}[\Sigma(\theta)^{-1}S] - \log|S| - p \tag{1.7}$$

where θ is a vector including all parameters. Other options exist and are preferred in other scenarios. Weighted Least Squares Mean and Variance adjusted (WLSMV), for example, is preferred when observed items are dichotomous (Finney & DiStefano, 2006). Maximum Likelihood with Robust standard errors (MLR) is an option which is robust to non normal observed data. MLR estimation aims at estimating standard errors robustly and does so by computing them with the sandwich estimator (Rosseel, 2010).

Recently, Bayesian approaches have become popular and are being used in applied SEM (Muthén & Asparouhov, 2012; Blodgett & Anderson, 2000; Arhonditsis et al., 2007). Bayesian methods come with many benefits: joint modeling approaches for missing data; convergence with lower sample sizes; robustness against data of non-normal distributions; distributional parameter estimates; and simplicity in model specification, to name a few (Gelman et al., 2008; Raftery, 1993; Muthén & Asparouhov, 2012).

1.5.2 Model identification

Identification of the measurement and structural models of a SEM must be accomplished to obtain accurate estimates (Bollen, 1989; Rindskopf, 1984). A measurement model is guaranteed to be identified if four conditions being met.

1. The latent variable is scaled.
2. A sufficient number of observed indicators load on each latent variable.
3. Each latent factor has at least one observed variable loading onto it which does not. share a residual correlation with an observed item on the another factor.
4. No item has factor loadings on more than one latent variable (*simple structure*).

To accommodate these restrictions, I follow advice by Rindskopf (1984) and set some guidelines which will guarantee model identification: 1) The latent variable is unmeasured and thus its units must be determined by the researcher. Two approaches are popular, *standardized latent variable* and *marker variable*. The *standardized latent variable* approach constrains the variance of the latent factor to be 1 with mean 0. Alternatively, the *marker variable* approach constrains a factor loading to be 1 for each latent variable. The latent variables then take up the scaling of the items with factor loadings of 1. 2) Each latent variables needs to have at least three observed indicators loading onto it. 3) Each latent factor needs to have at least one observed variable loading onto it which does not share a residual correlation with an observed item on the another factor. 4) No observed item has a factor loadings on multiple latent variables (Kenny & Milan, 2012; Rindskopf, 1984; Wu & Estabrook, 2016).

The structural level of a SEM must also be identified. I continue to follow the advice of Rindskopf (1984) in laying out guidelines which guarantee identification:

1. There are less estimated parameters than unique elements of the observed covariance matrix.

2. The structural model must not have non-recursive pathways.[2]

1) Regarding the Degrees of Freedom (DF) of the model, a SEM will have $DF = (k(k-1)/2) - 1$, where k is the total number of observed indicators. This is the number of non-redundant elements of the covariance matrix of the observed items. This is the only identification criteria which changes inherently for the SASEM. It is important to consider the additional parameter estimate(s) of the spatial effects. Each spatial parameter estimate (ρ and λ_s) counts as an additional added parameter. 2) The model must not have non-recursive pathways, estimated as covariances or linear slopes.

The aforementioned identification guidelines for both the measurement and structural models are relatively strict but will guarantee identification. In the measurement level 2-4 can be relaxed if further conditions are assessed. In the structural model, special cases allow for relaxation of both conditions. as model identification is not the focus of this paper, see Rindskopf (1984) for further explanation.

[2]The terminology for recursive and non-recursive models is counter-intuitive. A recursive model has unidirectional effects. Non-Recursive models have pathways which contain feedback loops. For example, Y is regressed on X, X is regressed on Z, Z is regressed on Y.

1.5.3 Structural interaction effect extensions to SEM

Interaction effects (sometimes referred to as moderated effects in behavioral sciences) are of interest in behavioral research as a means of understanding complex relationships between three or more variables. If the strength of a relationship between two variables differs as a function of a third variable, we have established moderation (e.g., Cohen et al., 2003). Consider an example regarding a weight loss study. A researcher aims to predict weight in kilogram (kg) from caloric intake. As caloric intake increases, weight increases. However, this effect is moderated by caloric expenditure. If a participant is frequently exercising they will have a diminished relationship with caloric intake and weight. The interaction effect of caloric intake and caloric expenditure explains this relationship between the three variables. Testing of this effect is established by making a product variable of caloric intake and caloric expenditure, then including this additional term as a predictor to estimate this interaction effect (e.g., in OLS regression).

In traditional SEM, estimation of latent interaction effects is not as simple as multiplying two or more variables together because the product is formed by two latent constructs. Multiple approaches have been developed to extend structural interaction effects into SEMs. Notable approaches are: The Product Indicators approaches (PI); distributional analytic approaches; method of moments approaches; and Bayesian techniques have shown success. The PI approach was first. As the name implies, PI establishes a latent product term by estimating the measurement model of the latent product variable from a set of products of observed variables (Kenny & Judd, 1984; Jöreskog et al., 1996; Marsh et al., 2004; Wall & Amemiya, 2001; Kelava & Brandt, 2009). Distributional analytic approaches introduced mixture modeling techniques to directly account for the nonlinear relationship in full information maximum likelihood estimators without product indicators (Klein & Moosbrugger, 2000; Klein & Muthén, 2007; Kelava et al., 2014). Method of moments approaches introduced multi-stage estimation and relaxed normality assumptions (Wall & Amemiya, 2003; Mooijaart & Bentler, 2010). Finally, Bayesian techniques with user defined prior distributions provide the least biased and reliable procedure for estimating structural interaction effects in SEM (Muthén & Asparouhov, 2012; Lee et al., 2007; Kelava et al., 2014; Brandt et al., 2018).

1.5.3.1 PI approaches to latent interaction effects

PI approaches were originally established by Kenny & Judd (1984) providing an intuitive solution to ML estimated latent interaction or polynomial effects. Eq. (1.8) provides the desired form of the structural equation of SEM when a single endogenous latent variable is predicted by two exogenous latent variables and an interaction term of the two exogenous variables. The product term is designated by $\gamma_3\xi_1\xi_2$.

$$\eta = \alpha + \gamma_1\xi + \gamma_2\xi + \gamma_3\xi_1\xi_2 + \zeta \tag{1.8}$$

The latent product term in PI approaches is identified via products of observed variables. Under this approach, the observed variables are multiplied together, a latent product term is then included, just as the other latent variables are through the estimation of linear factor loadings. As an example, consider the following scenario. Three latent variables are estimated, two exogenous ξ_1 and ξ_2 and a latent endogenous term η. Each latent construct is comprised of three observed items, x_1, x_2 and x_3, and load on ξ_1 while x_4, x_5 and x_6 load on ξ_2 and y_1 through y_3 load onto η. We hypothesize a latent interaction between ξ_1 and ξ_2. Using the PI approach we produce a latent product $\xi_1\xi_2$ by multiplying the observed exogenous variables together. In our example we could do that by $x_1 \times x_4 = x_7$, $x_2 \times x_5 = x_8$, and $x_3 \times x_6 = x_9$. Finally, x_7, x_8, and x_9 are now included in the model, loading onto the latent variable designated as the interaction term. In this scenario the assumption that error terms are independent is violated, considering x_7 through x_9 are products of the other observed variables. In the original implementation, the measurement model for the PIs was constrained using nonlinear constraints from the parameters of the measurement model from the indicators x_1 through x_6 (Kenny & Judd, 1984). Later these constraints were relaxed (Wall & Amemiya, 2001; Marsh et al., 2004).

The PI approach is intuitive, but there are drawbacks to this approach. The approach with ML estimation still assumes multivariate normality on the product term and indicators. This assumption will always be violated with PI approaches; even if all observed variables are normally distributed, their products will not be (Aroian, 1944). Consequently, estimates of standard errors and fit indices can be severely inaccurate. Biased standard errors result in increased error rates. This has been further confirmed by simulation studies (Kelava et al., 2014; Brandt et al., 2014). In addition, the PI approaches provide rather inaccurate results if models are any more complex than the example in the previous paragraph (Brandt et al., 2018).

1.5.3.2 Distributional analytic approaches

There are three notable distributional analytic approaches which aimed at creating less biased estimates of latent interaction/polynomial effects and improving estimation efficiency. The Latent Moderated Structures (LMS) approach was established by Klein & Moosbrugger (2000), and the Quasi-Maximum Likelihood (QML) approach by Klein and Muthén (2007).

The LMS approach employs a likelihood function which accounts for the non-linear relationships of interaction/polynomial latent effects. Broadly speaking, the likelihood function is estimated numerically and optimized using the Expectation Maximization (EM) algorithm (Dempster et al., 1977). The QML approach takes the non-normal distributions lower moments and estimates a normal distribution in its place that matches these moments. The multivariate normal distribution of the indicator variables are approximated by multiplying a conditionally normal distribution and unconditionally normal distribution. This procedure is akin to mixture modeling, in that the non-normal interaction variables distribution is a product of normal distributions. Finally, these two approaches were extended to account for non-normal distributions of the variables (Nonlinear Structural Equation Mixture Modeling approach, NSEMM; Kelava et al., 2014).

The original two distributional analytic approaches, LMS and QML, impose multivariate normality assumptions regarding the distributions of the latent exogenous predictors and residuals of the observed variables and latent level disturbances (Klein & Moosbrugger, 2000). The major benefit of LMS over QML is true maximum likelihood estimates, which indicates efficient derived standard errors and nested log-likelihood model testing procedures. However, ML also has its drawbacks; one being assumptions of normality, as previously discussed. QML boasts more robustness to departures from normality and faster computational efficiency over LMS (Kelava et al., 2011). Compared to the PI approaches, the distributional analytic show less bias, and more coverage of the actual effect when observed variables is normally distributed (Brandt et al., 2014). The NSEMM approach was introduced to overcome the problem LMS and QML showed but provided somewhat lower efficiency (Kelava et al., 2014).

The disadvantage of these approaches is that their implementations are limited and extension (e.g., to spatial models) is very complicated. Theoretic derivation of complex likelihood functions needs to be achieved, for applied users this cannot be expected. QML only exists in an experimental standalone software. LMS and NSEMM can be used via Mplus (which is a closed software system) and the R package nlsem. In each implementation, models are limited to those that do not include spatial modeling components.

1.5.3.3 Method of moments approaches

Two main approaches within the method of moments literature have been established to estimate latent interaction effects. Two stage Method of Moments (2SMM) is an iterative process, which first estimates a measurement model, then calculates factor scores. Finally, the procedure then estimates the structural level of the equation which introduces an interaction term (Wall & Amemiya, 2003). The other being Method of Moments established by Mooijaart & Bentler (2010), which takes non-normality directly into account by considering higher order moments, skew, kurtosis, and hyperskewness (Mooijaart & Bentler, 2010).

The 2SMM approach is a two stage process. First, the measurement model of the latent variables is estimated using typical procedures. Factor scores are then extracted via Bartlett's method, which is the product of the vector of observed variables and the inverse of the diagonal matrix of variances of factor scores ψ (DiStefano et al., 2009). In the second stage, the factor scores and variances are employed to estimate the parameters of the structural model. This procedure is grounded in the error-in-variable regression framework, which takes into account that the factor scores are derived in the first step and thus need to have a variance component modeled in the next stage.

The other notable approach is aptly named Method of Moments (Mooijaart & Bentler, 2010). In this method, the assumption of normality governing the error variances is relaxed. The non-normality of the interaction term is taken into consideration and the estimator aims at minimizing the difference between model implied higher order moments and observed higher order moments. Under the MM approach, only normality of the latent exogenous predictors is implied, whereas the non-normality due to the interaction effect is explicitly taken into account. It is worth noting that the method of moments approach suffers from the most severe bias and inflated type I error rates when observed data is non-normal or more than a single interaction effect is specified (Brandt et al., 2014).

The main disadvantage of the 2SMM approach is that it is a limited information approach that at least theoretically provides less efficient estimates than full information methods (e.g., Brandt et al., 2019). In addition, it is only available in experimental software scripts and an implementation is still missing.

1.5.3.4 Bayesian approaches to latent interaction effects

Bayesian estimation techniques provide a flexible modeling framework, which can explicitly model the non-normal distributions introduced by latent interaction and polynomial effects. Due to the Bayesian procedure of sampling posterior distributions, created through prior distributions (priors) and observed distributions (the data), the problems encountered with classical SEM procedures and interaction effects is not a problem. Early Bayesian SEM interaction adaptations were implemented by Lee et al. (2007). They established a Bayesian SEM capable of handling latent interactions.

The typical problems associated with frequentist estimation of latent interaction/polynomial effects does not apply to Bayesian models, established by Lee et al. (2007). Bayesian SEM provides equally accurate estimates as frequentest methods. When latent interaction effects are specified Bayesian SEM performs similarly to LMS. This requires large sample sizes (n > 400) and priors are uninformative.Bayesian methods do exhibit sensitivity to systematically non-normal observed variables, however extensions to accommodate this exist (Kelava et al., 2014).

Two distributions are *conjugate* if they are from the same family. Lee et al. (2007) suggest using conjugate priors as they provide the same distributional form as the posterior which is convenient in the production of the corresponding conditional distribution. They recommend the following prior specifications

$$\begin{aligned}
\phi &\sim \text{Wishart}^{-1}(R_0, \rho_0) \\
\psi_x &\sim \text{Gamma}^{-1}(\alpha_x^*, \beta_x^*) \\
\psi_\xi &\sim \text{Gamma}^{-1}(\alpha_\xi^*, \beta_\xi^*) \\
\Lambda &\sim \text{Normal}(\Pi_0, \Pi_1) \\
\gamma &\sim \text{Normal}(\Pi_0, \Pi_2) \\
\alpha &\sim \text{Normal}(\Pi_0, \Pi_3)
\end{aligned} \tag{1.9}$$

where $R_0,\ \rho_0,\ \alpha_x^*,\ \beta_x^*,\ \alpha_\xi^*,\ \beta_\xi^*$, and Π_{0-3} are the hyperparameters associated with each distribution. Choices of hyperparameters will vary depending on the situation (Lee et al., 2007). If a researcher wants the prior to be informative, small variance parameters ($\rho_0,\ \beta_x^*,\ \beta_\xi^*,\ \Pi_{1-3}$) can be established around the anticipated parameter value ($R_0,\ \alpha_x^*,\ \alpha_\xi^*,\ \Pi_0$). If we want to provide less information, a large variance can be specified (Gelman et al., 2013) [3].

Lee et al. (2007) provide simulation results for the Bayesian SEM with interaction/polynomial effects. In summary, when prior information is accurate, this approach provides accurate estimates of all effects in the model. Further, even under very small sample sizes (N = 150), the model was capable of accurately estimating all 47 parameters. This is a powerful advantage of Bayesian SEM. Behavioral research can be time consuming and/or expensive to collect participants and SEM notoriously requires large sample sizes in ML estimation (Fan et al., 1999).

[3] The Wishart and inverse Wishart^{-1} distribution are not available in the STAN modeling software. For users of STAN the LK_j prior is an equivalent option

One of the main advantages of the Bayesian nonlinear SEM is that it is directly extendable to account for spatial effects. Simulation results so far showed that Bayesian implementations preform well even under small sample sizes (n = 49).

1.5.4 Spatial Extensions to SEM

Just like simple linear models, SEMs may exhibit issues associated with spatial dependence. For example, predicting a country's latent pro-climate change action perspectives from political features like liberalism, environmentalism, and social dominance orientation would no doubt exhibit spatial dependence. However, under the classic SEM framework this cannot be explicitly modelled.

Several attempts to extend considerations of spatial dependence to factor analysis have been made. Christensen & Amemiya (2002) provided an early advancement, which allowed for the investigation of spatial relationships in factor scores using Confirmatory Factor Analysis (CFA). However, it required the spatial relationship to be operationalized using an artificial grid, which is restrictive (Christensen & Amemiya, 2001, 2002, 2003). Wang & Wall (2003) established an Exploratory Factor Analysis (EFA) method for estimating the factor covariance matrix in the presence of spatial dependence measured via distance. Liu et al. (2005) established a method for including distance based spatial modeling for confirmatory models. This provided the foundation for Stakhovych et al. (2012), which pioneered the ability to include spatial weighting (both contiguity and distance) in determining the spatial auto-correlation of factor means in Bayesian CFA. These advancements, while focusing on latent variables, only allow for the construction of spatially related measurement models. However, the next logical step is to focus on the actionable use of the established latent variables via SEM, which Oud & Folmer (2008) consequently did with the production of a frequentist spatial SEM.

1.5.4.1 Stakhovych et al. (2012): Spatial CFA

The Bayesian spatial CFA, as developed by Stakhovych et al. (2012), provides a framework for implementing spatial effects in latent variable models. While this is a meaningful advancement, their method only allows for the estimation of latent effects, not the prediction of such (as in SEM). However, they do afford the ability to estimate the spatial auto-correlation of each latent variable within higher order spatial units. As an example Stakhovych et al. (2012) use regions as spatial units, within the higher order spatial unit of countries. This is reminiscent of multi-group CFA, which allows for the estimation of unique factor loadings between groups. The model by Stakhovych et al. (2012) also allows for unique factor loadings as well as spatial auto-correlation for each higher order spatial unit (again, countries in their example). The non-spatial portion of the model equation is given by

$$
\begin{aligned}
x_{ij} =& \alpha + \Lambda_c \xi_{ij} + \varepsilon_{ij} \\
\varepsilon_{ij} \sim& N(0_k, \Theta_i) \\
\xi_{ij} \sim& N(\nu_i, \Theta_i)
\end{aligned}
\tag{1.10}
$$

where the observed scores x_{ij} for regions $i = 1...I$ and observations $j = ...J_i$ are modeled as an intercept (α), factor scores ξ_{ij}, and error term ε_{ij}. The spatial component of the model is given by

$$
\begin{aligned}
vec(V) =& (P \otimes W)vec(V) + Vec(U) \\
vec(U) =& (\nu_1 ... \nu_i ... \nu_I)' \\
vec(U) \sim& N_{IL}(0_{IL}, \Sigma \otimes I_I)
\end{aligned}
\tag{1.11}
$$

where the I by L matrix of factor means $V = (\nu_1 ... \nu_i ... \nu_I)'$ are partitioned into L vectors (V_l) of I by 1 dimensions, so that V_I is the Ith column of $V = (V_1 ... V_i ... V_I)'$ and corresponds to the means of the Ith factor. Every V_I has the structure of $V_i = \rho_I W V_i + U_i$. Where ρ_I is the spatial auto-regressive coefficients and W is the I x I spatial weights matrix representing the regions, and U_I is the $(Iby1)$ vector of error terms and is $\sim N_I(0_I, I_I$. The Kronecker product $\otimes$ is taken of P an L by L diagonal matrix of spatial weights (ρ_I), and W to expand the weight matrix across the higher order spatial units. It is assumed that factor score means ν_i are assumed to exhibit heterogeneous variances and are spatially correlated between the $i...I$ regions. The relationship between factor loadings can vary between higher order regional units $c = 1...C$ by introducing "country" specific factor loadings Λ_c.[4].

To test their model, a simulation study was conducted. The simulated model recovered population level effects very accurately. However, only a single data set was simulated and analyzed. Information regarding the accuracy of their model under different circumstances is not available. It is unclear how this model can perform in different situations, like smaller sample size or more connections in W. A further limitation of the model by Stakhovych et al. (2012) is the lack of flexibility in which portion of the model spatial effects can be included. In this formulation spatial effects are estimated for each latent variable. However, they are not included in a structural equation which includes additional latent factors. This makes a scenario where researchers believe a single observed item exhibits spatial dependence impossible. To accommodate this, a spatial lag at the measurement level would be necessary.

In their example, a model is estimated with ten latent constructs from a total of 44 observed items. They measure each of these variables from five countries in Europe, breaking each country down to multiple sub-regions. Using the sub-regions as spatial units, they estimate a multiple group factor model where each nation has its own parameter estimates. They then summarize the overall spatial auto-correlation of each latent factor across the entire sample. For example, a significant spatial auto-correlation parameter $\rho = 0.89$ for *conformity* was observed in Spain. This is interpreted to mean that in Spain conformity values are similar in nearby regions and different in further regions (Stakhovych et al., 2012).

Priors for the spatial CFA model follow closely with the measurement model priors chosen by

[4]Stakhovych et al. (2012) refer to lower order spatial units as regions, and higher order units as countries. However, their model is applicable to counties in states, or schools in school districts, etc.

Lee et al. (2007). However, the additional spatial auto-regressive parameter ρ is chosen to have a uniform prior distribution, $\rho \sim U(1/\lambda_{min}, 1/\lambda_{max})$. Where λ is the vector of Eigenvalues of the W matrix (Stakhovych et al., 2012).

1.5.4.2 Oud & Folmer (2008): Spatial SEM

Oud & Folmer (2008) developed a class of SEMs which introduce spatial effects. Their first model of the standard spatial error model includes a spatial error term, which accounts for spatial dependence in the residual of the structural equation, thus allowing for the possibility of IID residuals, which is assumed. The second model, the latent lag model, allows the researchers to test the coefficient regarding the magnitude of spatial dependence in the endogenous variable. Both models parameters are estimated with a modified maximum likelihood function (Oud & Folmer, 2008).

The observed spatial error model accounts for residual spatial relationships of the unexplained variance in the structural equation. The spatial error SEM structural equation is presented in Eq. (1.12).

$$\eta = \gamma x + \varepsilon \quad \text{and} \quad \varepsilon = \lambda_s W \varepsilon + \zeta \tag{1.12}$$

Where x is an observed exogenous predictor whose effect is summarized by the linear coefficient γ, ε is variance unexplained by the prediction, λ_s represents the spatial autocorrelation coefficient (not to be confused with factor loadings λ) summarizing the degree of spatial dependence in the residual term, and ζ represents the unexplained variance in the structural model once the spatial dependence is accounted for.

This model provides a much needed extension of the SEM framework to account for residual spatial dependence. In the equation for the spatial error SEM, the omission of ξ is replaced x symbolizes the predictor. Under Oud, and Folmer (2008) model specification, the spatial error SEM only allows for exogenous observed variables and is not structured to estimate exogenous latent variables. This is a drawback, as it only allows for the estimation of a single latent outcome variable, thus restricting its applications to those whom aim to predict a latent variable from observed variable.

The latent spatial lag model introduces an additional coefficient into the model via a lag of the dependent variable. There is no spatial error term λ in this formulation, suggesting that the spatial relationship is exclusively presented in the endogenous latent variable. The structural equation for the standard spatial lag model is presented in Eq. (1.13).

$$\eta = \rho \eta W + \gamma x_1 + \zeta \tag{1.13}$$

Where ρ is the spatial autocorrelation coefficient which summarizes the degree of spatial dependence present in the dependent variable and ζ is the error term. This model is appropriate for situations where the researcher hypothesizes that all spatial dependence is accounted within the dependent variable itself and not in the residual term (Oud & Folmer, 2008). Again, this model is only capable of estimating a single latent endogenous outcome variable η and not latent exogenous predictors.

1.5.4.3 Summary

Overall, both of the aforementioned models (Spatial CFA and SEM) are a great step towards a cohesive SEM framework to account for and hypothesize with spatial dependence; however, limitations remain. First, Oud & Folmer (2008) have specified a model which does not include latent exogenous variables, only a single endogenous one. In practical situations this is burdensome. If a researcher needs to use multi-faceted predictors, sum scores or averages could be used but can bias estimates of relationships and is not recommended theoretically (DiStefano et al., 2009) or statistically (Bollen, 1989). Next, the use of the modified ML function restricts estimation to linear effects, as latent interaction effects are not possible using this model. Finally, as with the model by Stakhovych et al. (2012) measurement level spatial effects, disturbance lags, or structural predictions are not possible in these models.

1.6 Conclusions

Spatial and network auto-regressive approaches provide a means of analyzing dependent data. This class of analysis provides rich interpretations of model parameters by calculation of spillover effects. Interpretation of spillover effects provide a means of investigating unique hypothesis which cannot be explored otherwise.

Psychological and behavioral phenomena are frequently conceptualized as multi-dimensional constructs which are measured by multiple items. Latent variable techniques provide a set of approaches to measure and make predictions with these multi-faceted constructs.

Behavioral research also frequently calls for the use of interaction effects. It is commonly hypothesized that variables exhibit a conditional relationship with other variables in their prediction of outcomes.

The combination of each of these phenomena has not yet been fully explored. While some combinations exist (i.e. spatial CFA (Stakhovych et al., 2012) or spatial regression with interaction effects LeSage & Pace (2009)), all 3 have not been included into a unified framework which allows for the simultaneous use of each.

In the following chapter I present a solution to this problem. A combined framework which incorporates latent variables, latent structural predictions, latent structural interactions, and spatial (or network) auto-regressive effects.

Chapter 2

A New Approach for Latent Spatial and Interaction Effects

I now present a novel framework for fitting Bayesian spatial auto-regressive SEMs with latent interaction effects. This framework extends previous efforts to bring spatial models into a Bayesian latent variable framework capable of accurately recovering latent structural effects. Specifically, the extensions include: a.) Expansion of the framework by Stakhovych et al. (2012) and Oud & Folmer (2008) to include a structural level equation which can include exogenous latent predictors. b.) Flexible specification to include spatial effects in either the structural or measurement level equations. c.) Simultaneous estimation of both spatial and latent interaction effects.

2.1 Bayesian Spatial Auto-regressive SEM

The Bayesian Spatial Auto-regressive Structural Equation Model (SASEM) framework supports specification of spatial effects throughout the model equations. Spatial effects can be specified at the structural or the measurement levels. That is, spatial effects can be specified on a subset of observed variables in the measurement equation or at the structural level. Further, the SASEM can support disturbance or endogenous lags at either level.

In this presentation, I will focus on a single latent dependent variable (endogenous variable, η) to keep model presentation simple. The extension to more than a single endogenous latent variable is straightforward.

2.1.1 Measurement model

The general SASEM measurement models for $j = 1 \ldots J$ observed variables y_j of the single endogenous latent variable η and the $k = 1 \ldots K$ observed variables x_k of the $p = 1 \ldots P$ exogenous latent variables $\xi = (\xi_1, \ldots, \xi_P)'$ is given by (suppressing the person subscript i)

$$\begin{aligned} y_j &= \tau_{yj} + \rho_{yj} W y_j + \lambda_{yj}\eta + \varepsilon_j, \qquad \varepsilon_j = \phi_{\varepsilon j} W \varepsilon_j + \mu_{\varepsilon j} \\ x_k &= \tau_{xk} + \rho_{xk} W x_k + \lambda_{xk}\xi + \delta_k, \qquad \delta_k = \phi_{\delta k} W \delta_k + \mu_{\delta k} \end{aligned} \tag{2.1}$$

where the relationship between y_j and η is summarized by the intercept τ_{yj}, the factor loading λ_{yj} and associated error term ε_j. The relationship between x_k and ξ is summarized by the intercept τ_{xk}, the $P \times 1$ vector of factor loadings $\lambda_{xk} = (\lambda_{xk1}, \ldots, \lambda_{xkP})$ for the kth indicator variable and the error term δ_k. Dependent spatial lags are given by $\rho_{yj} W y_j$ and $\rho_{xk} W x_k$. Disturbance spatial lags are given by $\phi_{\varepsilon j} W \varepsilon_j$ and $\phi_{\delta k} W \delta_k$, resulting in an error sub-model which includes spatially dependent error matrices $\mu_{\varepsilon j}$ and $\mu_{\delta k}$.

2.1.2 Structural model

The general nonlinear SASEM structural equation for one endogenous latent variables η and P latent exogenous variables $\xi = (\xi_1, \ldots, \xi_P)'$ is provided by

$$\eta = \alpha + \rho_\eta W \eta + \gamma_1 \xi + \gamma_2 h(\xi) + \zeta, \qquad \zeta = \phi_\zeta W \zeta + \mu_\zeta \tag{2.2}$$

where α is an intercept. $h()$ is a function that creates an M dimensional vector that is used to specify nonlinear polynomial terms (such as ξ_1^2, ξ_1^3) or other product terms between predictors (such as $\xi_1 \xi_2$). Details can be found in Brandt et al. (2019); Bollen (1995). γ_1 and γ_2 are P and M dimensional vectors that include the regression coefficients for the linear and nonlinear effects of the latent exogenous variables, respectively. A latent endogenous spatial lag is included by $\rho_\eta W \eta$. A disturbance term spatial lag $\phi_\zeta W \zeta$ is also specified on the uncorrected error term μ_ζ. The disturbance sub-model accounts for omitted spatially dependent variables resulting in the conditionally independent structural error term ζ.

An example for a simple model with two latent exogenous variables ξ_1 and ξ_2 and their interaction effect $h(\xi_1, \xi_2) = \xi_1 \xi_2$ is provided by

$$\eta = \alpha + \rho_\eta W\eta + \gamma_{11}\xi_1 + \gamma_{12}\xi_2 + \gamma_{21}\xi_1\xi_2 + \zeta, \qquad \zeta = \phi_\zeta W\zeta + \mu_\zeta \tag{2.3}$$

where γ_{11}, γ_{12} are linear effects of ξ_1, ξ_2 and γ_{21} is a latent interaction effect.

2.1.3 Specification of SASEM sub-models

The general SASEM framework allows for the specification of auto-regressive effects throughout the model. This flexibility provides a means of matching theoretical beliefs about the spatial process to the analysis. Two choices must be made: which level of the model to place the lag and which type of lag to specify.

The choice of where to specify lags will be dictated by theory and prior expectations about the route of spatial dependence. In a spatial context, if all of the observed y items are spatially dependent to some degree then the resulting latent construct is spatially dependent. In this situation, an endogenous lag can be specified at the structural level to account for the spatial dependence of η and estimate the spatial summary ρ_η. If spatial dependence is believed to be solely from omitted spatially dependent latent variables, a structural disturbance lag can be specified. This controls for spatially dependent omitted latent factors which bias estimates. Both situations can be present simultaneously at least theoretically in which both structural endogenous and disturbance lags can be specified (LeSage, 2014).

From a theoretical stance simultaneous endogenous and disturbance lags seem desirable. However, from a practical perspective some work has suggested the simultaneously specified endogenous and disturbance lag scenario is potentially problematic (LeSage, 2014). LeSage (2014) explains in a spatial regression context ($y = \rho Wy + \beta_1 x_1 + ... + \beta_k x_k + \varepsilon,\ \varepsilon = \phi W\varepsilon + \mu$) that the presence of simultaneous lags forces the relationship of predictors $x_1 ... x_k$ with y must be proportionately equal. When this does not hold true the resulting estimates of ρ_η are inaccurate. This assumption becomes burdensome when researchers hypothesize interaction effects or include categorical and continuous covariates in the same model as it is unlikely all variables will exhibit similiar effects.

Spatial dependence may plausibly be present in an individual subset of observed items. For example, only some of the items include aspects that depend on the geographic location (e.g., culture-relevant information or questions), whereas other items are more general. To accommodate this situation lags at the measurement level may be specified. Consider an example where a researcher measures 6 observed y items and three observed x items each for two latent exogenous factors ξ_1 and ξ_2. The observed items y_1, y_2, and y_3 are spatially dependent but the other y variables are not. The researcher believes that the y items are spatially dependent, and no spatially omitted variables explain the relationship between η and y_1 to y_3. In this situation, three spatial parameters are specified ρ_{y1}, ρ_{y2}, and ρ_{y3} at the measurement level. This results in a spatial estimate for each of the spatially dependent y variables. Additionally the associated latent variable (η in this example) will not be spatially dependent. Measurement level lags are not constrained to endogenous variables and dependent on theory can also be specified on the exogenous variables x. If a researcher believes that spatially dependent omitted observed variables exist which could plausibly correlated with η or ξ in its prediction of the associated observed indicator(s) y or x a disturbance lag is specified on the associated observed error term ε or δ. Specification of a spatial lag on a single observed item which is also regressed on an endogenous latent variable is synonymous with the specification of Oud & Folmer (2008).

Implementations of the SASEM framework with measurement level auto-regressive effects can provide great utility for network science applications. Consider the analysis discussed earlier by Valente et al. (1997) measuring contraceptive use opinions in a dependent network of South African women. Using the SASEM framework, the researchers could operationalize a multivariate representation of contraceptive use via multiple observed indicators as opposed to a single item. This represents the outcome variable η. It may be plausible to account for covariates which are not dependent on the network but relevant to the outcome. Specification of spatial lags at the measurement level can be used but omitted for non-dependent variables like genetic factors. This provides a means of controlling for and estimating auto-correlated effects for some variables but not others.

2.1.4 Comments on identification

As with traditional SEM modeling, identification must be considered to for parameter estimation. To guarantee model identification I recommend adhering to the conditions outlined in section 1.5.2. For example, it is necessary to constrain at least some of the factor loadings in the factor loading matrix for the exogenous variables to zero if more than a single exogenous variable is specified Bollen (1989). The only additional consideration which the SASEM imposes is in counting the number of total parameter estimates. Recall that to be identified a model must estimate less than $(k(k-1)/2)$ parameters, where k is the total number of observed parameters. Each specified spatial auto-regressive coefficient (ρ, and ϕ) counts as an additional parameter.

While theoretically all spatial auto-regressive coefficients can be specified simultaneously in the proposed framework, certain situations can occur where the model might be identified. For example, if a spatial coefficient ρ_η is included, not all respective coefficients $\rho_{y1}, \ldots, \rho_{yJ}$ might be identified simultaneously (and one needs to be fixed to zero). Similar identification rules as for the scaling of the factors are necessary, but need additional work.

2.1.5 Specification of variables' distributions and priors

The SASEM was designed to be estimated with Bayesian Monte-Carlo Markov Chain (MCMC) techniques. First the relevant distributions for error terms, exogenous variables and parameters (prior distributions) must be established. Distributions of error terms and latent exogenous variables are given by

$$\begin{aligned}
\varepsilon_j &\sim Normal(0, \sigma^2_{\varepsilon j}), \quad \text{for } j = 1 \ldots J \\
\delta_k &\sim Normal(0, \sigma^2_{\delta k}), \quad \text{for } k = 1 \ldots K \\
\zeta &\sim Normal(0, \sigma^2_{\zeta})
\end{aligned} \tag{2.4}$$

where $Normal(\mu, \sigma^2)$ is the normal distribution with mean μ and variance σ^2, and $MVNormal(\mu, \Sigma)$ is the multivariate normal distribution with mean vector μ and covariance matrix Σ. All error terms are assumed normally distributed with mean 0, and finite variances $\sigma^2_{\varepsilon j}$, $\sigma^2_{\delta k}$, and σ^2_{ζ}, respectively.

In the Bayesian framework prior distributions must be specified for each parameter. For the traditional SEM parameters I follow the recommendations by Lee et al. (2007) and Gelman et al. (2013) that are provided by

$$\begin{aligned}
\Phi &\sim \mathrm{LK}_j(R_0, \rho_0) \\
\sigma_{\varepsilon j} &\sim \mathrm{Cauchy}(0, \beta^*_{\varepsilon j})^+, \quad \text{for } j = 1 \ldots J \\
\sigma_{\delta k} &\sim \mathrm{Cauchy}(0, \beta^*_{\delta k})^+, \quad \text{for } k = 1 \ldots K \\
\sigma_{\zeta} &\sim \mathrm{Cauchy}(0, \beta^*_{\zeta})^+ \\
\lambda_{yj} &\sim \mathrm{Normal}(\mu_{\lambda yj}, \sigma^2_{\lambda yj}), \quad \text{for } j = 1 \ldots J \\
\lambda_{xkp} &\sim \mathrm{Normal}(\mu_{\lambda xkp}, \sigma^2_{\lambda xkp}), \quad \text{for } k = 1 \ldots K, p = 1 \ldots P \\
\gamma_{1p} &\sim \mathrm{Normal}(\mu_{\gamma 1p}, \sigma^2_{\gamma 1p}), \quad \text{for } p = 1 \ldots P \\
\gamma_{2m} &\sim \mathrm{Normal}(\mu_{\gamma 2m}, \sigma^2_{\gamma 2m}), \quad \text{for } m = 1 \ldots M \\
\alpha &\sim \mathrm{Normal}(\mu_{\alpha}, \sigma^2_{\alpha}) \\
\tau_{yj} &\sim \mathrm{Normal}(\mu_{\tau yj}, \sigma^2_{\tau yj}), \quad \text{for } j = 1 \ldots J \\
\tau_{xk} &\sim \mathrm{Normal}(\mu_{\tau xk}, \sigma^2_{\tau xk}), \quad \text{for } k = 1 \ldots K
\end{aligned} \tag{2.5}$$

where LK_j is a specific prior for correlation matrices and $\mathrm{Cauchy}(0, a)^+$ is the half Cauchy distribution. R_0, ρ_0, $\beta^*_{.}$, $\mu_{.}$, $\sigma^2_{.}$ are the associated hyperparameters for the parameters that are estimated. Depending on the actual model specification some of the parameters are fixed.

For all of the spatial parameters I recommend the advice of LeSage & Parent (2007) and Stakhovych et al. (2012) to use uniform distributions bound between 0 and 1 ($\rho,\ \phi \sim U[-1,1]$) in conjunction with row normalized W.

$$\begin{aligned}
\rho_{yj} &\sim \text{Uniform}(0,1), \quad \text{for } j = 1 \ldots J \\
\rho_{xk} &\sim \text{Uniform}(0,1), \quad \text{for } k = 1 \ldots K \\
\rho_{\eta} &\sim \text{Uniform}(0,1) \\
\phi_{yj} &\sim \text{Uniform}(0,1), \quad \text{for } j = 1 \ldots J \\
\phi_{xk} &\sim \text{Uniform}(0,1), \quad \text{for } k = 1 \ldots K \\
\phi_{\eta} &\sim \text{Uniform}(0,1)
\end{aligned} \tag{2.6}$$

Researchers who have strong prior information suggesting the spatial effect is positive may choose to specify a lower bound of 0.

2.1.6 Assumptions

Assumptions of the Bayesian SASEM include those typically associated with linear models and SEM, as well as additional spatial assumptions. Conditionally independent observations are assumed as is the case in spatial regression (LeSage, 1999). Cases are assumed independent after accounting for their spatial dependence. Residuals are assumed homoscedastic and normally distributed with a known variance. In other words, the SACSEM model assumes conditionally independent and identically distributed error terms (δ, ε, ζ). As mentioned earlier, the independence of the observed error terms ε and δ can be relaxed on a theoretical basis by estimating the covariance between item-specific disturbances. The *No island* assumption states there is no row in the W matrix that sums to zero. In other words, no case is considered outside of the geographic region (island) or network. The *Isotropy* assumption states that the summary of the spatial effect(s) are expressed as an average across the cases. This means that the spatial effects ρ^* or λ_s summarize the spatial process for the entire set of cases as an average process. For example, if a researcher has evidence to suggest that the spatial effect is strong in a subset of cases or branch of a network, but weaker in the rest, the resulting spatial estimate will poorly reflect the phenomena.

2.1.7 SASEM summary

The general Bayesian SASEM model provides a set of models for estimating SEMs with spatially dependent data. These models allow for researchers to estimate latent variables from sets of observed indicators while simultaneously providing a means to control for and estimate spatial effects in the presence of structural level interaction terms. In a network context the SASEM framework has strong empirically relevant applications. Modeling the complex intricacies of networks of agents lends itself very well to the psychological tradition of measuring constructs as latent variables. Instead of making predictions on and with univariate variables, the SASEM framework allows network scientists to predict complex unobserved features. This is a major advantage over potential dimension reduction alternatives like sum-scores (DiStefano et al., 2009; Bollen, 1989).

2.2 Interpretation

Interpretation of the SASEM model parameters differs between that of traditional linear models and SEMs. The presence of an *endogenous spatial lag* introduces complexity to parameter estimate interpretation. This is due to the aforementioned *spillover effects*. Interpretation of *spillover effects* are the primary reason for the popularity of the spatial models in network studies (Valente et al., 1997).

In traditional linear models each predictor variable has a single slope describing its relationship to the outcome, the slope does not vary between cases. The predicted value of case i is simply $\alpha + \gamma\xi_i$.

The addition of an endogenous lag at the structural or measurement level induces variation in the interpretation of the relationships between predictors and outcomes. When the W matrix is a normalized contiguity matrix the number of neighbors to case i will effect the predicted value.

When W is a scaled inverse distance matrix the predicted score of case i will vary as a function of how near it is to other cases.

To interpret slope and spatial parameter estimates we must summarize the n by n cross-partial derivatives matrix $\partial_y/\partial_x^{r\prime}$ associated with the analysis (LeSage & Pace, 2009). The cross-partial derivatives matrix is computed by:

$$\partial_y/\partial_x^{r\prime} = (I_n - \rho W)^{-1} I_n \beta_r \tag{2.7}$$

where r is the number of predictor variables. This matrix summarizes the spillover effect in which the predictor variables value for case i effect the dependent variable value for case $j \neq i$. The effect of the spatial lag can be expressed as a near infinite process $I_n + \rho W + \rho^2 W^2 + \rho^3 W^3 + \ldots$ through higher order relationships. W^2 is the second order set of neighbors, or neighbors of my neighbors. Because $W^{>1}$ has non zero diagonals (by definition cases are neighbors to their neighbors), a feedback loop is present (LeSage & Pace, 2014b).

Direct interpretation of $\partial_y/\partial_x^{r\prime}$ is possible, but depending on n can be potentially burdensome. The mean of row i provides the expected change in the outcome for a one unit increase in the rth predictor of all neighbors to i when controlling for all other covariates. Interpreting each row of the $\partial_y/\partial_x^{r\prime}$ matrix will result in $n * r$ effects to denote. To accommodate this researchers commonly summarize the matrix by computing three summaries of *spillover effects*: Indirect spillover, direct spillover, and total spillover.

Indirect spillover is computed using the off diagonal of $\partial_y/\partial_x^{r\prime}$. The mean of the off-diagonal provides the mean change all other regions have on case i for a one unit increase in the predictor r in all other regions while all other covariates are constrained to 0 (Golgher & Voss, 2016). The *direct spillover* of the rth variable is the expected mean change across all regions for the outcome variable in a particular region due to an increase of one unit in the rth variable while controlling for all other covariates. The *direct spillover* is calculated via the mean of the diagonal elements of $\partial_y/\partial_x^{r\prime}$. Finally the *total spillover* is the sum of the *direct* and *indirect spillover* of predictor r. The *total spillover* is the expected change in the outcome of case i for a one unit increase in predictor r

in all cases, while controlling for the effect of all other covariates.

The strength of both direct and indirect spillover will be controlled by the overall estimate of the spatial effect ρ as well as the rth variables slope. When ρ is near zero, *indirect* and *direct spillover* will be smaller, and larger as ρ is larger. The same can be said for β, as β increases the spillover will increase. If ρ is 0 all elements of $\partial_y/\partial_x^{r\prime}$ are the same, as is the case in traditional regression.

Under the general SASEM framework, interpretation of *spillover effects* will differ dependent on where auto-regressive effects are specified. Only endogenous lags will result in the necessity to calculate spillover effects. Disturbance lags result in an error sub-model which does not produce reciprocal effects of predictors in the model. When an endogenous lag is specified at the structural level *spillover effects* must be calculated regarding the slopes Γ of the exogenous predictors ξ. If lags of the endogenous observed variables y are made at the measurement level, impacts must be calculated to directly interpret the associated factor loading estimates.

2.3 Research Questions

Some characteristics must be explored to understand the boundaries of SASEM performance. [1] First, I frame the need for directly modeling spatial effects by exploring the bias in parameter estimates induced by ignoring spatial dependence with a traditional SEM model.

Previous models have not estimated latent interaction effects in the presence of spatial effects, which brings the accuracy of the estimates into question in situations where spatial dependencies are present in the data. Thus, I ask does the SASEM accurately and efficiently recover spatial and interaction parameters when they are simultaneously present?

Prior work has shown that the degree of connectedness in W does not bias parameter estimates in regression models at reasonable sample sizes, however, it was found that bias is induced at lower sample sizes (Anselin & Florax, 1995). The SASEM is highly parameterized and thus may exhibit

[1] When I use the term *performance* I refer to the general ability of a model to make accurate estimates. The elements of *performance* will be quantified and explained in more detail in a later section.

sensitivity to W specification. Therefor, I ask does changing the number of average connections per case in W effect the performance of SASEM?

Finally, it is reasonable to believe that some constructs are spatially dependent at the latent level while others may only have spatial dependence within a subset of observed items. The latent construct produced from all spatially dependent indicators would itself be spatially dependent. However, in some circumstances there may be a subset of spatially dependent observed indicators as well as some which are not. However, theory and prior expectations may not provide a concrete expectation as to which observed indicators may be spatially dependent. Therefore it is important to consider potential model misspecification. How is model performance effected when data is generated from a population model with structural level spatial effects, but analyzed with a measurement level spatial effect? The inverse situation is also relevant; How is model performance effected when data is generated from a population model with a measurement level spatial effect? The following list provides a summary of the research questions.

1. Does ignoring spatial dependence present in the structural model affect estimates in the measurement or structural model? To what degree? Does this effect differ when the spatial dependence is within the measurement model or the population model?

2. Can we accurately recover parameter estimates of structural level interaction effects in the presence of spatial effects?

3. Does varying the number of connections (neighbors) within a contiguity based W result in systematic changes to model accuracy or efficiency?

4. Do the spatial SEM models accurately and efficiently recover the linear estimates of β, γ, and λ, as well as spatial estimates ϕ and ρ when the data generating model provides spatial effects in the measurement model and the model is specified for estimation of a structural level spatial effect? What about the inverse scenario?

A secondary consideration for each research question is how model performance changes as a function of other manipulations of interest, like sample size and magnitude of the spatial effect.

These manipulations are not unique research questions themselves, but provide a more complete picture of model performance under each research question. The next section defines how I assess model performance, then provides the methods and results of the Monte-Carlo investigations of the research questions.

Chapter 3

Monte-Carlo Study

In this chapter, I describe the methods and results of four Monte-Carlo studies designed to investigate the aforementioned research questions regarding the general SASEM framework. The aim of study one is to highlight the consequences of ignoring spatially dependent effects in SEM. Study two investigates the accuracy and efficiency of parameter estimates in the presence of latent spatial and interaction effects. The aim of study three is to explore the effect of variations in the average connectedness of W on parameter estimates. The fourth study explores the impact of model misspecification. Specifically, I investigate how the accuracy and efficiency of parameter estimates vary as a function of spatial effect model misspecification (e.g., analyzing a spatial effect in the measurement model that should have been specified in the structural model).

3.1 Data Generation for All Studies

The general structural model for data generation was specified as

$$\eta = \rho_\eta (I_n - \rho_\eta W)^{-1}(0.3\xi_1 + 0.3\xi_2 + 0.15\xi_1 \cdot \xi_2 + \zeta) + 0.3\xi_1 + 0.3\xi_2 + 0.15\xi_1 \cdot \xi_2$$
$$+ \zeta + \phi_\zeta (I_n - \phi_\zeta W)^{-1}\zeta$$

with standard normal distributed uncorrelated latent factors ξ_1, ξ_2 (i.e., with means 0 and variances 1). The intercept α was set to 0. The linear effects γ_{11}, γ_{12} were set to 0.3 for both exogenous factors, and their interaction effect γ_{21} was set to 0.15. These values were chosen in line with typical effect sizes in psychology and past literature which establishes these values as a realistic linear and interaction effects (Chaplin, 1991; Kelava & Nagengast, 2012). The structural spatial lags (ϕ_ζ and ρ_η) are present in some data generating conditions and constrained to 0 in others.

The measurement model for the data generation was given by

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0 \\ \rho_{y2}(I_n - \rho_{y2}W)^{-1}(1 \cdot \eta + \varepsilon_2) \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \eta + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix} \tag{3.1}$$

and

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} + \begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{pmatrix} \tag{3.2}$$

Without loss of generality, all intercepts were set to zero. The spatial lag in the measurement model was only included for y_2 (in some of the models). The residuals ε_j and δ_k were mutually uncorrelated and normally distributed with mean 0 and variance .25. This implies reliabilities of 0.80 for all observed items. This reliability coincides with commonly utilized values in SEM Monte-Carlo work and with typically observed behavioral science reliabilities (Kelava & Nagengast, 2012; Brandt et al., 2019).

Four different SASEM population sub-models were derived for the studies by constraining some of the coefficients.

D1: Model without spatial effects; i.e., $\rho_{y2} = \rho_\eta = \phi_\zeta = 0$.

D2: Model with endogenous lag in the measurement model; i.e., $\rho_{y2} \neq 0; \rho_\eta = \phi_\zeta = 0$.

D3: Model with endogenous lag in the structural model; i.e., $\rho_\eta \neq 0; \rho_{y2} = \phi_\zeta = 0$.

D4: Model with endogenous lag and disturbance lag in the structural model; i.e., $\rho_\eta \neq 0; \phi_\zeta \neq 0; \rho_{y2} = 0$.

3.1.1 Data generating conditions for all studies

Some population level parameters are varied in all studies to provide a more detailed understanding of the research questions. The spatial parameters (ρ_η, ρ_{y2}, and ϕ_ζ) are simulated as 0.3 and 0.6 to coincide with small and medium effects for those parameters that are not zero in population models D2 to D4 (Cohen, 1988). Specifications of W are simulated as either an inverse distance matrix W_d^* or contiguity matrix W_c^*. Both representations of W are used to accommodate spatial econometric uses, which tend to use W_c^* and network auto-correlation techniques, which rely on W_d^*. W reflects a two dimensional space and the size of W is directly linked to sample size. Three sample size conditions are utilized: $n = 49$, $n = 196$, and $n = 400$. Sample sizes are chosen as squared values to coincide with a square spatial representation. Figure 3.1 provides a visual representation of the space in which each simulated case is generated. A consistent square representation was used to control for potentially confounding effects of irregularly shaped geographic regions in spatial analysis (Duczmal et al., 2006) and to avoid the near infinite spatial orientation specifications.

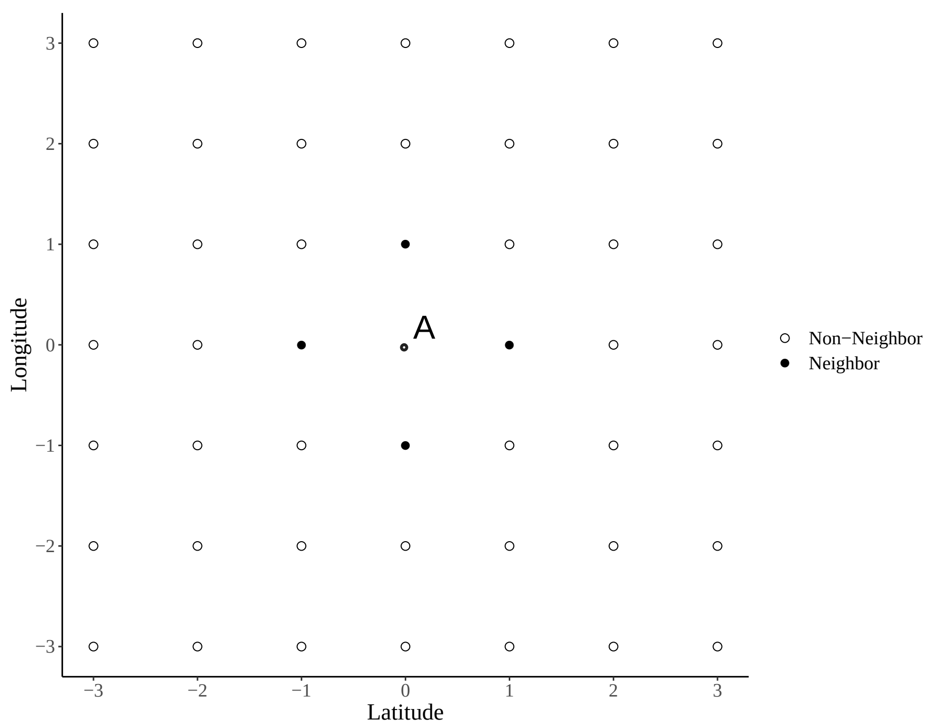

Figure 3.1: Visual representation of neighbors to case A under n = 49 condition

In the contiguity representation of W, cases are established as neighbors if they share a horizontal or vertical boundary with another case. In the W_d^* representation, euclidean distance $d_{p,q} = \sqrt{\sum_{i=1}^{n}(q_i - p_i)^2}$ is calculated between each case (q and p) then inverted ($\frac{1}{d_{p,q}}$) to reflect larger numbers for closer neighbors. In all data generating models, W is row normalized to constrain auto-regressive estimates between -1 and 1 (see section 1.4).

All simulation conditions are fully crossed and further simulation conditions unique to each study are discussed within each section. Under each simulation condition 500 iterations are conducted (Muthén & Asparouhov, 2012; Oberle, 2015). R version 3.5.4 (R Core Team, 2019) was used for data generation. The R package mvtnorm version 1.0-11 (Genz et al., 2019) was used for all multivariate data generation.

3.2 Analysis Models for All Studies

The general nonlinear SASEM to analyze the data was specified as

$$\eta = \rho_\eta W \eta + \gamma_1 \xi_1 + \gamma_2 \xi_2 + \gamma_3 \xi_1 \cdot \xi_2 + \phi_\zeta W \zeta + \mu_\zeta \tag{3.3}$$

The measurement model for the data analysis was given by

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0 \\ \rho_{y2} W y_2 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ \lambda_{y2} \\ \lambda_{y3} \end{pmatrix} \cdot \eta + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix} \tag{3.4}$$

and

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \lambda_{x1} & 0 \\ \lambda_{x2} & 0 \\ 0 & 1 \\ 0 & \lambda_{x3} \\ 0 & \lambda_{x4} \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} + \begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{pmatrix} \tag{3.5}$$

Four sub-models were used in the studies by constraining some of the parameters to zero. The following constraints were set:

A1: Model without spatial effects; i.e., $\rho_{y2} = \rho_\eta = \phi_\zeta = 0$.

A2: Model with endogenous lag in the measurement model; i.e., $\rho_\eta = \phi_\zeta = 0$.

A3: Model with endogenous lag in the structural model; i.e., $\rho_{y2} = \phi_\zeta = 0$.

A4: Model with endogenous lag and disturbance lag in the structural model; i.e., $\rho_{y2} = 0$.

In all four Monte-Carlo studies, the SASEM analysis sub-models are used that correspond with the data generating models regarding the number of observed and latent items, their associated loading structure, and structural interactions. In all analysis models, W is row normalized to bound spatial estimates between -1 and 1, and to coincide with data generating W.

Prior specifications for the analysis models are given by (LeSage & Parent, 2007; Gelman et al., 2013; Lee et al., 2007).

$$
\begin{aligned}
\Phi &\sim \text{LK}_j(I_2, 2) \\
\sigma_{\varepsilon j} &\sim \text{Cauchy}(0, 2.5)^+, \quad \text{for } j = 1 \dots 3 \\
\sigma_{\delta k} &\sim \text{Cauchy}(0, 2.5)^+, \quad \text{for } k = 1 \dots 6 \\
\sigma_{\zeta} &\sim \text{Cauchy}(0, 2.5)^+ \\
\lambda_{yj} &\sim \text{Normal}(0, 1), \quad \text{for } j = 1 \dots 2 \\
\lambda_{xk} &\sim \text{Normal}(0, 1), \quad \text{for } k = 1 \dots 4 \\
\gamma_p &\sim \text{Normal}(0, 1), \quad \text{for } p = 1 \dots 3 \\
\alpha &\sim \text{Normal}(0, 1) \\
\rho_{y2} &\sim \text{Uniform}(0, 1) \\
\rho_{\eta} &\sim \text{Uniform}(0, 1) \\
\phi_{\eta} &\sim \text{Uniform}(0, 1)
\end{aligned}
\tag{3.6}
$$

R version 3.5.4 (R Core Team, 2019), STAN version 2.18.0 (Stan Development Team, 2018), and Rstan version 2.19.2 (Stan Development Team, 2019) were used to conduct all analysis. All analysis models across each study are designated to have 4 independent chains and 4,000 iterations, half of which are designated burn in.

3.3 Outcomes

Performance of the models is operationalized through convergence rates, parameter bias, and coverage. Convergence is considered to be met when the $\hat{R}$ estimate of a parameter is less than 1.1 and Effective Sample Size (ESS) is greater than $5m$, where m is the number of MCMC chains specified (Gelman et al., 2013; Vehtari et al., 2019). Bias is presented as the percent deviation of the population value from the mean posterior estimate and is calculated as $100\left(\frac{\text{Mean Posterior Estimate} - \text{Population Value}}{\text{Population Value}}\right)$. Population parameters that are 0 cannot be expressed as percent deviations; instead, they are presented as absolute bias: Mean Posterior Estimate – Population Value. Coverage is calculated as the proportion of simulation iterations that fall within the central 95% density of a parameter's posterior estimate.

3.4 Study 1

To provide insight into the consequences of ignoring spatially dependent data in SEM, Study 1 investigates the impact of omitting spatial effects in spatially dependent data. In Study 1, all conditions are fully crossed (i.e., sample size, spatial parameters, W representation, and data generating models). For data generation, all models D1-D4 are used.

For data analysis, model A1 is used. The model is properly specified regarding all non-spatial effects. Prior distributions outlined in 4.2 are used for all analysis models. No spatial parameters are estimated in this model to investigate the consequences of omitting these parameters.

3.4.1 Expectations

I predict ignoring spatial dependence will result in biased parameter estimates depending on the nature of the spatial omission. Specifically, when population level spatial dependence exists at the structural ($\rho_\eta \neq 0$) level, γ_1, γ_2, and γ_3 estimates will be biased. That is, unexplained spatial dependence in η will likely erroneously be explained by γ_1, γ_2, and γ_3. When population level $\phi_\zeta \neq 0$ the structural level disturbance ζ is not conditionally independent, this will bias estimates of γ_1, γ_2, and γ_3. When spatial dependence is ignored at the measurement level, factor loadings will likely be biased in addition to γ_1, γ_2, and γ_3. Finally, I anticipate sample size and magnitude of population level spatial dependence to moderate the bias effect. Specifically, bias will increase as sample size decreases and/or spatial dependence increases.

3.4.2 Results

The unabridged results for all parameters are provided in the Appendix. Results for population model D2 are provided by Table A.1, D3 in Table A.2, and D4 in Table A.3 and Table A.4. Results for the D1 population model are provided in each of the aforementioned tables for convenient comparisons.

3.4.2.1 Convergence

Across data generating conditions and parameters, convergence rates for the non-spatial SEM were greater than 95% with one exception. Under the simultaneous structural lag condition when $\phi_\zeta > 0$, convergence rates are lower, as population level ϕ_ζ increases convergence rates of ϕ_ζ decrease. With the exception of ϕ_ζ in the D4 model, all parameters exhibit convergence rates $> 95\%$. Convergence rates slightly decline with lower sample sizes.

3.4.2.2 Bias

Across data generating models bias generally increases as population level spatial parameters (ρ_{y2}, ρ_η, and ϕ_ζ) increase. Bias decreases as sample size increases.

D2 Fig. 3.2 depicts the bias for the parameters of interest under the D2 population model.[1] Estimates are unbiased in the presence of un-modeled spatial auto-correlation in the measurement term of item y_2. Increasing population level dependence solely impacts the bias of $\sigma_{\varepsilon 3}$. Under W_C^*, n = 49, and population level $\rho_{y2} = 0$ conditions the observed bias of $\sigma_{\varepsilon 2} = 0.35\%$. Increasing population $\rho_{y2} = 0.6$ increases the observed bias of $\sigma_{\varepsilon 2}$ to 4.35%. In higher sample size conditions, $\sigma_{\varepsilon 2}$ maintains increased bias. Under n = 400 and population level $\rho_{y2} = 0$, the observed bias of $\sigma_{\varepsilon 2} = -0.47\%$ increasing $\rho_{y2} = 0.6$ observed bias of $\sigma_{\varepsilon 2} = 4.05\%$. W specifications do not exhibit a systematic relationship with bias. Sample size dictates parameter bias more than any other conditions, with higher sample sizes resulting in less bias.

[1]Parameters of interest were selected to reduce the burden of the size of the results. Parameters chosen are typically the aim of analysis in applied work, as well as parameters that were affected by the simulation conditions.

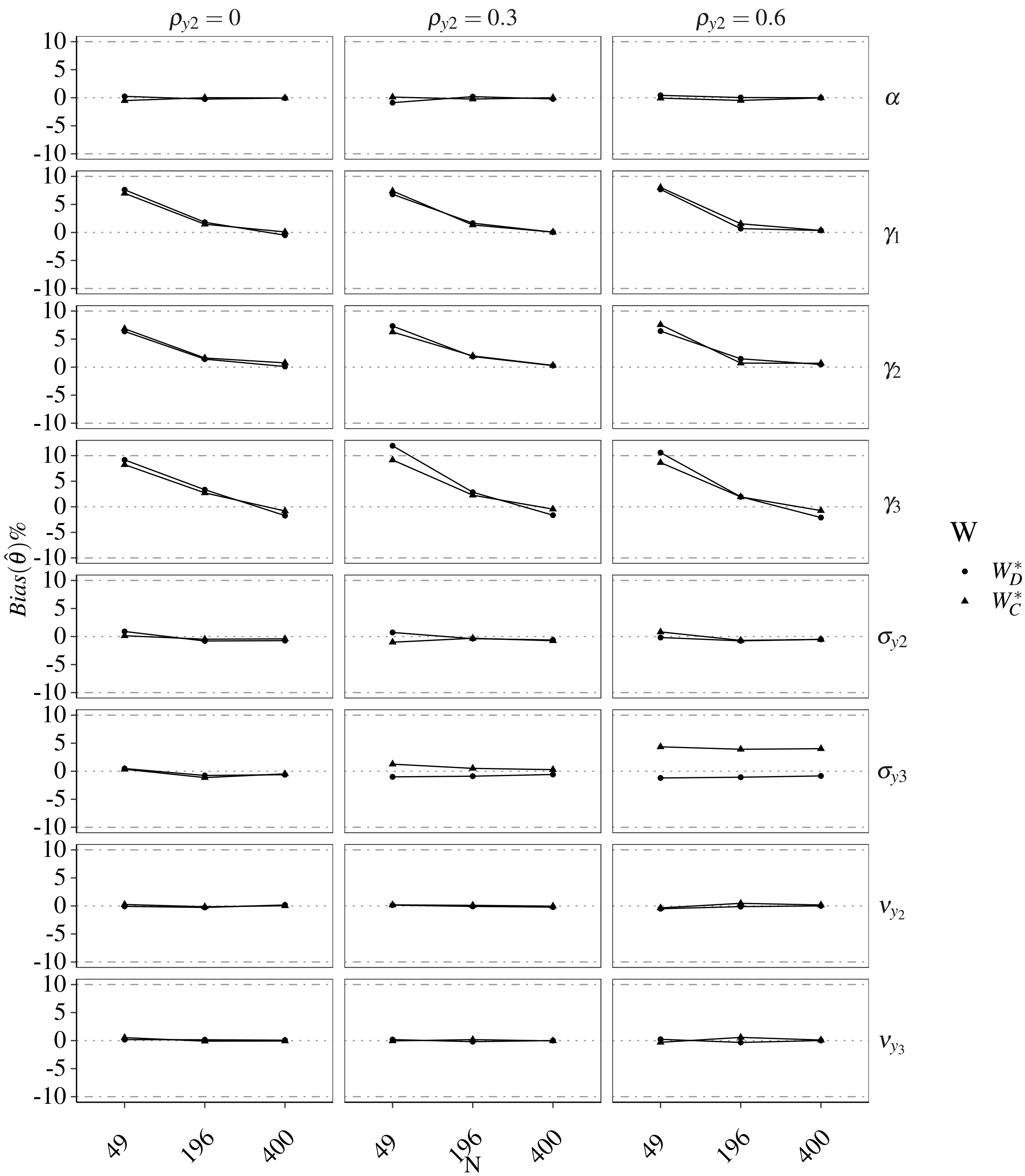

Figure 3.2: Line plot of $Bias(\hat{\theta})\%$ under the measurement spatial lag population model D2, analyzed with non-spatial SEM A1.

D3 Fig. 3.3 provides a summary of bias for the parameters of interest under the D3 population model. Parameter estimates under population model D3 are robust to the ignored spatial dependence. Only $\sigma_{\varepsilon 2}$ exhibits sensitivity to the misspecification. In the W_C^*, n = 49, and population $\rho_\eta = 0$ the bias of $\sigma_{\varepsilon 2} = 0.14\%$, increasing population ρ_η to 0.6 the observed bias of $\sigma_{\varepsilon 2} = 2.45\%$. Sample size impacts the bias of the structural slopes. In the W_C^*, n = 49, and population $\rho_\eta = 0$ conditions, the bias of $\gamma_1 = 6.98\%$, $\gamma_2 = 6.84\%$, and $\gamma_3 = 8.24\%$. Increasing the sample size to n = 400, the observed bias of $\gamma_1 = 1.08\%$, $\gamma_2 = -1.17\%$, and $\gamma_3 = 0.59\%$. In the W_C^*, n = 400, and population $\rho_\eta = 0$ the bias of $\gamma_1 = 0.09\%$, $\gamma_2 = 0.76\%$, and $\gamma_3 = -0.82\%$.

D4 Fig. 3.4 provides bias for the parameters of interest under the D4 population model. Structural slope estimates γ_1, γ_2 and γ_3 exhibit increased bias with increased un-modeled dependence. Under W_C^*, n = 49, and population ρ_η and $\phi_\zeta = 0.3$, bias of $\gamma_1 = 7.47\%$, $\gamma_2 = 8.00\%$, and $\gamma_3 = 8.65\%$. Increasing population level ρ_η to 0.6 increases the observed bias of $\gamma_1 = 14.22\%$, $\gamma_2 = 16.33\%$, and $\gamma_3 = 19.49\%$. Increasing ϕ_ζ does not impact this relationship. When ρ_η and $\phi_\zeta = 0.6$ the bias of $\gamma_1 = 15.22\%$, $\gamma_2 = 13.56\%$, and $\gamma_3 = 18.31\%$. Increasing sample size mitigates the relationship. When ρ_η and $\phi_\zeta = 0.6$ and n = 400, the observed bias $\gamma_1 = 7.78\%$, $\gamma_2 = 7.54\%$, and $\gamma_3 = 8.33\%$. W specification does not systematically affect parameter bias.

3.4.2.3 Coverage

Across data generating conditions and parameters, coverage is very high (> 95) when spatial parameters ρ_{y2}, ρ_η, and ϕ_ζ are zero. Data generating model and population level spatial effect magnitude produce the largest impact on coverage rates.

D2 The D2 population model exhibits consistently high coverage rates (coverage > 93%) across all parameters with the exception of $\sigma_{\varepsilon 3}$ and τ_{y3}. In the W_C^*, n = 49, and population $\rho_{y2} = 0$ conditions the coverage rate of $\sigma_{\varepsilon 3} \approx 95\%$, increasing ρ_{y2} to 0.6 decreases the coverage rate of $\sigma_{\varepsilon 3}$ to 92.10%. This relationship is maintained at higher sample sizes. Under n = 400, and population level $\rho_{y2} = 0$, the coverage rate of $\sigma_{\varepsilon 3} \approx 95\%$. Increasing ρ_{y2} to 0.6 decreases the coverage rate

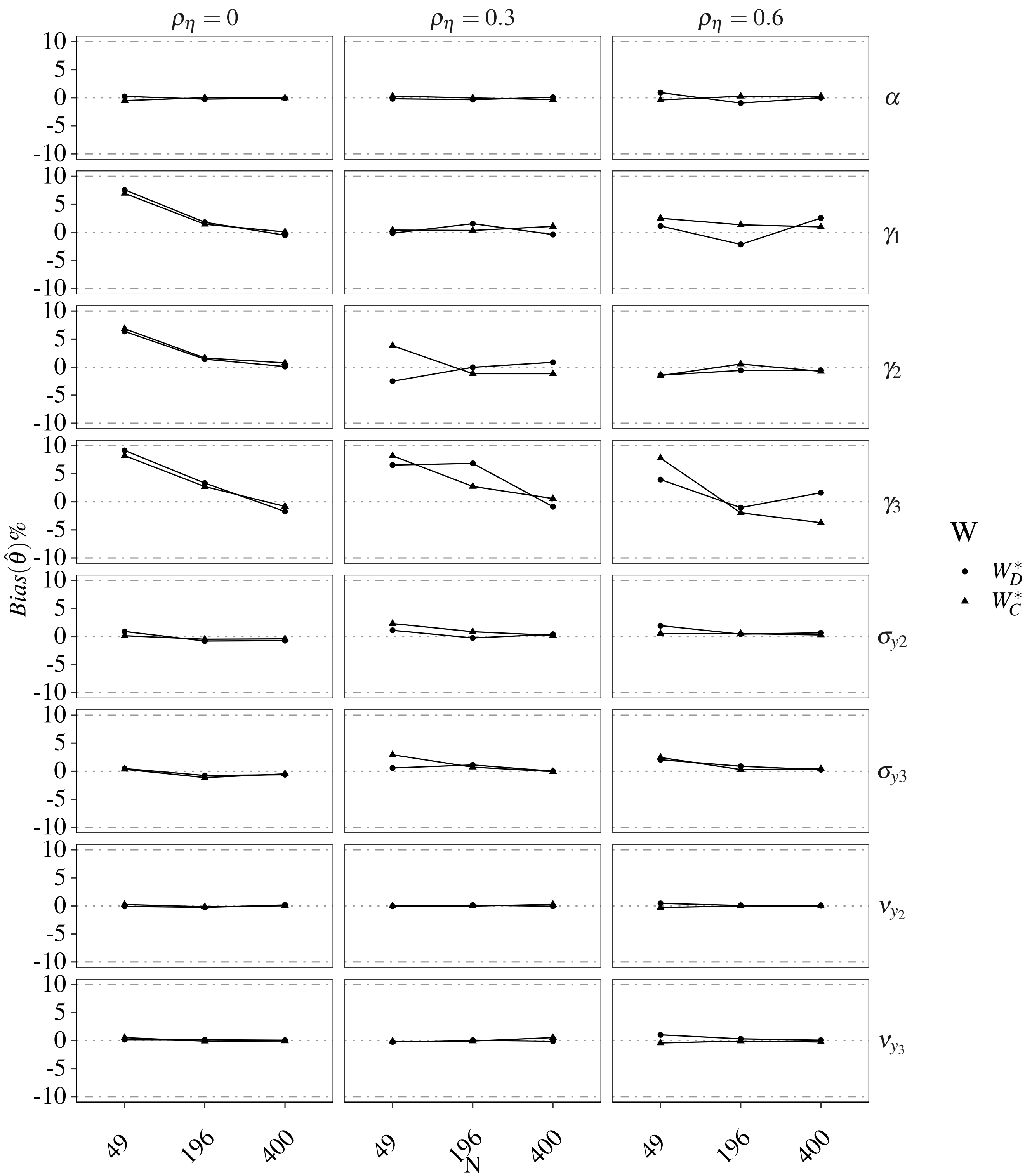

Figure 3.3: Line plot of $Bias(\hat{\theta})\%$ under the endogenous structural lag population D3 model analyzed with non-spatial SEM A1.

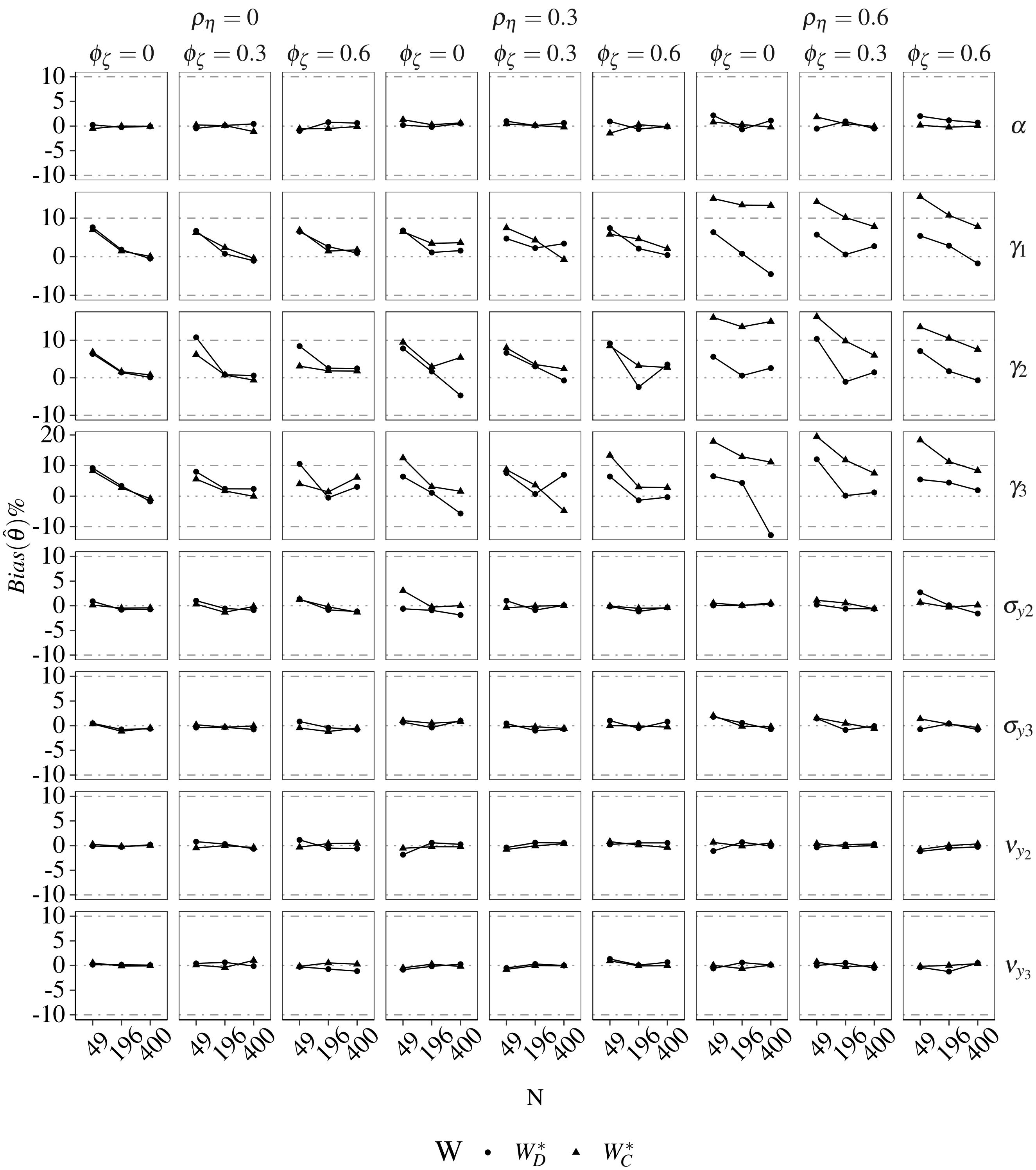

Figure 3.4: Line plot of $Bias(\hat{\theta})\%$ under the simultaneous structural spatial lags population model (D4) when $\phi_\zeta = 0.3$, analyzed with non-spatial SEM A1.

of $\sigma_{\varepsilon 3}$ to 79.04%. τ_{y3} exhibits decreased coverage rates from un-modeled dependence as well. In the W_C^*, n = 49, and population $\rho_{y2} = 0$ conditions, the coverage rate of $\tau_{y3} \approx 94\%$, and increasing ρ_{y2} to 0.6 decreases coverage rates of τ_{y3} to 86.77%. Under n = 400 and population level $\rho_{y2} = 0$, the coverage rate of $\tau_{y3} = 96.21\%$, and increasing ρ_{y2} to 0.6 decreases the coverage rate to 87.87%. The magnitude of the population level spatial effect of y_2 does not systematically affect the coverage rates of other parameters.

D3 Across all parameters, coverage rates for population model D3 > 93% with one exception. α exhibits the strongest decrease in coverage rates as a function of non zero population level ρ_η. Under W_C^*, n = 49, and population level $\rho_\eta = 0$, the coverage rate of α is 95.5%; when population level ρ_η is increased to 0.3, the coverage rate of α decreases to 91.07%, and when population $\rho_\eta = 0.6$ the coverage rate of $\alpha = 86.93\%$. Increasing sample size does not impact the coverage rates of other parameters, but it decreases the coverage rate of alpha. W specification does not systematically impact coverage rates. Under W_C^*, n = 400, and population level $\rho_\eta = 0.6$, the coverage rate of $\alpha = 83.69\%$. All other parameter coverage rates are unaffected by sample size and W specification conditions.

D4 The D4 population model also consistently exhibits coverage rates > 93% across all parameters. The exceptions to this are α, γ_1, γ_2, and γ_3. As the magnitude of the omitted dependence increases, coverage rates of the structural effects decrease. Under W_C^*, n = 49, and population level $\phi_\zeta =$ and $\rho_\eta = 0.3$, the coverage rate of $\alpha = 94.78\%$, $\gamma_1 = 96.27\%$, $\gamma_2 = 98.01\%$, and $\gamma_3 = 96.52\%$. Increasing ρ_η to 0.6 decreases coverage rates of α to 81.09%, $\gamma_1 = 92.79\%$, $\gamma_2 = 91.79\%$, and $\gamma_3 = 93.78\%$. Increasing ϕ_ζ to 0.6 does not impact the coverage rates. Increasing sample size does not impact coverage rates for slopes, but it decreases rates for α. Under W_C^*, n = 400, and $\phi_\zeta =$ and $\rho_\eta = 0.3$, the coverage rate of $\alpha = 75.76\%$. Varying W specification does not systematically impact coverage rates.

3.5 Study 2

The objective of Study 2 is to establish model performance of the SASEM to estimate latent interactions and spatial effects simultaneously. To investigate this research question, data is generated under a SASEM sub-model, then analyzed with a correctly specified SASEM analysis sub-model.

All three SASEM data generating sub-models D2 to D4 are used in Study 2. No additional conditions beyond those established in Section 3.1.1 are included.

All three SASEM analysis sub-models A2 to A4 are used for analysis. All models are correctly specified. Prior distributions outlined in Section 4.2 are used for all analysis models.

3.5.1 Expectations

I hypothesize that spatial and interaction effect parameter estimates in the SASEM sub-models will be unbiased when a single endogenous lag is present at either the structural or measurement levels. Specifically, I anticipate the simultaneous endogenous and disturbance lag analysis model (A4) will exhibit biased γ_1, γ_2, and γ_3 estimates. The work by LeSage (2014) establishes in a spatial regression context that simultaneous disturbance and endogenous lags constrict linear estimates of predictors to be the same in magnitude. In Study 2 this will never be true at the population level due to the magnitude of γ_3 being half of the magnitude of γ_1 and γ_2.

3.5.2 Results

Full results for Study 2 are provided in the Appendix. Population model A2 is provided by Table A.5, A3 is provided by Table A.6, and A4 provided by Tables A.7 and A.8.

3.5.2.1 Convergence

Across data generating conditions and parameters, convergence rates for all SASEM sub-models were larger than in Study 1 ($> 97\%$) with one exception. Under the simultaneous structural lag condition when $\phi_\zeta > 0$, convergence rates are lower; as ϕ_ζ increases, convergence rates of ϕ_ζ decrease. Convergence rates slightly decline with low sample sizes.

3.5.2.2 Bias

Measurement Level Spatial Lag: D2 and A2 Fig. 3.5 provides the bias by simulation condition for model A2. Model A2 failed to accurately recover the spatial auto-regressive coefficient ρ_{y2}. In the W_C^* condition, A2 consistently over estimated ρ_{y2} resulting in positively biased estimates. In the W_D^* specification, A2 consistently estimated $\rho_{y2} \approx 0.5$ regardless of population ρ_{y2} value. Structural slopes γ_1, γ_2, and γ_3 remain unbiased. Under W_C^*, n = 49, and population level $\rho_{y2} = 0$, the bias of $\gamma_1 = 7.71\%$, $\gamma_2 = 6.49\%$, and $\gamma_3 = 9.38\%$. When population level $\rho_{y2} = 0.6$, the bias of $\gamma_1 = 7.67\%$, $\gamma_2 = 6.67\%$, and $\gamma_3 = 10.85\%$. Increasing sample size decreases the bias observed in the interaction effect. When n = 400, under population level $\rho_\eta = 0$, the bias of of γ_1, γ_2, and $\gamma_3 < 0.50\%$. Increasing population level $\rho_{y2} = 0$ has little effect, with all three estimates observing bias $< 2\%$.

Endogenous Structural Spatial Lag: D3 and A3 The endogenous structural lag analysis model A3 exhibits low bias in both spatial and non-spatial parameter estimates, but with a caveat. Regarding ρ_η estimates, under W_C^*, n = 49, and population level $\rho_\eta = 0$, absolute bias = 19.99%; under $\rho_\eta = 0.3$ bias is 15.45%, and $\rho_\eta = 0.6$ bias is -3.21%. Increasing sample size reduces the bias in ρ_η. At n = 400 and population $\rho_\eta = 0$, bias = 6.82%, when $\rho_\eta = 0.3$ bias is 0.33%, and when $\rho_\eta = 0.6$ bias is -0.20%. Structural slope estimates remain unbiased. Under W_C^*, n = 49, and population $\rho_\eta = 0.3$, the bias of γ_1 = 1.70%, γ_2 = 0.21%, and γ_3 = -6.12%. In the $\rho_\eta = 0.6$ condition bias of γ_1 = 2.93%, γ_2 = -0.90%, and γ_3 = 4.05%. Increasing sample size decreases structural bias. Under W_C^*, n = 400, and population level $\rho_\eta = 0$, the bias of γ_1 = 0.59%, γ_2 = 1.09%, and γ_3 = 1.28%. Increasing ρ_η to 0.6 under n = 400, bias of γ_1 = 0.45%, γ_2 = -0.37%, and γ_3 = -2.87%.

Simultaneous Structural Lag Model: D4 and A4 Fig. 3.7 provides a summary of bias by simulation conditions for model A4. Spatial parameters of model A4 are biased. Under W_C^*, n =

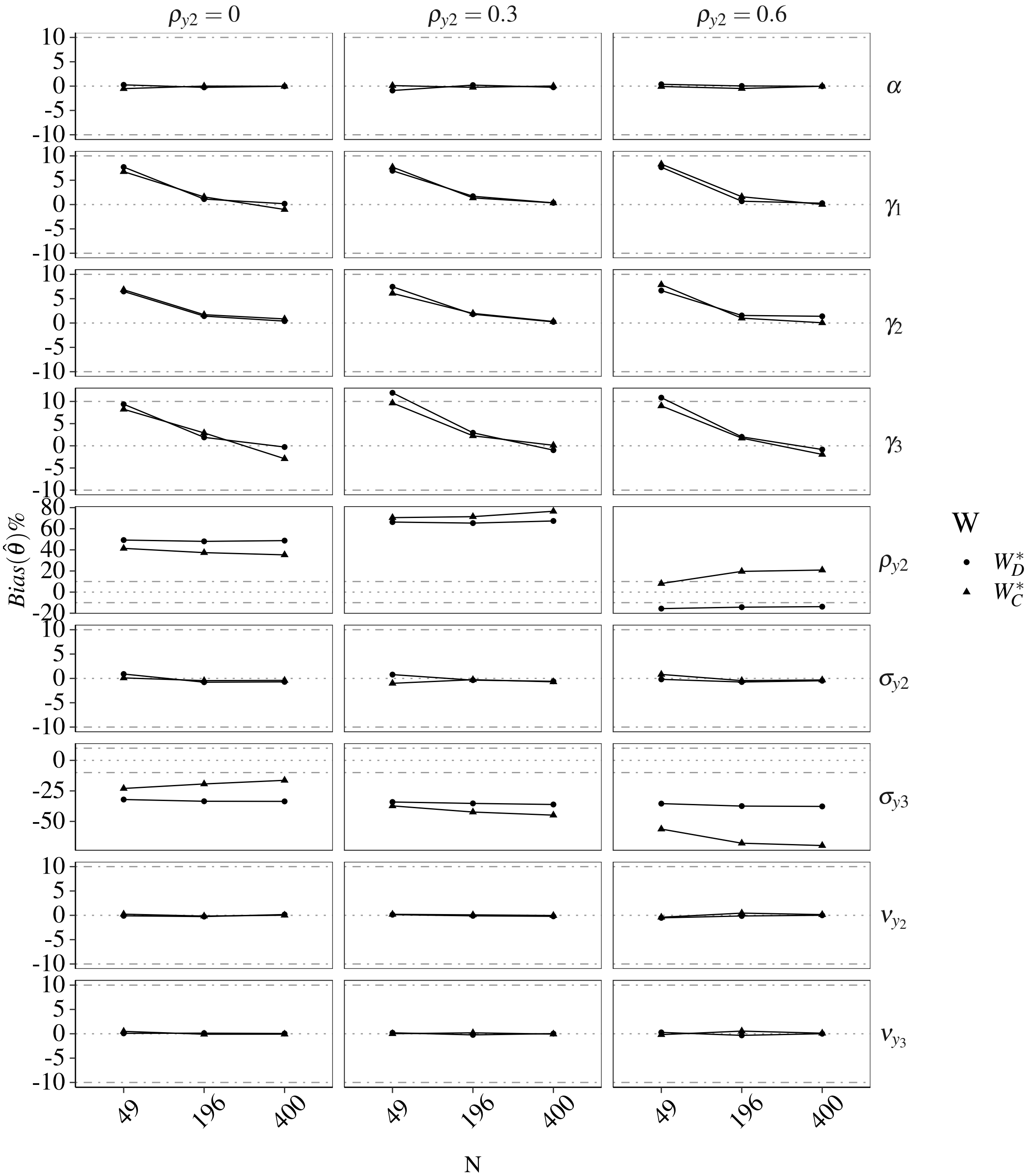

Figure 3.5: Line plot of $Bias(\hat{\theta})\%$ under the measurement spatial lag population and analysis models (A2, D2).

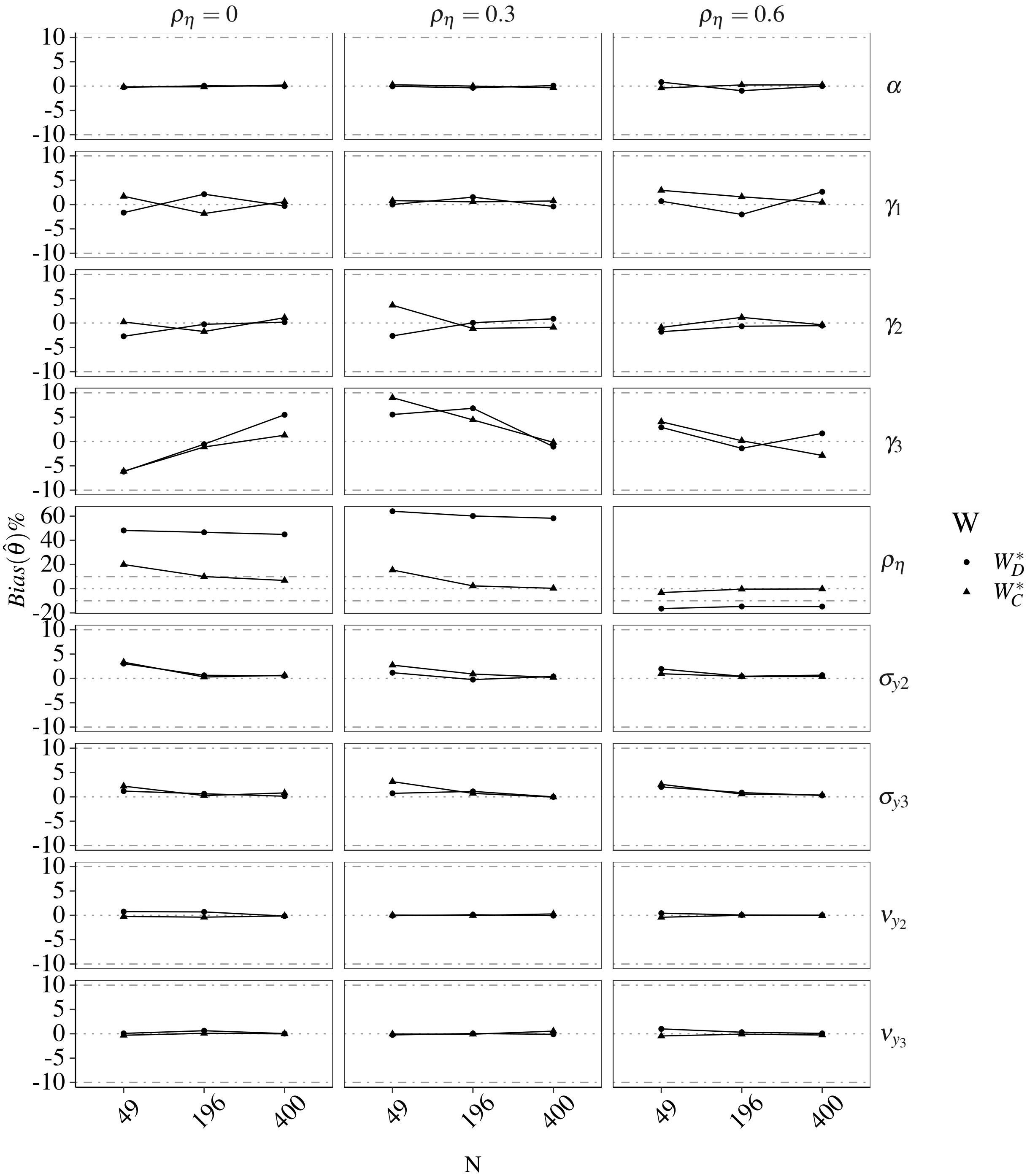

Figure 3.6: Line plot of $Bias(\hat{\theta})\%$ under the endogenous structural lag population and analysis models (A3, D3).

49, and $\rho_\eta = 0$ and $\phi_\zeta = 0.3$ the bias of $\rho_\eta \approx 17.5\%$, under $\rho_\eta = 0.3$ bias $\approx -20\%$, and when $\rho_\eta = 0.6$ bias is -30%. Increasing sample size diminishes the magnitude of the bias, under n = 400, $\rho_\eta = 0$, and $\phi_\zeta = 0.3$ absolute bias of $\rho_\eta = 14\%$, when $\rho_\eta = 0.3$ bias is -11.17%, and when $\rho_\eta = 0.6$ bias is -4.83%. ϕ_ζ estimates vary by sample size and W specification, but less from population level ϕ_ζ or ρ_η than anticipated. Under W_C^*, and n = 49 the estimate of $\phi_\zeta \approx 0.36\%$ (bias = 18.64%), when n = 196 $\phi_\zeta = 0.29$ (bias = 2.20%), and when n = 400 $\phi_\zeta = 0.29$ with bias = -11.90%.

Structural estimates remain unbiased. Under W_C^*, n = 49, ρ_η, and $\phi_\zeta = 0.3$, the bias of γ_1 = 5.22%, γ_2 = 4.73%, and γ_3 = 4.95%. Increasing ρ_η to 0.6 yields bias of γ_1 = 7.13%, γ_2 = 9.62%, and γ_3 = 11.36%. Under ρ_η and $\phi_\zeta = 0.6$ bias of γ_1 = 9.33%, γ_2 = 7.20%, and γ_3 = 11.01%. Increasing sample size decreases structural bias. Under W_C^*, n = 400, $\phi_\zeta = 0.3$, $\rho_\eta = 0$, the bias of γ_1 = -2.46%, γ_2 = 0.05%, and γ_3 = -5.17%. Increasing ρ_η to 0.6 yields bias of γ_1 = 0.76%, γ_2 = 0.30%, and γ_3 = 1.07%.

3.5.2.3 Coverage

Across data generating conditions and parameters, coverage is > 93 when spatial parameters ρ_{y2}, ρ_η, and ϕ_ζ are zero. Data generating model and population level spatial effect magnitude produce the largest impact on coverage rates. Across models and conditions as sample size increases, coverage slightly decreases.

D2 and A2 Coverage under model D2 is consistently high across all parameters (coverage > 93%) with 2 exceptions. Model D2 erroneously estimates ρ_{y2} to be ≈ 0.50 regardless of the population value. This results in coverage > 90% in the $\rho_{y2} = 0.3$ and 0.6 conditions, and coverage of 0% to 30% under the $\rho_{y2} = 0$ condition. Further, $\sigma_{\varepsilon 3}$ also exhibits decreased coverage rates. Coverage is unaffected when population level $\rho_{y2} = 0$; however, when $\rho_{y2} > 0$, coverage of the parameter is drastically decreased ($\approx$ 85% under n = 49, $W = W_D^*$). While $\sigma_{\varepsilon 3}$ is biased, the other endogenous residual variances ($\sigma_{\varepsilon 1}$ and $\sigma_{\varepsilon 2}$), factor loadings (λ_{y1}, λ_{y2}, and λ_{y3}) and intercepts (τ_{y1},

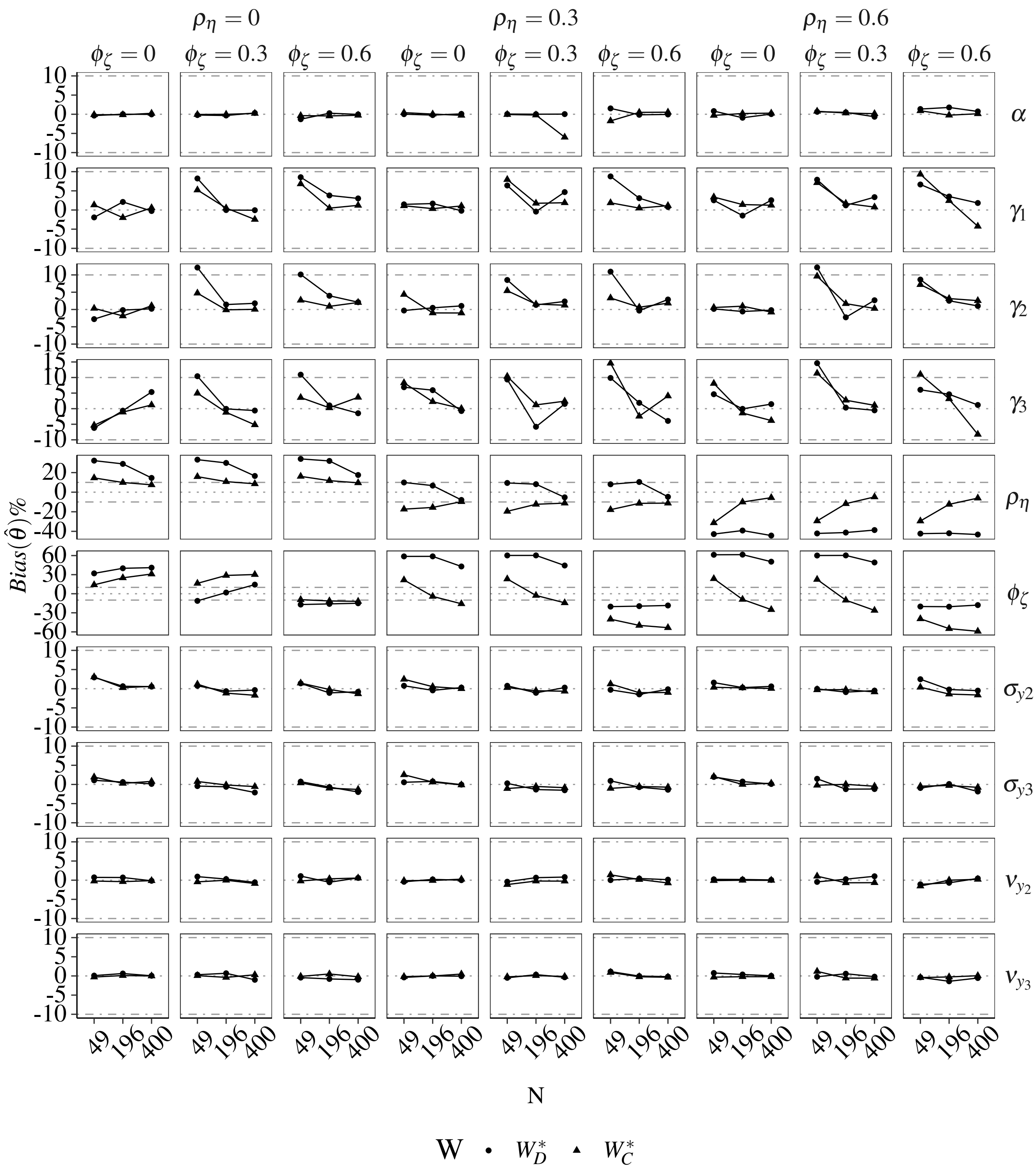

Figure 3.7: Line plot of $Bias(\hat{\theta})\%$ under the simultaneous structural spatial lag population model analyzed with the simultaneous structural lag model (A4, D4).

τ_{y2}, and τ_{y3}) have acceptable coverage rates($\approx$ 95%).

D3 and A3 The D3 model exhibits coverage rates > 92% across all parameters with one exception. Under population $\rho_\eta = 0$, coverage of ρ_η is 0% for all conditions. However, under $\rho_\eta = 0.3$ or 0.6, coverage rates are comparable with all other parameters in the model ($\approx$ 95%). The latent interaction effect γ_3 coverage rates are favourable in all conditions, the lowest of which (93.18%) is observed under W_D^*, n = 196, and $\rho_\eta = 0.3$ conditions.

D4 and A4 The simultaneous structural lag model (D4) across all parameters exhibits the lowest overall coverage rates of the spatial SEM models. The spatial parameters ρ_η and ϕ_ζ exhibit variation in coverage rates ranging from 0% to 100%. When population level $\rho_\eta = 0$ and $\phi_\zeta = 0.3$, estimate coverage rates are 22% and 100% respectively. However, when $\rho_\eta > 0$ coverage is acceptable, ranging from 91% to 100%. The latent interaction term γ_3 has comparable coverage to the other models (A2 and A3), with minimum coverage of 93.35% when population $\phi_\zeta = 0.3$, and 93.48% under population $\phi_\zeta = 0.6$. The magnitude of ϕ_ζ does not appear to systematically affect coverage rates.

3.6 Study 3

The goal of Study 3 is to explore model performance as a function of the number of connections in W_C^*.

3.6.1 Data generation and additional conditions

All three SASEM data generating sub-models D2 to D4 outlined in Section 3.1 are used. In addition to the conditions outlined in Section 3.1.1, Study 3 includes alternative specifications of W_C^*. Three connectedness conditions are established as high, middle, and low. The low condition corresponds with the econometric practice of establishing *Rook contiguity*, the middle condition

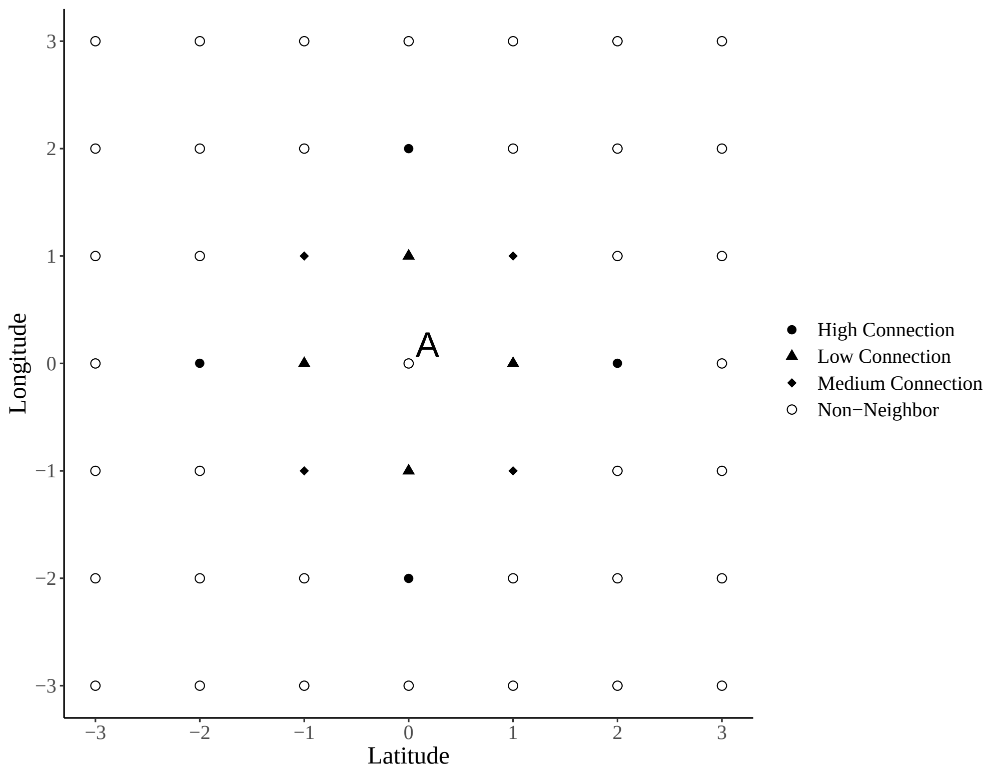

Figure 3.8: Visual representation of connection conditions under n = 49

establishes *Queen contiguity*, and the high condition extends *Queen contiguity* to include second order neighbors (neighbors of my neighbor).

W matrices were constructed by specifying the latitude and longitude of the cases, then calculating the euclidean distances cases are apart. A cutoff threshold is then applied to construct the final W_C^* matrices. Fig. 3.8 provides a visual representation of the connection conditions under $n = 49$. Triangles represent the neighbors to case A in the low condition, diamonds are added as neighbors in the medium condition, and all black shapes are neighbors to A in the high condition.

Table 3.1 provides a summary of the average number of neighbors in each specification of W_C^* by sample size and connection conditions. The number of connections in the W_D^* condition are already fully saturated. Alternative conditions of W_D^* are not included.

Table 3.1: Average number of connections by sample size and connection conditions

	Sample Size		
Connection Condition	49	196	400
Low	3.43	3.71	3.80
Medium	6.37	7.16	7.41
High	9.22	10.59	11.01
W_D^{*}[1]	17.22	37.91	55.71

[1] The W_D^* condition does not have neighbors. Every case is specified as a distance to every other case. Instead the mean row sum is displayed.

3.6.2 Analysis models

All three SASEM analysis sub-models A2 to A4 are used in Study 3. All models are correctly specified.

3.6.3 Expectations

I hypothesize parameter estimates will be unbiased but require larger sample sizes under higher connection conditions of W_C^*. LeSage & Pace (2014a) has shown that in a regression context modification of W does not result in particularly different parameter estimates of slopes. Stakhovych & Bijmolt (2009) shows that unbiased estimates of spatial regression models with W_D^* require larger sample sizes. The SASEM model is highly parameterized and can include unique spatial effects via the measurement model; therefore, it may not exhibit the robustness to saturated W specifications observed by LeSage & Pace (2014a).

3.6.4 Results

Full results for all parameters and conditions are provided in the Appendix. Population model D2 is provided by Table A.9, D3 is provided by Table A.10, and D4 is provided by Table A.11 and Table A.12.

3.6.4.1 Convergence

Convergence rates in Study 3 follow the trends observed in Study 2, meaning, high convergence rates with a few exceptions. Parameters of interest consistently converge $> 95\%$, with the exception of ϕ_ζ ($> 92\%$) and ρ_η ($> 93\%$) in model A4. Experimental conditions do not systematically impact convergence rates with the exception of sample size. Increases of sample size correspond with increased convergence rates.

3.6.4.2 Bias

A2 Model A2 does not reveal a consistent relationship between bias and connection conditions in W. A2 consistently estimates $\rho_{y2} \approx 0.4$ regardless of the population value or W specification. Structural slopes and endogenous factor loadings remain unbiased. In A2, a systematic relationship between the number of connections and bias is not apparent. Generally speaking, as the average number of connections in W increases, so does the magnitude of bias. W_D^* specifications exhibit the most bias across parameters.

A3 Model A3 exhibits a relatively consistent relationship between bias and connection conditions in W. Regarding ρ_η estimates, the higher the average number of connections, the higher the observed bias. The highest bias is observed by W_D^* and lowest by W_C^*. The other parameters appear to be robust to specification of W with no consistent relationships.

A4 Congruent with Study 2, A4 exhibits unacceptably high bias regarding both auto-regressive estimates ρ_η and ϕ_ζ. Regarding auto-regressive effects, higher connection conditions observe similar bias with the exception of W_D^*. Under the distance specification and $\rho_\eta = 0$ or 0.3, ρ_η estimates are consistently the higher when compared to other W conditions. The inverse is true when population level $\rho_\eta = 0.6$; regardless of the true value of ϕ_ζ, estimates of ρ_η are consistently higher in the W_D^* condition as compared to other W specifications.

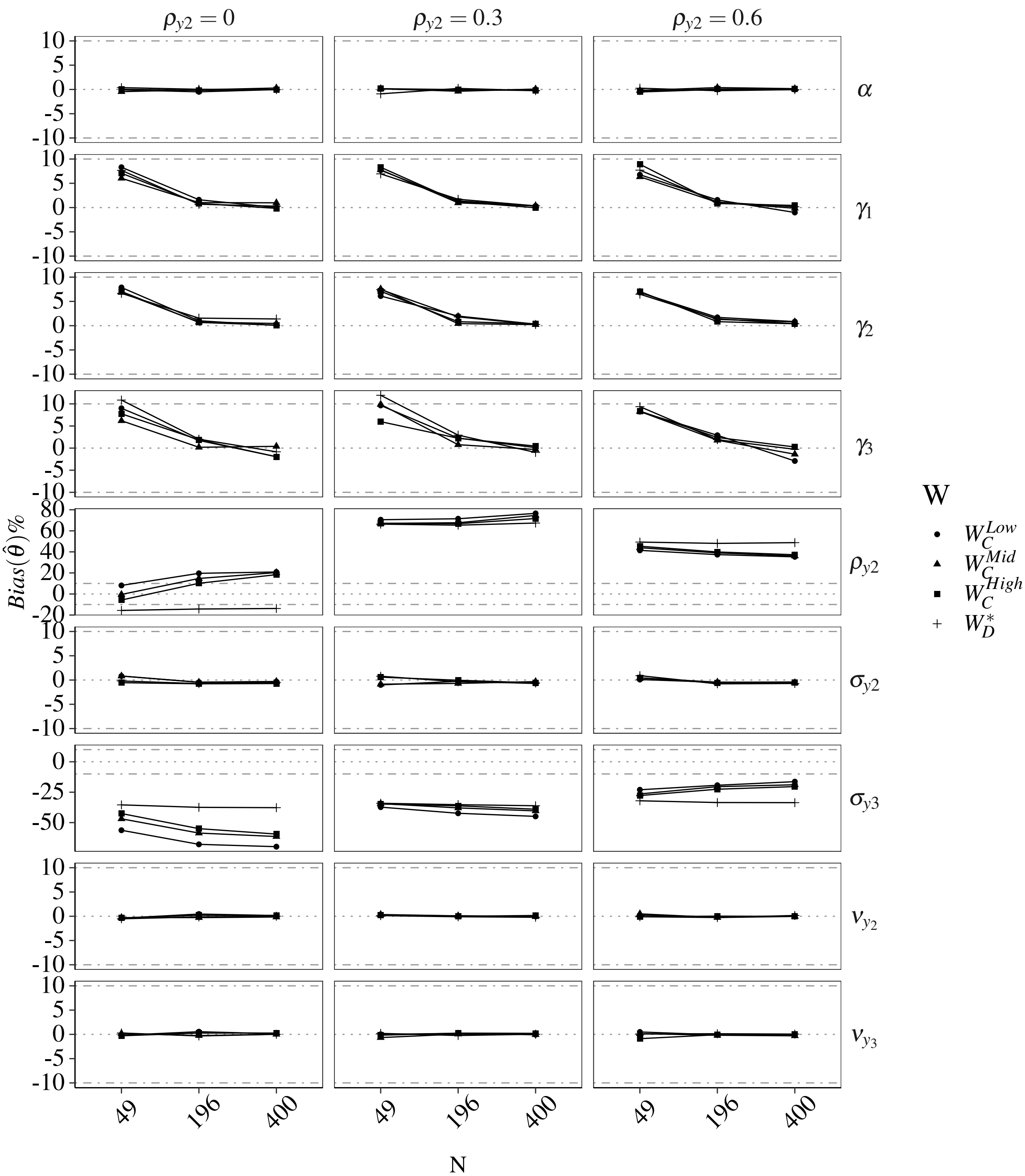

Figure 3.9: Line plot of $Bias(\hat{\theta})\%$ under the measurement spatial lag population (A2) and analysis models (D2) with additional W connection specifications.

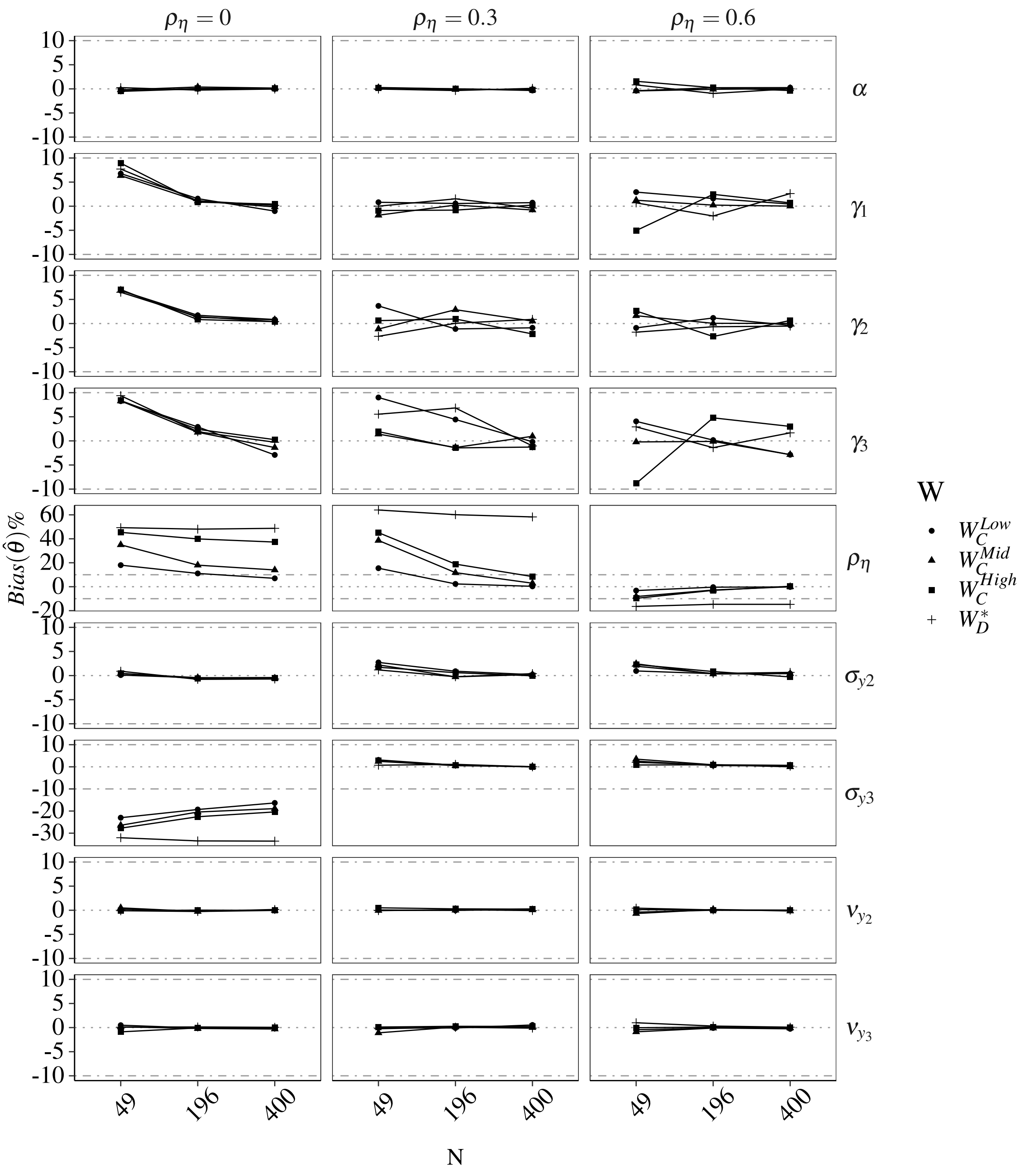

Figure 3.10: Line plot of $Bias(\hat{\theta})\%$ under the endogenous structural lag population (A3) and analysis models (D3) with additional W connection specifications.

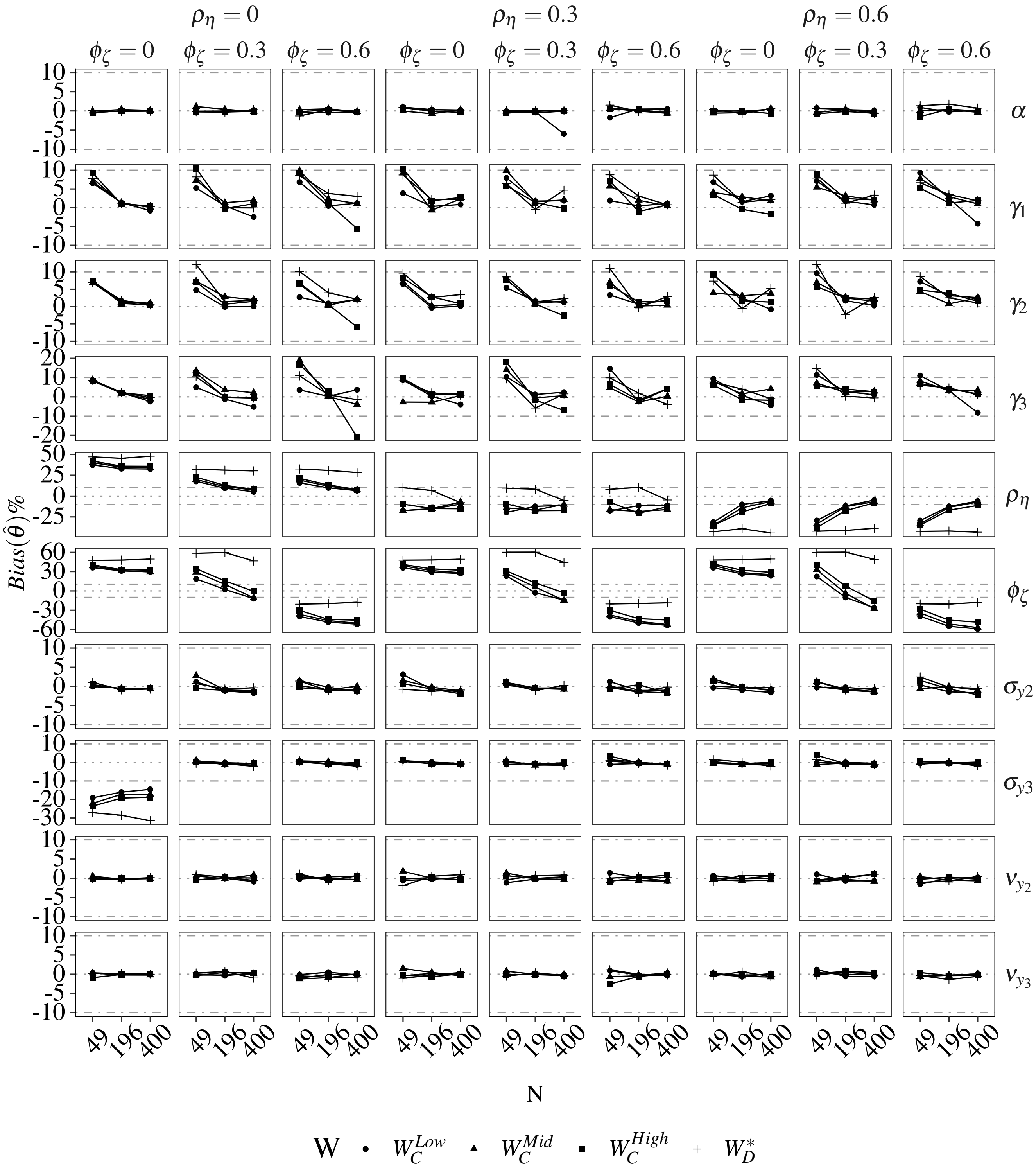

Figure 3.11: Line plot of $Bias(\hat{\theta})\%$ under the simultaneous structural spatial lag population (A4) and analysis models (D4) with additional W connection specifications.

3.6.4.3 Coverage

Across all models and conditions, coverage rates of parameters exhibit a slight decrease with increased sample size.

D2 and A2 In model A2 the alternative W specifications solely impact the coverage rates of the spatial auto-regressive estimates. When population $\rho_{y2} = 0$, coverage rates for $\rho_{y2} = 0\%$. As population level ρ_{y2} increases, coverage does as well. When population $\rho_{y2} \neq 0$ coverage rates increase as the average number of connections increase. That is, the highest observed coverage rates of ρ_{y2} are under the W_D^* specification, while the lowest are observed under W_C^{Low}.

D3 and A3 Congruent with model A2 alternative specifications of W solely impact the coverage rates of the spatial auto-regressive estimates. When population $\rho_{y2} = 0$, coverage rates for $\rho_{y2} = 0\%$. As population level ρ_{y2} increases coverage does as well. When population $\rho_{y2} \neq 0$ coverage rates increase as the average number of connections increase. That is, the highest observed coverage rates of ρ_{y2} are under the W_D^* specification, while the lowest are observed under W_C^{Low}. Coverage rates of other parameters do not vary as a function of alternative W specifications.

D4 and A4 In line with the other models (A2 and A3) W specifications only impact the coverage rates of spatial parameters. Under $\phi_\zeta = 0.3$, coverage of spatial parameters exhibits large variation. ρ_η coverage rates vary between 16.33% and 100.00%, where the highest coverage rates are observed in W_D^* and lowest in the $W_C^L ow$ specification. $\phi_\zeta = 0.3$ coverage rates vary between 57.65% and 100.00%. Again, the highest coverage rates are observed in W_D^* and lowest in the $W_C^L ow$ specification.

3.7 Study 4

The goal of Study 4 is to investigate the consequences of spatial parameter misspecification. To achieve this, data is generated under a SASEM sub-model defined in Section 3.1, then analyzed with a misspecified spatial effect SASEM sub-model.

3.7.1 Data generation and additional conditions

In Study 4 data are generated under each of the SASEM data generating sub-models defined in Section 3.1. There are no additional conditions to that described in Section 3.1.1.

3.7.2 Analysis models

In Study 4 all three SASEM analysis sub-models are utilized. However, they are mispecified regarding the spatial effect. Data is generated under the D2 population model, and analyzed with A3, and A4. Population model D3 is analyzed with A2 and A4, and population model D4 is analyzed with A2 and A3.

3.7.3 Expectations

I expect results for Study 4 to vary depending on the data generating model and misspecified analysis model combination. Table 3.2 provides a summary of the model misspecification conditions.

Data Generating		Analysis	
Label	Sub-Model Spatial Lag	Label	Sub-Model Spatial Lag
D2	Measurement	A3	Structural Endogenous
		A4	Structural Endogenous and Disturbance
D3	Structural Endogenous	A2	Measurement
		A4	Structural Endogenous and Disturbance
D4	Structural Endogenous and Disturbance	A2	Measurement
		A3	Structural Endogenous

Table 3.2: Summary of Study 4 model spatial effect misspecification conditions

In the D2 data generating condition, I anticipate increased bias and type 1 error rates in ρ_η when the population level spatial effect $\rho_{y2} \neq 0$. Specifically, in analysis model A3 it is likely the omitted measurement level spatial effect will result in a spatially dependent η. The resulting structural endogenous lag estimate ρ_η will measure the spatial effect in η. In analysis model A4 I anticipate biased ρ_η and ϕ_ζ estimates. The resulting un-modeled spatial dependence at the measurement level will be present in both the error estimate ϕ_ζ and endogenous lag ρ_η. This will likely result in increased bias and type 1 error rates of both parameters. It is worth mentioning the problems associated with simultaneous disturbance and endogenous lags discussed earlier will also likely bias the structural slope estimates γ_1 and γ_2 with the latent interaction effect γ_3 suffering the most.

In data generating condition D3 when the spatial data generating process $\rho_\eta \neq 0$ in analysis model A2, I anticipate increased bias and type 1 error rates in estimates of ρ_{y2}, γ_1, γ_2, and γ_3. Specifically, unmodelled structural level spatial effects will result in biased estimates of the relationships between ξ and η as well as a deviation from the population value of $\rho_{y2} = 0$. In analysis model A4, I anticipate the persistent bias discovered in Study 2, as well as increased type 1 error rates and low power.

In data generating condition D4, the population level spatial process is present in both the endogenous variable η and disturbance term μ. In condition analysis model A2 when population level ρ_η, $\phi_\zeta \neq 0$, I anticipate increased bias in ρ_{y2} and structural slopes γ_1, γ_2 and γ_3 because cases are no longer independent (Kenny & Judd, 1986). For analysis model A4 I anticipate the least bias and error rates. I predict when population level $\phi_\zeta \neq 0$ model A4 will exhibit positively biased ρ_η estimates, which in turn will model spatial dependence resulting in unbiased estimates of structural slopes. In addition to these predictions, I expect increased bias under lower sample sizes and increased bias when the spatial process is 0 at the population level.

3.7.4 Results

Full result tables for Study 4 are presented in the Appendix: Table A.14 and Table A.13 for D2 population model; Table A.15 and Table A.16 for D3 population data; and Table A.17 and Table A.18 for D4 population model.

3.7.4.1 Convergence

Convergence rates in Study 4 follow the trends observed earlier studies, showing acceptable convergence rates with exceptions. Misspecified conditions exhibit increased bias. Specifically, in analysis model A4 ϕ_ζ exhibits a lower percent of converged simulated iterations (93%) as compared to non-spatial parameters ($\approx$ 99.6% on average) in the same conditions. Parameters of interest consistently converge ($\approx$ 99%) with the exception of ϕ_ζ and ρ_η in model A4. Simulation conditions do not systematically impact convergence rates.

3.7.4.2 Bias

Across population and analysis models, bias systematically decreases as sample size increases. W_D^* specifications are also systematically more biased than W_C^* conditions. Overall, analysis model A3 is the least biased.

D2 and A3 Fig. 3.12 provides the observed bias by simulation conditions for parameters of interest. Under W_C^* and n = 49 conditions, model A3 exhibits deviations in ρ_η estimates from the population value of 0, observing an absolute bias of 47.90% to 51.05%. Regarding non-spatial parameter estimates, model A3 exhibits consistently unbiased estimates. Structural slope estimates are unbiased. In n = 49 and W_C^* conditions γ_1 and γ_2 $bias \approx 7\%$ and do not increase with increased population level ρ_{y2} values. When n = 400 the structural slope bias drops to $< 1\%$. The structural interaction effect estimate γ_3 is more biased, but still in the acceptable range. In the n = 49, W_C^* and $\rho_{y2} = 0$ conditions $bias(\gamma_3) = 9.74\%$. Increasing population level ρ_{y2} slightly increases the bias to 10.24%. Increasing sample size decreases the observed bias, under n = 400, W_C^* and $\rho_{y2} = 0$ $bias(\gamma_3) = -2.54\%$, when $\rho_{y2} = 0.6$ $bias(\gamma_3) = -0.77\%$.

D2 and A4 Parameter estimates in model A4 exhibit bias from the misspecification of population level ρ_{y2}. Fig. 3.13 provides bias results by simulation condition for parameters of interest. Structural slope estimates are unbiased; under the n = 49, W_C^*, and population $\rho_{y2} = 0$ conditions,

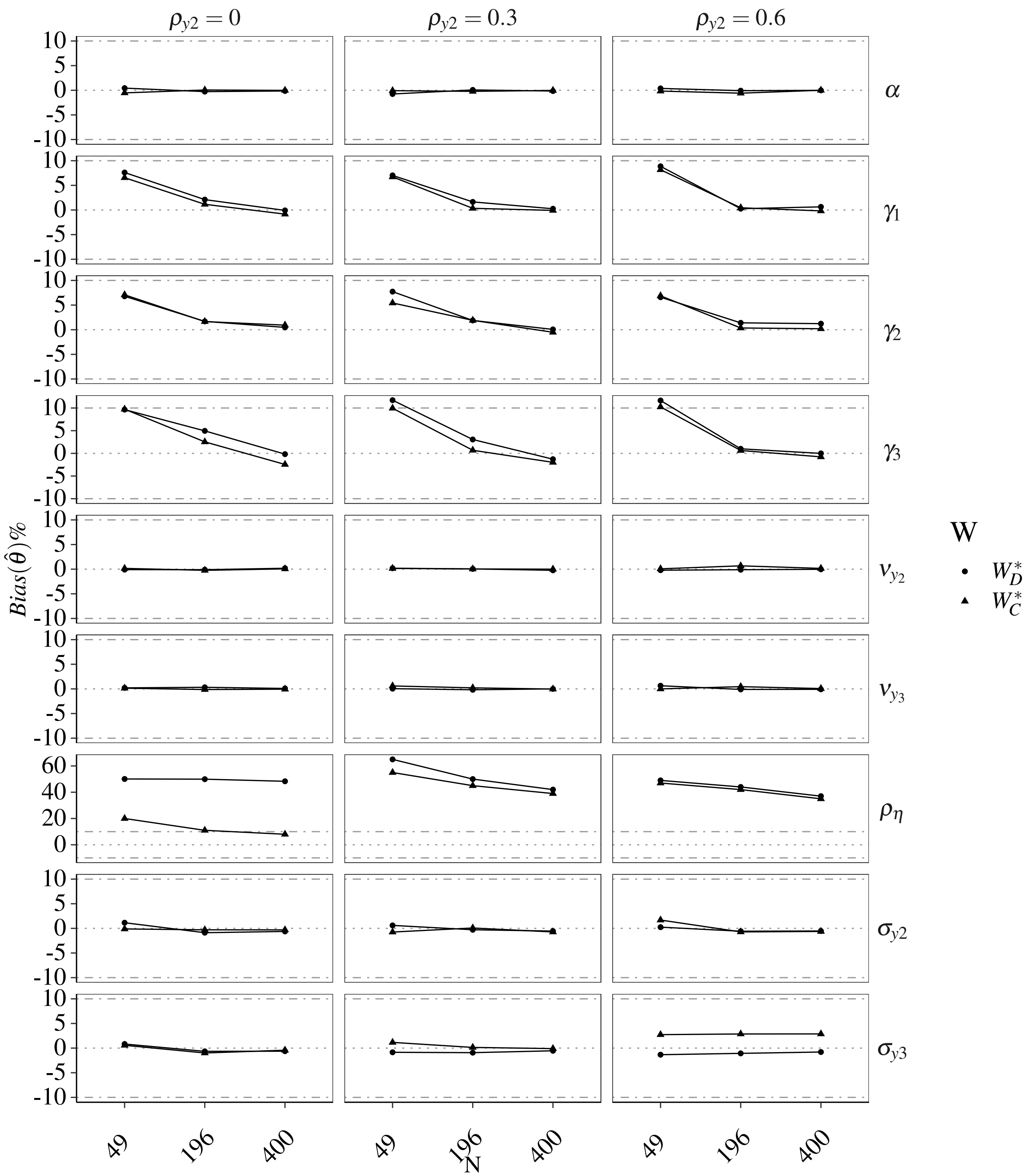

Figure 3.12: Line plot of $Bias(\hat{\theta})\%$ under the measurement lag population model (D2) analyzed with endogenous structural lag model (A3).

the bias of γ_1 and $\gamma_2 \approx 5\%$ and does not increase with increased population level ρ_{y2} values. The structural slope bias drops to $< 2\%$ when n = 400 . The interaction effect estimate γ_3 is more biased than γ_1 and γ_2. In the n = 49, W_C^*, and $\rho_{y2} = 0$ conditions $bias(\gamma_3) = 2.66\%$. Increasing ρ_{y2} results in observed bias of -3.65%. Increasing sample size diminishes the observed bias. Under n = 400, W_C^*, and population $\rho_{y2} = 0$ $bias(\gamma_3) = 1.59\%$, when population $\rho_{y2} = 0.6$ $bias(\gamma_3) = -3.19\%$.

Spatial parameter estimates consistently deviate from the population values of 0. Under W_C^*, n = 49, and population level $\rho_{y2} = 0$, the bias of $\rho_\eta = 18.17\%$ and $\phi_\zeta = 37.28\%$. ρ_η estimates are consistently erroneously $\approx$ .20, while $\phi_\zeta \approx$ .38. They do not vary as a function of population level ρ_{y2}. However, as sample size increases the estimate change. when n = 400, estimates of $\rho_\eta \approx 0.06$ and ϕ_ζ estimates exhibit variation. Under $\rho_{y2} = 0$ $\phi_\zeta = .31$ which results in 31% absolute bias, when $\rho_{y2} = 0$ ϕ_ζ estimates $\approx$ .42.

D3 and A2 Under W_C^* and n = 49 conditions, model A2 exhibits deviations in ρ_{y2} estimates from the population value of 0 with consistent estimates varying solely through sample size and W conditions. Under W_C^*: when n = 49 the ρ_{y2} estimates are consistently ≈ 0.45; when n = 196 ρ_{y2} estimates are consistently ≈ 0.40; and when n = 400 ρ_{y2} estimates are consistently ≈ 0.37. Under W_D^*, the ρ_{y2} estimates are consistently ≈ 0.50 regardless of sample size or population level ρ_η conditions. Fig. 3.14 provides a summary of bias by simulation conditions for parameters of interest.

Regarding non-spatial parameter estimates, model A2 exhibits consistently unbiased estimates with one exception, $\sigma_{\varepsilon 3}$. $\sigma_{\varepsilon 3}$ is consistently underestimated when $\rho_\eta > 0$. Under W_C^*, n = 49 and population $\rho_\eta = 0$, $\sigma_{\varepsilon 3}$ is unbiased (-1.05%). When $rho_\eta = 0.3$ the magnitude of bias increases $bias(\sigma_{\varepsilon 3} = -22.10\%$, and when population $\rho_\eta = 0.6$ bias increases to -23.85%. The bias of $\sigma_{\varepsilon 3}$ decreases with increased sample size. At n = 400 and population $\rho_\eta = 0.6$, the observed bias is -18.98%. Structural slope estimates are unbiased. Under n = 49 and W_C^* conditions, γ_1 and γ_2 observed bias is $< 1\%$ and increases slightly when $\rho_\eta > 0$, with observed bias of $<$ 3% when population $\rho_\eta = 0.6$. The structural interaction effect estimate γ_3 is more biased, but

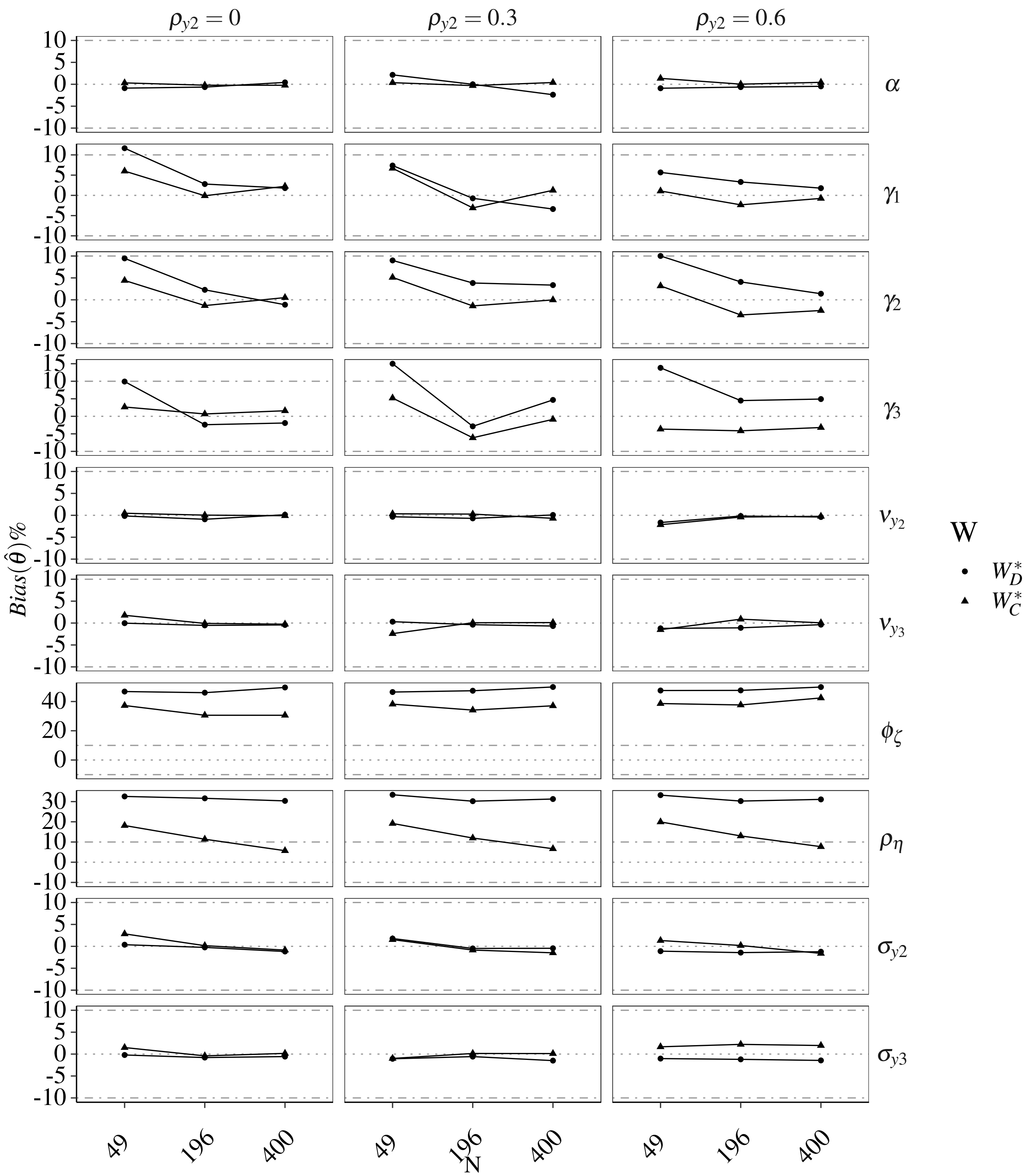

Figure 3.13: Line plot of $Bias(\hat{\theta})\%$ under the measurement lag population model (D2) analyzed with simultaneous structural lag model (A4).

still acceptable. In the n = 49, W_C^* and $\rho_\eta = 0$ conditions the bias of $\gamma_3 = -5.25\%$. Increasing population level ρ_η slightly increases the bias to 8.56%. Increasing sample size decreases the bias of γ_3, under n = 400, W_C^* and $\rho_\eta = 0$ observed bias of $\gamma_3 = 1.29\%$, when $\rho_\eta = 0.6$ observed bias of $\gamma_3 = -3.63\%$.

D3 and A4 Model A4 performs similarly under population model D3 as D2. Fig. 3.15 provides the bias by simulation conditions for parameters of interest. Structural slope estimates are unbiased, in n = 49 and W_C^* conditions γ_1 and γ_2 $bias \approx 7\%$ and are stable across population level ρ_η values. When n = 400 the structural slope bias drops to $< 1.5\%$.regarding the interaction effect, in the n = 49, W_C^* and $\rho_\eta = 0$ conditions $bias(\gamma_3) = 10.51\%$ and is relatively stable across population level ρ_η estimates. Increasing sample size decreases the bias to $\approx 1.5\%$.

Spatial parameter estimates consistently deviate from the population values. Under W_C^*, n = 49, and population level $\rho_\eta = 0$, ρ_η estimates are consistently erroneously $\approx .15$, while $\phi_\zeta \approx .30$. The ϕ_ζ estimates do not vary as a function of population level ρ_η. However, as sample size increases the estimates change. When n = 400, and W_C^* is used, estimates of $\rho_\eta \approx 0.30$ and ϕ_ζ estimates $\approx .30$. Under $\rho_\eta = 0$ $\phi_\zeta = .31$ which results in 31% absolute bias, when $\rho_\eta = 0$ ϕ_ζ estimates $\approx .42$.

D4 and A2 Model A2 exhibits biased estimates under $\phi_\zeta \neq 0$ population conditions. Fig. 3.16 provides the bias by simulation conditions for parameters of interest. ρ_{y2} estimates are consistent across population values of ϕ_ζ and ρ_η. Sample size induces variation in ρ_{y2} estimates, under W_C^* when n = 49 estimates are consistently ≈ 0.45, when n = 196 they are $\approx= .44$, and when n = 400 ≈ 0.44. Estimates are even more homogeneous under W_D^* specifications, regardless of sample size, population level ϕ_ζ, or ρ_η ρ_{y2} is consistently estimated as $\approx= .50$.

Regarding non-spatial parameter estimates, structural effects and $\sigma_{\varepsilon 3}$ exhibit variations in bias. Structural slope estimates are biased when $\phi_\zeta \neq 0$. Under n = 49, W_C^*, and ρ_η & $\phi_\zeta = 0.3$, γ_1 and γ_2 bias $\approx 7.5\%$. When ρ_η & $\phi_\zeta = 0.6$ bias increases to $\approx 15\%$. The structural interaction γ_3 is more sensitive and increases from $\approx 8\%$ to $\approx 18.5\%$ from an increase of population level ρ_η &

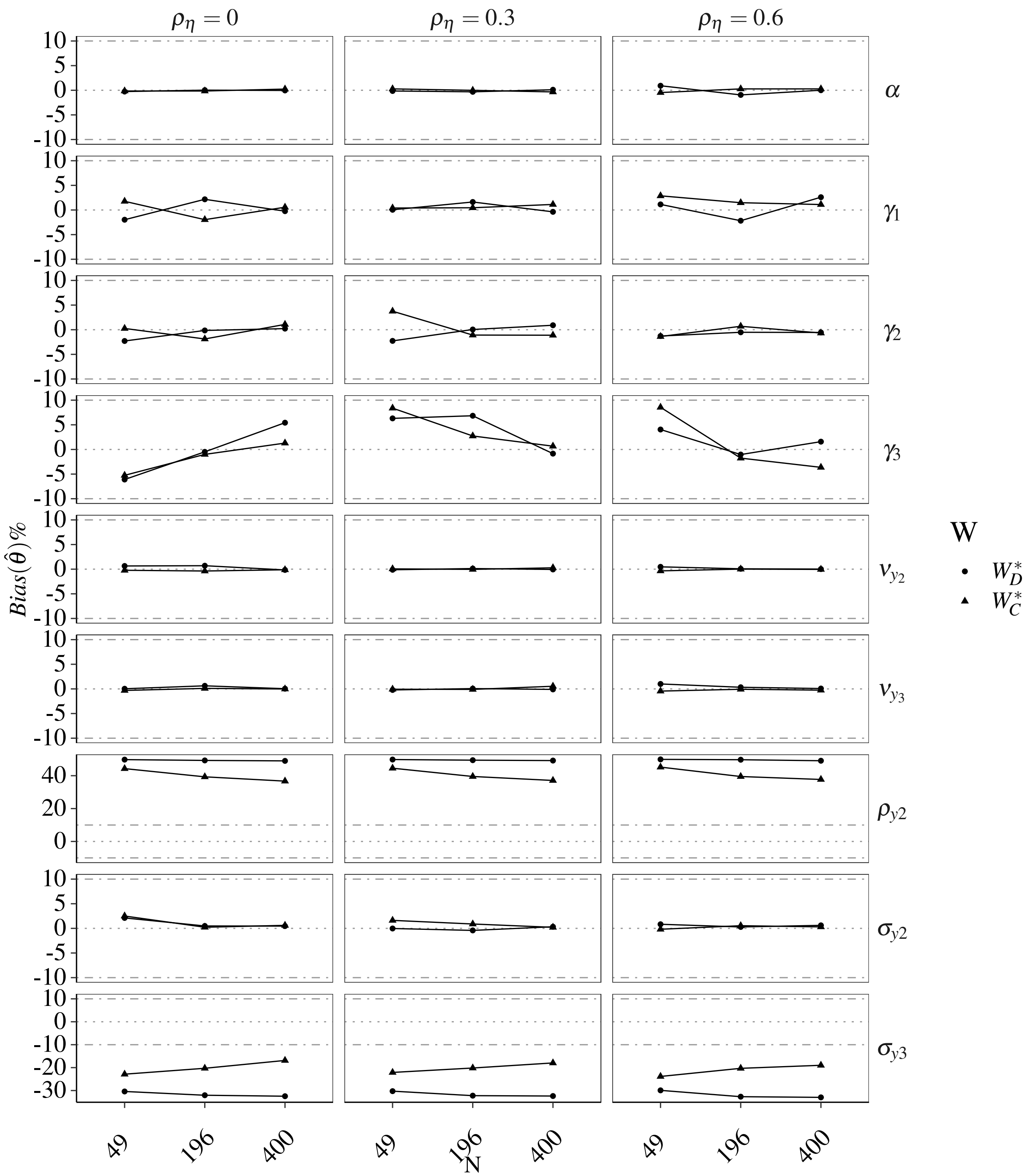

Figure 3.14: Line plot of $Bias(\hat{\theta})\%$ under the endogenous structural lag population model (D3) analyzed with measurement lag model (A2).

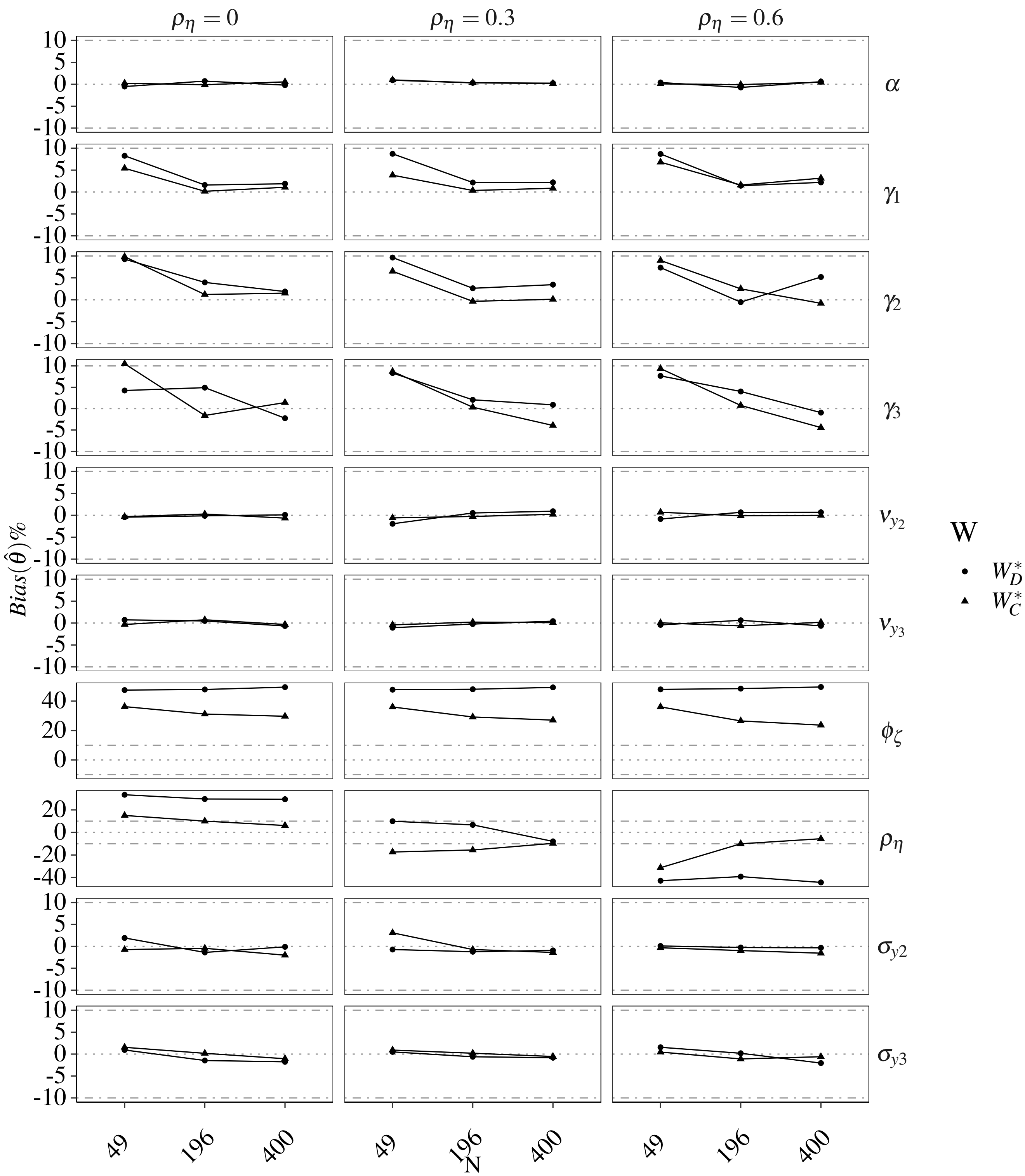

Figure 3.15: Line plot of $Bias(\hat{\theta})\%$ under the endogenous structural lag population model (D3) analyzed with simultaneous structural lags model (A4).

$\phi_\zeta = 0.3$ to ρ_η & $\phi_\zeta = 0.6$. Under W_D^*, the increase of bias observed in W_C^* does not occur with bias only reaching $> 10\%$ in the n = 49 condition.

D4 and A3 Fig. 3.17 provides a summary of bias by study conditions for model A3 under population model D4. Under W_C^*, n = 49, $\phi_\zeta = 0$, and $\rho_\eta = 0.6$, ρ_η estimate bias = -12.21%. Increasing ϕ_ζ to 0.6 results in a large increase in bias of $\rho_\eta \approx -9\%$. The non-spatial parameter estimates are consistently unbiased across simulation conditions. Structural slopes exhibit the most sensitivity to the misspecification. Under W_C^*, n = 49, and ρ_η & $\phi_\zeta = 0.3$, the bias of γ_1 = 4.24%, γ_2 = 6.26%, and γ_3 = 8.05%. Increasing ϕ_ζ to 0.6, yields bias of γ_1 = 7.27%, γ_2 = 7.85%, and γ_3 = 10.11%. When Increasing ρ_η & $\phi_\zeta = 0.6$, bias of γ_1 = 5.82%, γ_2 = 7.43%, and γ_3 = 9.82%. Sample size diminishes the increase in bias from un-modeled ϕ_ζ, under W_C^*, n = 400, and ρ_η & $\phi_\zeta = 0.3$, the bias of γ_1 = 1.78%, γ_2 = 2.38%, and γ_3 = 0.55%. Increasing population parameters to ρ_η & $\phi_\zeta = 0.6$, the bias of γ_1, γ_2, and $\gamma_3 < 2.5\%$.

3.7.4.3 Coverage

Across population models and conditions as sample size increases, coverage rates see slight declines. The specification of W does not systematically affect coverage rates. Spatial parameters (ρ_η, ρ_{y2}, and ϕ_ζ), intercepts (α and τ), and observed variance (σ) estimates exhibit the most variation in coverage rates across models.

D2 and A3 Analysis model A3 is robust to the misspecification of the spatial parameter. Across parameters coverage rates do not differ between conditions with or without the inclusion of the mispecified spatial parameter. The spatial estimate ρ_η does not encompass the population value of 0 in any condition resulting in coverage of 0.00%. All other parameters are robust to the misspecification resulting in coverage rates > 92% with one additional exception. The endogenous observed item intercept τ_{y3} shows a consistent decrease in coverage as the population parameter ρ_{y2} increases. Under n = 49 and W_D^*, when $\rho_{y2} = 0$ the coverage of $\tau_{y3} = 94.39\%$ when $\rho_{y2} = 0.3$ coverage decreases to 88.53% and when $\rho_{y2} = 0.6$ coverage is 86.14%.

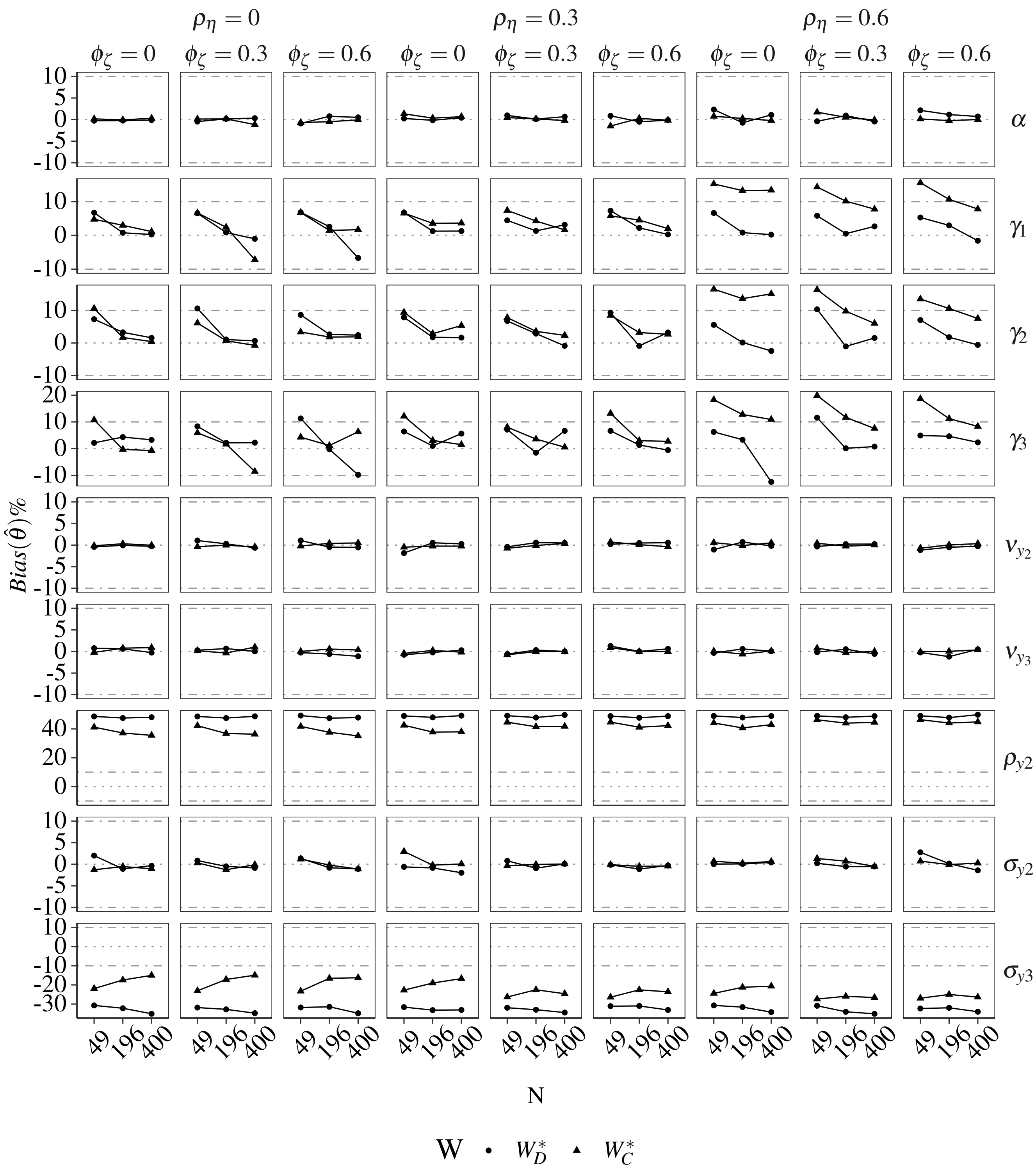

Figure 3.16: Line plot of $Bias(\hat{\theta})\%$ under the simultaneous structural spatial lag population model (D4) analyzed with the measurement lag model (A2).

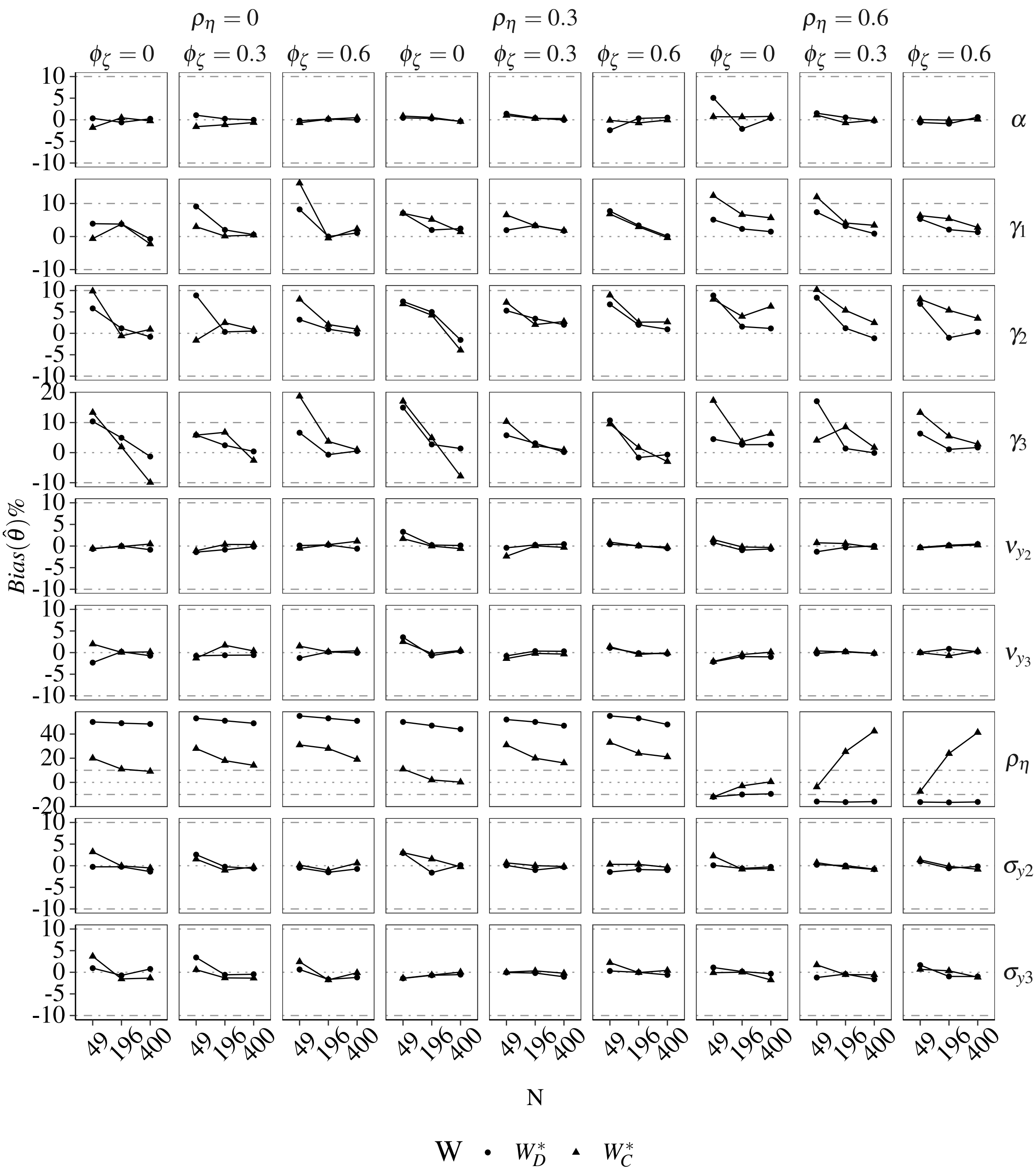

Figure 3.17: Line plot of $Bias(\hat{\theta})\%$ under the simultaneous structural spatial lag population model (D4) analyzed with the endogenous structural lag model (A3).

D2 and A4 Under population model D2, analysis model A4 yields similar trends in coverage rates to that of A3. Across parameters, coverage rates are not impacted by the misspecification, with the exception of the spatial parameters. Both ρ_η and ϕ_ζ exhibit coverage rates of 0% under population $\rho_{y2} = 0$. The coverage rate of the observed intercept τ_{y3} decrease as the population parameter ρ_{y2} increases. Under n = 49 and W_D^*, when $\rho_{y2} = 0$ the coverage of $\tau_{y3} = 95.57\%$ when $\rho_{y2} = 0.3$ coverage decreases to 91.13% and when $\rho_{y2} = 0.6$ coverage is 88.18%.

D3 and A2 Under population model D3, analysis model A2 shows robustness to misspecification. In the n = 49 and W_C^* condition parameters exhibit high coverage (>93%) with one exception. Regarding the structural intercept α, as the magnitude of the misspecified spatial parameter ρ_{y2} increases coverage decreases. When $\rho_\eta = 0$ the coverage of $\alpha = 95.20\%$ when $\rho_\eta = 0.3$ coverage decreases to 90.80% and when $\rho_{y2} = 0.6$ coverage is 86.67%. In all conditions, coverage of ρ_{y2} = 0%.

D3 and A4 Model A4 also shows acceptable coverage rates for non-spatial parameters. Under n = 49 and W_C^* specification non-spatial parameters exhibit high coverage rates (>91%) with a single exception. The structural intercept α decreases as the population level spatial effect increases. When $\rho_\eta = 0$ the coverage of $\alpha = 99.46\%$, when $\rho_\eta = 0.3$ coverage decreases to 98.92% and when $\rho_\eta = 0.6$ the coverage of $\alpha = 95.70\%$. The spatial parameters ρ_η and ϕ_ζ are unacceptably low. Under population level $\rho_\eta = 0$, the coverage of ρ_η = 0%, when $\rho_\eta = 0.3$ coverage climbs to 99.64%, and under $\rho_\eta = 0.6$ coverage is 86.02%. Population level $\phi_\zeta = 0$ in all conditions and is erroneously estimated consistently as 0.36, resulting in a universal coverage rate of 0%.

D4 and A2 Model A2 coverage rates of non-spatial parameters exhibit robustness to the spatial structural misspecification. Under the W_C^* specification and n = 49, non-spatial parameter estimates exhibit acceptable coverage rates regardless of the population level spatial effects. One exception is α which shows decreased coverage rates as population level ρ_η increases. Under n = 49, W_C^*, population level $\rho_\eta = 0$ and $\phi_\zeta = 0.3$, coverage of α is 100%, when $\rho_\eta = 0.3$ coverage decreases to

94.03%, and when $\rho_\eta = 0.6$ coverage of $\alpha = 79.40\%$. The other exception is $\sigma_{\varepsilon 3}$, which exhibits very low coverage rates. Under the same set of conditions coverage of $\sigma_{\varepsilon 3}$ is 90.13% and does not vary between conditions of ρ_η or ϕ_ζ. However, as sample size increases coverage rates decrease. When sample size is 400, W_C^*, population level ρ_η & $\phi_\zeta = 0.3$ coverage of $\sigma_{\varepsilon 3}$ drops to 67.83%. The spatial estimate ρ_{y2} exhibits 100% coverage across all conditions. Population level ϕ_ζ does not systematically impact coverage rates between conditions.

D4 and A3 Model A3 exhibits high coverage rate across parameters and conditions with the exception of the structural intercept α. α exhibits decreased coverage rates as population level ρ_η increases. Under n = 49, W_C^*, and population level $\rho_\eta = 0.3$ the coverage rate of α is 94.44%. When population level increases to $\rho_\eta = 0.6$, the coverage rate of α decreases to 80.15%. Population level structural disturbance dependence ϕ_ζ does not have a systematic effect on coverage rates. Coverage rates for ρ_η are 100% for all n = 49 conditions, and decrease as sample size increases. Under W_C^*, n = 400, and population level ρ_η & $\phi_\zeta = 0.6$ the coverage rate of $\rho_\eta = 83.94\%$. Population level ϕ_ζ does not systematically impact coverage rates between conditions.

Chapter 4

Empirical Example

The following example explores homicide rate data obtained from the United States (US) southern region to demonstrate an applied use of the endogenous lag SASEM with structural latent interaction effect. In this chapter, I will first briefly describe the data set and provide background information, then motivate the model choice, analyze the data, and provide interpretations of spillover effects.

4.1 Homicide data

The US southern homicide data provides rates of crime for 1,412 counties in the southern US region in the year 1990. Collected by Messner et al. (1999), the southern US homicide data has been frequently used as a typifying example in econometric and behavioral science literature (Kubrin & Weitzer, 2003; Morenoff et al., 2001; Goodchild et al., 2000). Cases in the data are counties in the southern US and variables are rates within each county. Variables of interest are rate of homicide, aggravated assault, burglary, rape, larceny, vehicle theft, and robbery. The expanded version of the data used here has been integrated with additional covariates of income inequality, originally collected in Land et al. (1990). Additional variables of interest are unemployment rate, Gini coefficient (Dorfman, 1979), and average income.

4.1.1 Predicting Violent Crime

Identifying risk factors for violent crime rates at the societal level provides valuable information for policy makers and government institutions. Research has established a link between income inequality and violent crime. As income inequality rises violent crime rates increase (Deller & Deller, 2010). Research has also noted a distinction in types of criminal acts. For example, violent crime and property crime are distinct constructs (Schreck et al., 2009). A unified analysis which accounts for the latent relationships between these variables and the inherent spatial dependence is not available yet.

4.1.2 Research questions

Regions which have higher income inequality experience greater property and violent crime rates (Patterson, 1991). It is reasonable to hypothesize an interaction effect between property crime and financial accessibility in the prediction of homicide rates.

Prior research suggests nearer regions have more similar violent crime rates compared to further regions (Bernasco & Elffers, 2010). Therefore, it is also reasonable to hypothesize the endogenous latent variable, violent crime, is spatially dependent. To illustrate this, I present choropleth plots of the observed violent crime variables. Specifically, Fig. 4.1 provides a plot of murder rates, Fig. 4.2 provides burglary rates, Fig. 4.3 provides aggravated assault rates, and Fig. 4.4 provides rape rates. In each figure, darker counties indicate higher values relative to the mean of the sample, whereas lighter counties have values lower than the mean of the sample. If a variable is not spatially dependent, the choropleth plot will yield no pattern, and appear seemingly random. This is not the case, each of the violent crime variables reasonably exhibits pockets of similar scores. This supports the hypothesis that the data are spatially dependent.

To investigate the relationship between financial accessibility and property crime on violent crime, I present a set of relevant research questions:

1. Does a county's financial accessibility and property crime rates predict violent crime?

2. Is the relationship in counties with low financial accessibility stronger between property crime and violent crime compared to high accessibility?

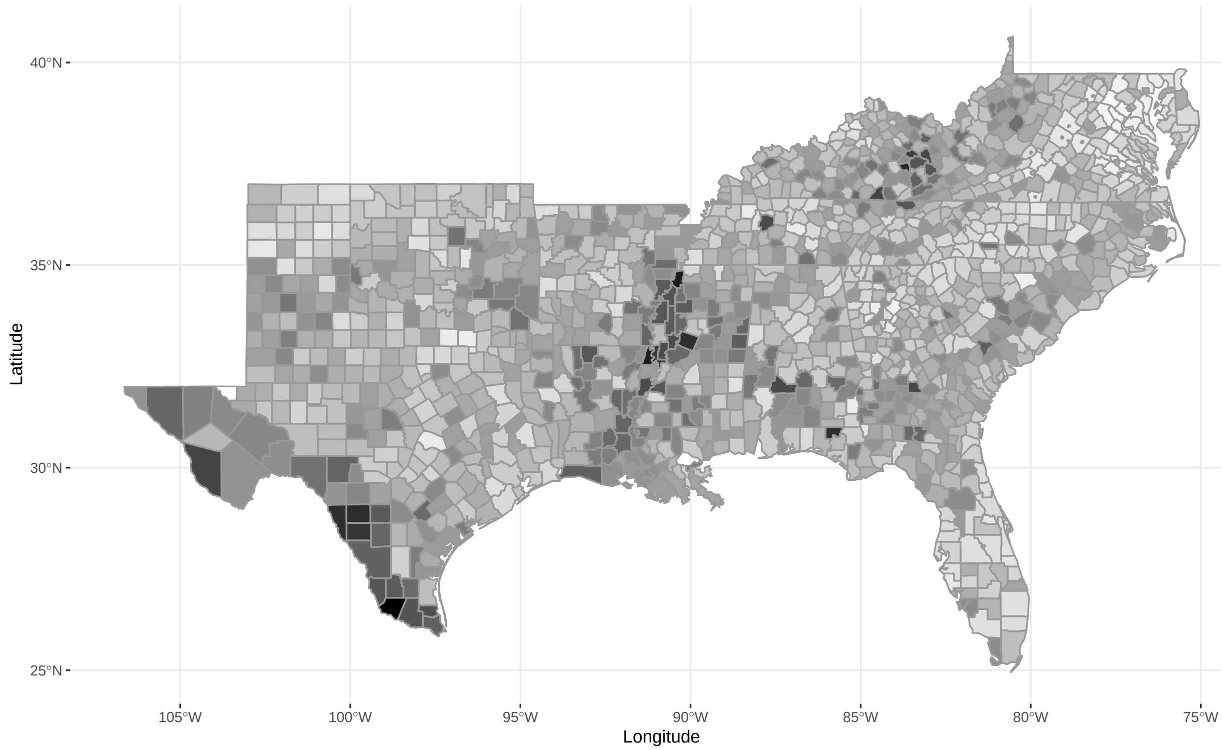

Figure 4.1: Choropleth plot of Murder Rate by county, darker counties indicate rates higher than the mean while lighter counties are lower.

3. In counties with low financial accessibility is the relationship stronger between property crime and violent crime compared to high accessibility?

4.2 Methods

4.2.1 Definitions and descriptive statistics

Cases are counties in the US south. Variables are averages of rates within each county. All rates are expressed as occurrences per 100,000 people, with the exception of unemployment rate which is defined below. All variables used here were recorded in the year 1990. The following crime variable definitions were adapted from Messner et al. (1999) and the financial accessibility variables from Land et al. (1990).

Homicide rate is the number of homicides in each county. Aggravated assault is defined as violent physical assault with or without a weapon. Assault rate is the rate of aggravated assaults

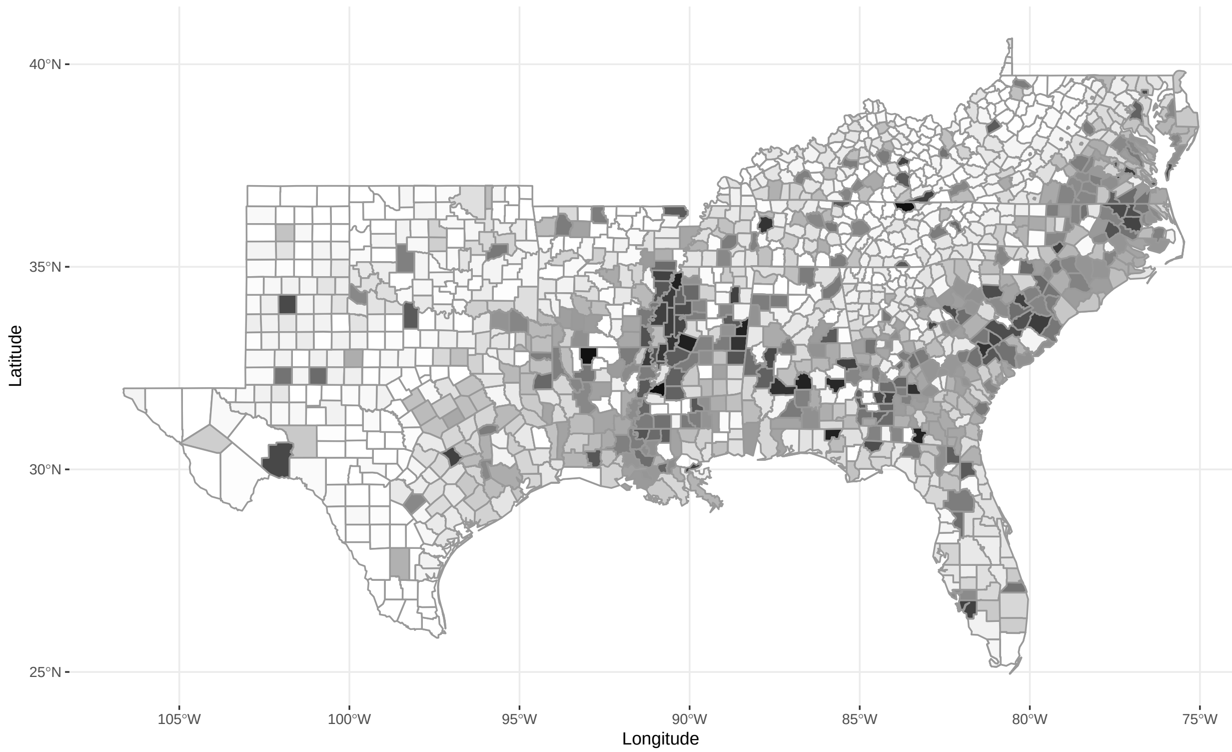

Figure 4.2: Choropleth plot of burglary rate by county, darker counties indicate rates higher than the mean while lighter counties are lower.

recorded in each county. Burglary is defined as forced or illegal entry and theft. Burglary rate is the rate of recorded burglaries by county. Rape rate is the rate of reported rapes in each county. Larceny is defined as non-violent theft of personal property excluding motor vehicles. Larceny rate is the rate of non-violent theft within each county. Vehicle theft rate is defined as theft or attempted theft of a vehicle which is motorized. Robbery rate is defined as the rate of non-violent theft without the use of physical force.

The Gini coefficient is a measure of income inequality, where 0 indicates perfect income equality, and 1 represents perfect inequality (Dorfman, 1979). Average income is the mean income in the county. Unemployment rate is expressed as the proportion of residents in a county who are

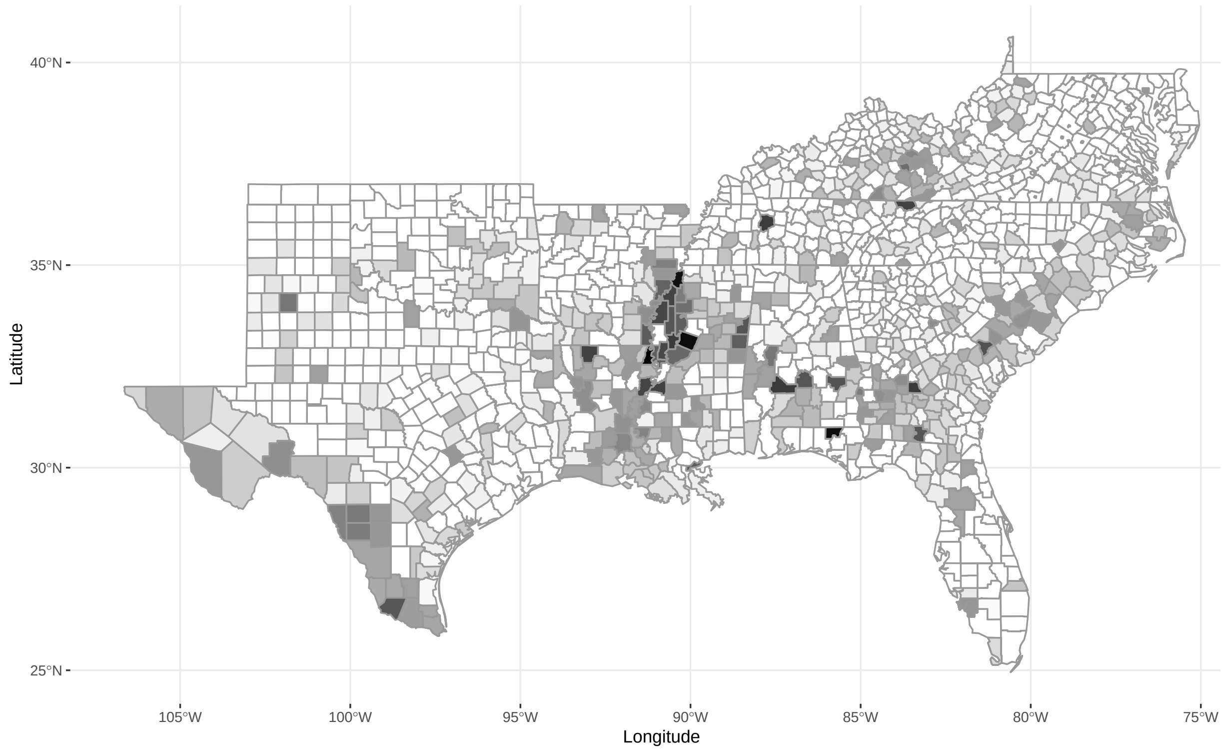

Figure 4.3: Choropleth plot of rape rate by county, darker counties indicate rates higher than the mean while lighter counties are lower.

unemployed but looking for work.

Table 4.1 provides the sample means ($\bar{x}$), standard deviations (σ), medians, and minimum and maximum observed values by variable. In the appendix Table A.19 provides a correlation matrix of the variables.

4.2.2 Factor structure

The first stage in latent variable modeling is to identify the measurement model. That is to identify which items share common variance in explaining latent factors (Bollen, 1989). However, when researchers have prior knowledge of the factor structure it is common practice to move straight to confirmatory analysis (SEM, for example; Brown, 2014).

Schreck et al. (2009) provide evidence suggesting violent and property crime are distinct factors. Evidence also suggests income inequality is a distinct factor as well (Deller & Deller, 2010; Rosenfeld et al., 2001). Therefore, I assume three distinct latent factors within the variables of

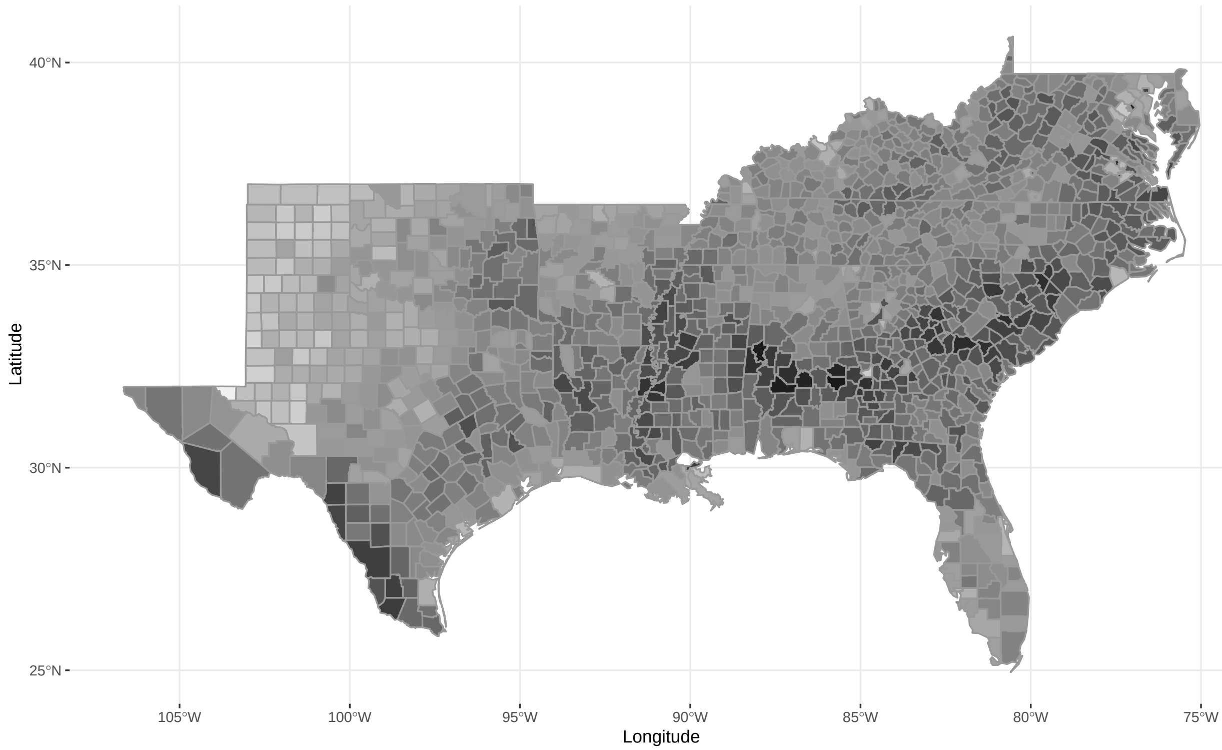

Figure 4.4: Choropleth plot of assault rate by county, darker counties indicate rates higher than the mean while lighter counties are lower.

interest; specifically, violent crime, property crime, and financial accessibility. Violent crime is comprised of homicide rate, aggravated assault rate, burglary rate, and rape rate. Property crime is comprised of larceny rate, vehicle theft rate, and robbery rate. Financial accessibility is comprised of Gini coefficient, average income, and unemployment rate.[1]

4.2.3 Specification of *W*

The *W* specification was selected to be a rook single order neighborhood matrix to reflect the spatial process of violent crime. This was done to align with the idea that crime is likely to spillover to neighboring counties more which share larger borders.

[1] Note that these (statistical) factors can be seen as composites. For the sake of simplicity and due to the illustrative character of this example, standard reflective measurement models were used and not formative factor models (see also the discussion and recommendation to use reflective measurement models in Howell et al., 2017).

Table 4.1: Extended US southern homicide data descriptive statistics for all 1412 counties.

Factor	Observed Variable	$\bar{x}$	σ	Median	Min.	Max.
Violent Crime						
	Rape Rate	3.40	0.33	3.44	2.41	4.33
	Burglary Rate	6.46	0.36	6.45	5.69	7.01
	Aggravated Rate	5.42	0.48	5.43	4.09	6.32
	Murder Rate	1.34	0.61	1.46	0.00	3.13
Property Crime						
	Larceny Rate	7.59	0.20	7.62	7.21	8.41
	Vehicle Theft Rate	5.30	0.48	5.37	4.25	6.73
	Robbery Rate	4.37	0.78	4.54	2.49	6.61
Financial Accessibility						
	Gini Coefficient	0.61	0.04	0.61	0.55	0.74
	Average Income	10.86	0.18	10.83	10.44	11.22
	Unemployment Rate	0.30	0.04	0.29	0.24	0.41

[1] $\bar{x}$ is the sample mean.
[2] σ is the sample standard deviation.
[3] Median is the sample median.
[4] Min. is the sample minimum.
[5] Max. is the sample maximum.
[6] All rates are expressed in occurrences by 100,000 people.

4.2.4 Model and prior specification

With spatially dependent endogenous observed items, the resulting latent factor will also be spatially dependent. The endogenous spatial lag model will be employed to account for the likely endogenous dependence and provide interpretations of spillover effects. The endogenous lag SASEM is specified to have the aforementioned observed items loading onto their respective factors. The single endogenous latent variable is Violent Crime. The exogenous latent variables are Property Crime and Financial Accessibility. A latent interaction effect is specified between Property Crime and Financial Accessibility. No cross loadings, residual correlations, or latent covariances are estimated. The marker variable approach was used to establish latent variable scaling. For the Violent Crime factor, homicide rate served as the marker. For property crime, robbery was selected. For financial accessibility, average income was used. The structural level equation is given by:

$$\eta_{\text{Violent}} = \alpha + \rho_{\text{Violent}} W \eta_{\text{Violent}} + \gamma_1 \xi_{\text{Property}} + \gamma_2 \xi_{\text{Financial}} + \gamma_3 \xi_{\text{Property}} \cdot \xi_{\text{Financial}} + \zeta \tag{4.1}$$

Regarding prior specifications, I follow the advice of Lee et al. (2007) for non spatial parameters and LeSage & Parent (2007) for the spatial parameter ρ_{Violent}. I use the following prior specifications defined earlier, with uninformative hyperparameters outlined in Section 3.2.

$$\begin{aligned}
\Phi &\sim \text{LK}_j(I_2, 2) \\
\sigma_{\varepsilon j} &\sim \text{Cauchy}(0, 2.5)^+, \quad \text{for} j = 1 \ldots 4 \\
\sigma_{\delta k} &\sim \text{Cauchy}(0, 2.5)^+, \quad \text{for} k = 1 \ldots 6 \\
\sigma_{\zeta} &\sim \text{Cauchy}(0, 2.5)^+ \\
\lambda_{yj} &\sim \text{Normal}(0, 1), \quad \text{for } j = 1 \ldots 3 \\
\lambda_{xk} &\sim \text{Normal}(0, 1), \quad \text{for } k = 1 \ldots 4 \\
\gamma_p &\sim \text{Normal}(0, 1), \quad \text{for } p = 1 \ldots 3 \\
\alpha &\sim \text{Normal}(0, 1) \\
\rho_{\text{Violent}} &\sim \text{Uniform}(0, 1)
\end{aligned} \tag{4.2}$$

R version 3.5.4 (R Core Team, 2019), STAN version 2.18.0 (Stan Development Team, 2018), and Rstan version 2.19.2 (Stan Development Team, 2019) were used to conduct the analysis. Four independent chains were specified. Chains ran for 4,000 iterations each, half of which was designated burn-in. All variables were standardized via a z-score transformation prior to analysis.

4.3 Results

Results for the analysis are given in Table 4.2. All parameters met the convergence criteria established by Vehtari et al. (2019) of $\hat{R} < 1.1$ and ESS > 20. All results are presented as 95% credible

Table 4.2: Result table for the endogenous lag SASEM

Parameter	$\bar{\bar{\theta}}$	2.5%	95.5%	ESS	$\hat{R}$
ρ_{Violent}	0.71	0.67	0.74	3599	1.00
α	0.02	-0.05	0.08	3360	1.00
γ_{Property}	0.29	0.20	0.41	3660	1.00
$\gamma_{\text{Financial}}$	0.41	0.33	0.45	2494	1.00
$\gamma_{\text{Prop.}\cdot\text{Fin.}}$	0.12	0.09	0.16	3751	1.00
λ_{Gini}	0.71	0.57	0.89	3892	1.01
$\lambda_{\text{Unemployment}}$	0.19	0.04	0.43	3597	1.00
λ_{Larceny}	0.32	-0.02	0.68	2994	1.01
λ_{Vehicle}	0.72	0.35	0.95	2457	1.00
$\lambda_{\text{Aggravated}}$	0.74	0.39	0.92	2633	1.00
$\lambda_{\text{Burglary}}$	0.66	0.32	0.92	3605	1.00
λ_{Rape}	0.76	0.43	0.98	3096	1.01
σ_{Income}	1.02	0.89	1.11	3946	1.00
σ_{Gini}	0.99	0.91	1.21	2969	1.01
$\sigma_{\text{Unemployment}}$	1.01	0.88	1.09	2998	1.01
σ_{Robbery}	1.00	0.93	1.06	2310	1.00
σ_{Larceny}	1.01	0.88	1.15	2755	1.01
σ_{Vehicle}	1.00	0.92	1.18	2384	1.00
σ_{Murder}	1.04	0.80	1.21	2918	1.00
$\sigma_{\text{Aggravated}}$	1.00	0.83	1.13	3102	1.00
σ_{Burglary}	1.03	0.94	1.15	3241	1.02
σ_{Rape}	0.98	0.89	1.08	2047	1.00
$\sigma_{\text{Financial}}$	1.01	0.95	1.09	3391	1.00
σ_{Property}	0.99	0.89	1.11	2883	1.00

[1] $\bar{\bar{\theta}}$ is the mean of the parameter estimate.
[2] 2.5% is the 2.5% percentile of the estimates posterior distribution.
[3] 95.5% is the 95.5% percentile of the estimates posterior distribution.
[4] ESS is the effective sample size.
[5] $\hat{R}$ the coefficients estimated $\hat{R}$ value for assessing convergence.
[6] Prop. $\cdot$ Fin. provides the interaction term of Property Crime and Financial Accessibility.

intervals. Factor loadings are all high and positive with the exception of Larceny (-0.02, 0.68). The spatial effect ρ_{Violent} is very high (0.67, 0.74). The latent structural coefficients are non-zero γ_{Property} (0.20, 0.41) and $\gamma_{\text{Financial}}$, including the latent interaction effect $\gamma_{\text{Prop.}\times\text{Fin.}}$ (0.09, 0.16).

Table 4.3: Spillover effect for structural effects.

	γ_{Property}	$\gamma_{\text{Financial}}$	$\gamma_{\text{Prop.}\cdot\text{Fin.}}$
Direct Impact	0.34	0.49	0.14
Indirect Impact	0.66	0.93	0.27
Total Impact	1.01	1.41	0.41

4.3.1 Model interpretation

4.3.1.1 Spillover effects

To interpret the structural effects, spillover must be computed for each structural effect. Direct, indirect, and total impacts are calculated by computing $\partial_y / \partial_x^{r'}$ for each structural estimate and taking the average of the off diagonal (indirect), and diagonal (direct) elements (see Section 2.2). Table 4.3 provides the direct, indirect, and total impact for each effect (i.e. spillover).

The direct impact for the property factor is 0.34. This means that a 1 standard deviation increase in the property factor in county i is associated with a 0.34 standard deviation expected increase in homicide rates in all $\neq i$ while controlling for financial accessibility at zero.

The indirect impact for the property factor is 0.66, which means a 1 standard deviation increase in all counties $\neq i$ would result in an expected 0.66 standard deviation increase in homicide rates in county i while controlling for financial accessibility at zero.

The total impact for the property crime rates is 1.01. This tells us that a 1 standard deviation increase in property crime rates in all counties correspond with an expected 1.01 standard deviation average increase in all counties.

4.3.1.2 Spillover marginal effects

Interpretation of the latent interaction effect is more complicated. To get a clear understanding of the interaction between property crime and financial accessibility, marginal slopes are computed. This process takes the point estimate of the structural slope values and calculates the slope at selected levels of the other variable. Fig. 4.5 shows the marginal slopes at, +1, -1, and 0 standard deviations from the mean of financial access. As financial accessibility increases, the relationship

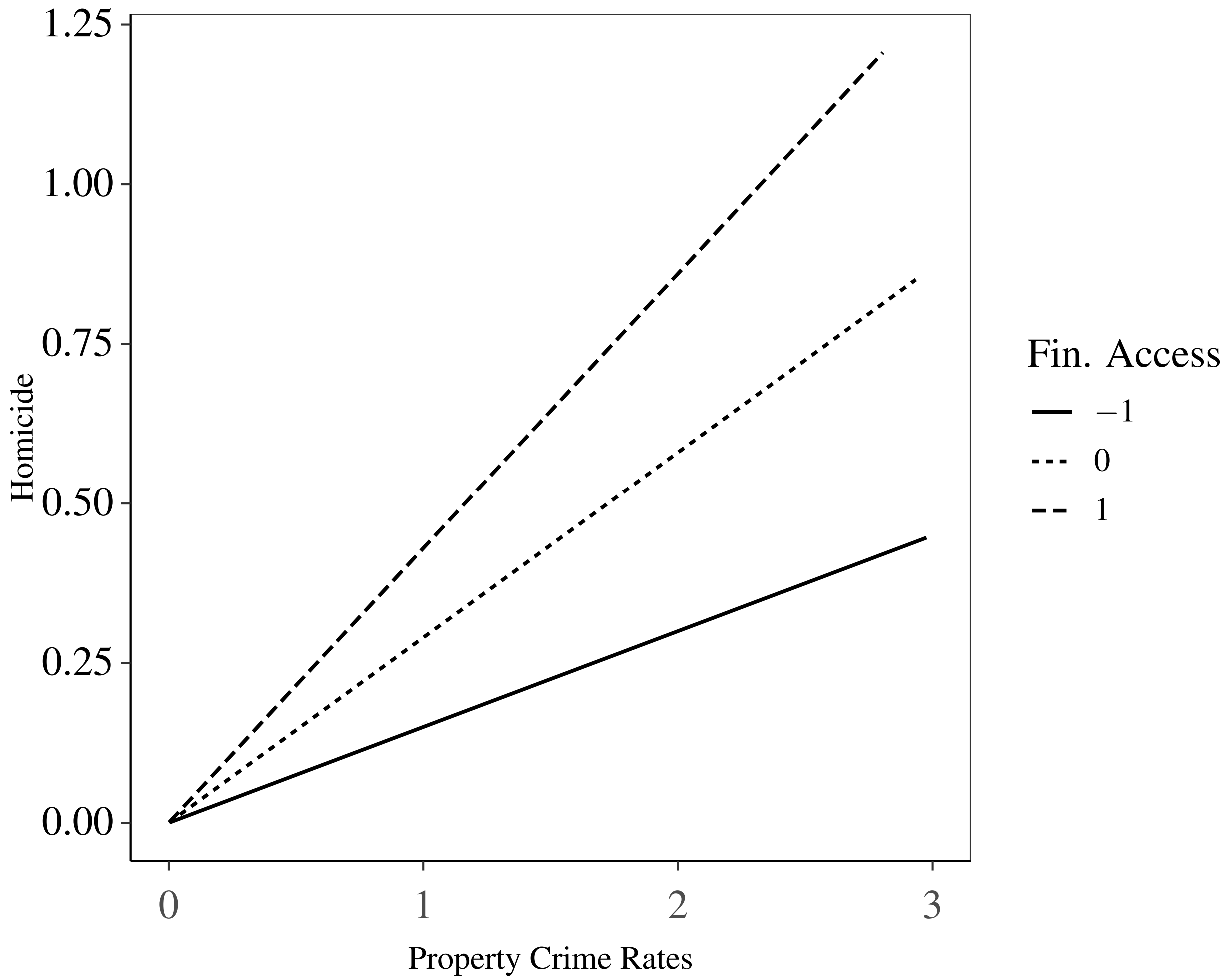

Figure 4.5: Simple slopes plot of financial inaccessibility at selected values of property crime.

between property crime rate and homicide rate increases. When financial accessibility is 1 standard deviation below the mean, the slope of property crime is .29 when financial accessibility is 0. When property crime rates are 1 standard deviation above the mean the slope of financial accessibility is 0.43.

I then use the marginal slopes to compute direct and indirect impacts. Table 4.4 provides the spillover effects for the marginal slopes. This step provides an understanding of the relationships these variables have with the spatial process.

The difference in the impacts provides a contrast of the magnitude of the spatial effect of property rates due to varying levels of financial accessibility. The direct impact explains that when financial accessibility is at a mean level, a 1 standard deviation increase in property crime rates in

Table 4.4: Spillover effect for the marginal slopes of property crime.

	$\gamma_{\text{Property}} = -1$	$\gamma_{\text{Property}} = 0$	$\gamma_{\text{Property}} = 1$
Direct Impact	0.17	0.34	0.51
Indirect Impact	0.33	0.65	0.97
Total Impact	0.51	1.01	1.48

county i corresponds with an expected 0.34 unit increase in homicide rates in all $\neq i$ counties.

In practical settings, it may be of interest to inspect the spillover effects of the observed items themselves. This is accomplished by multiplying the factor loading by the impact. The direct impact when the financial accessibility factor is 1 standard deviation above the mean, a 1 standard deviation increase in property crime rate in one county is associated with an expected $\lambda_{\text{Aggravated}} \cdot 0.51 = 0.74 \cdot 0.51 = 0.38$ standard deviation increase in aggravated assault.

4.4 Conclusions

The results of the analysis provide supporting evidence for all of the research questions:

1. Does a county's financial accessibility and property crime rates predict violent crime rates?

The results suggest both property crime rates $\gamma_{\text{Property}}[.29, .41]$ and financial accessibility $\gamma_{\text{Financial}}[.33, .45]$ have a positive relationship with homicide rates.

2. In counties with low financial accessibility is the relationship stronger between property crime and violent crime compared to high accessibility?

The results suggest yes. The latent interaction effect between financial accessibility and property crime is positive $\gamma_{\text{Prop.}\cdot\text{Fin.}}[.09, .16]$. Counties with higher financial accessibility exhibit a higher relationship between property crime rates and homicide rates.

3. Is violent crime positively spatially related in states nearer to one another, so that they have more similar violent crime rates compared to further states?

Again, the results suggest yes. The structural spatial effect was high and positive $\rho_{Violent}[0.67, 0.74]$.

All research questions were found to have supporting evidence. The strong positive spatial relationship suggests that homicide rates in part are strongly influenced by neighboring homicide rates. The positive interaction effect between financial accessibility and property crime rates suggest that property crime has a higher association with violent crime in regions where financial accessibility is higher. This relationship is moderated again by the spatial dimension.

Chapter 5

Discussion

In this dissertation, I presented a novel statistical model (SASEM) for analyzing spatially/network dependent latent constructs with latent level interaction effects. I presented a series of Monte-Carlo studies, which investigated different parameterizations of the SASEM. Study 1 investigated the impacts of ignoring dependent data. Study 2 explored the model performance under correctly specified situations. Study 3 investigated the effects on model performance of different weight matrix specifications. Study 4 explored model performance under different data generating scenarios. Finally, I provided an empirical example of the use of the endogenous lag SASEM with the extended US south homicide data Messner et al. (1999); Land et al. (1990). The empirical example provided examples of the rich interpretations which the SASEM model provides.

5.1 Monte Carlo study

The main results of the Monte-Carlo studies were that particularly one of the sub-models exhibits acceptable performance for applied uses across a wide range of different data conditions. The structural endogenous lag model A3 exhibited unbiased estimates of both spatial and non-spatial parameters under properly specified conditions. Under high sample sizes, model A3 was also robust to misspecifications of spatial parameters. In contrast, the measurement lag model A2 showed good performance for non-spatial parameters, but poor performance for spatial parameter estimates. The simultaneous structural lag model A4 also showed acceptable performance of non-spatial parameters, but poor performance of spatial parameter estimates.

In this section, the performance of each model is discussed in more detail. I then tie these results to existing literature. I provide recommendations for the use of the SASEM in applied settings. Finally, I discuss limitations and future directions.

5.1.1 Study 1

Study 1 provided an understanding of the impact of ignoring dependent data in a latent variable model. The results suggested that SEMs are more robust to ignored spatial dependency than anticipated. Estimates for typical parameters of interest, such as latent variable regression coefficients (γ) and factor loadings (λ), were mostly robust to omitted structural endogenous, disturbance, and endogenous measurement level dependent effects. In low sample sizes bias rose in these estimates; increasing sample sizes mitigated this effect almost completely.

5.1.1.1 Spatial Measurement Lag Model (D2)

When dependence was present at the population measurement level of item y_2 (D2) but was ignored (A1), only the intercept ν_{y3} and residual variance σ_{y3} exhibited decreased coverage rates and increased bias. In the low sample size condition bias and decreased coverage rates were observed for ν_{y3} and σ_{y3}. Increasing sample size mitigated this effect, but not entirely. The omission related to the prediction of y_2 by η; however, the resulting bias is on the intercept and residual variance of item y_3. This implies that the omitted spatial dependence produced bias in other parts of the measurement model; here, it produced an endogenous variable η which in turn was spatially dependent, thus biasing estimates of the relationship between item y_3 and η.

This finding is different to prior research for regression analysis, which assert that slope estimates are biased when spatial dependence is ignored (Anselin, 1988b). Here, the expectation that the factor loading λ_{y2} is affected (which is the slope parameter for the relationship between η and y_2) was not confirmed. Literature on spatial effects in SEM has not explored the impact of omitted dependent effects at the measurement level. This study provided first evidence suggesting the route of bias may differ in SEM and regression.

5.1.1.2 Endogenous Lag Model (D3)

When dependence was specified as an endogenous structural lag, only α exhibited poor performance as a function of the magnitude of the omitted dependence. When omitted dependence was increased, coverage rates for α decreased while bias was unaffected. This suggests that estimates of the posterior distribution of α have less variation, but estimate an improper solution.

This finding defies expectations set forth by Anselin (1988b), who stated misspecification of outcome variable dependence in a regression context resulted in biased slope and intercept estimates (see above). If this were to hold true with the SASEM, the regression coefficients γ_1, γ_2, and γ_3 estimates would exhibit increased bias when population level $\rho_\eta \neq 0$. However, it is in line with the findings for the omission of spatial dependence in the measurement model (above) that resulted in biased intercepts. The decreased coverage rates of the intercept were anticipated and are in-line with the assertions of Anselin (1988b), who suggested estimates of the intercept become less stable under omitted dependent effects. Regarding the affected structural intercept, applied users of SEM frequently do not interpret α, as they are often constrained to 0 from latent variable scaling techniques Bollen (1989).

5.1.1.3 Simultaneous Structural Lag Model (D4)

When dependence was specified as simultaneous endogenous and disturbance lags at the structural level (D4) but ignored (A1) α γ_1, γ_2, and γ_3 exhibited increased bias and/or decreased coverage rates. Structural slope estimates were unbiased and exhibited acceptable coverage rates when $\rho_\eta > 0$ and $\phi_\zeta > 0$, but not when $\rho_\eta > 0$ and $\phi_\zeta = 0$. This suggests structural slope estimates are robust to omitted endogenous dependence ($\rho_\eta \neq 0$) when the assumption of $i.i.d.$ disturbance term is met. When the disturbance term is not $i.i.d.$ ($\phi_\zeta \neq 0$), structural estimates are sensitive to omitted endogenous dependence ($\rho_\eta \neq 0$).

These findings supports the work of Anselin (1988b), who stated misspecification in a regression context results in biased slope and intercept estimates. Further, work by Pace & LeSage (2008) explains simultaneous omitted dependence in both the outcome (endogenous variable(s) in SEM) and disturbance term is the most severe case for inducing bias in intercept and slope estimates. Model D4 in Study 1 corroborates these findings.

5.1.1.4 Summary

Study 1 aimed to provide insights into the impact of dependent omitted variables on the novel SASEM. To provide a better understanding, three routes of spatial dependence were established: measurement level endogenous dependence (D2); structural endogenous dependence (D3); and simultaneous structural endogenous and disturbance dependence (D4). Contrary to prior research, increased bias and decreased coverage rates in all structural estimates were only observed under omitted simultaneous endogenous and disturbance dependence (Anselin, 1988b; Pace & LeSage, 2008).

5.1.2 Study 2

Study 2 tested the performance of the SASEM sub-model interaction effect and spatial autoregressive coefficients for correctly specified models. The primary finding was that the endogenous structural lag model performed well by accurately recovering both aforementioned effects, even in the small sample size condition. All parameters converged at very high rates, were unbiased, and exhibited acceptable coverage rates. For both the measurement lag (A2) and simultaneous lag model (A4), interaction effects and other non-spatial coefficients performed well. However, the spatial parameters in A2 and A4 (ρ_{y2}, ρ_{η}, & ϕ_{ζ}) were consistently biased with low coverage rates.

In Section 2.2 I discuss the additional information obtained by interpreting model coefficients via *Spillover* effects. Interpretation of structural slopes in the presence of non-zero ρ_{η} estimates requires calculation of *Direct* and *Indirect Spillover* effects to properly interpret structural coefficients. This is the primary benefit of dependent modeling in applied situations. This means that if ρ_{η} estimates are biased, the resulting interpretations of the structural slopes become biased (LeSage & Pace, 2014a). Therefore, it is important that estimates of ρ_{η} are unbiased for structural slopes to be unbiased in practice,as unbiased estimates of the slopes themselves are not enough.

5.1.2.1 Spatial Measurement Lag Model (A2)

The measurement level lag model (A2) exhibited biased ρ_{y2} estimates and unbiased non-spatial parameters with one exception. A consistent trend of the A2 is to estimate $\rho_{y2} = 0.51$. This occurred consistently in the W_D^* conditions when population $\rho_{y2} > 0$, and in W_C^* when population $\rho_{y2} = 0.3$. This suggests the prior information is solely informing the estimate as a randomly sampled uniform distribution with bounds of 0 and 1, which results in a mean estimate of 0.5. Structural estimates are unbiased, even though ρ_{y2} estimates are. However, when population $\rho_\eta >$ 0, the estimates of σ_{y2} are negatively biased. In the same condition, ρ_{y2} estimates are positively biased. This suggests model A2 is erroneously explaining residual variance in item y_2 resulting in negatively biased σ_{y2}.

This finding supports the latent variable model specification of Lee et al. (2007) for Bayesian structural level interaction effects in SEM. However, the inability to recover ρ_{y2} is unexpected. This model should theoretically provide a means of accounting for specific item level auto-correlation effects. Further research is needed.

5.1.2.2 Endogenous Lag Model (A3)

The endogenous lag model exhibited optimal performance in estimating spatial and non-spatial parameters when n = 196 and greater. When the population spatial effect $\rho_\eta = 0$, ρ_η estimates were positively biased and exhibited only coverage rates of l0%. The bias was reduced with increased sample size and increased spatial dependence. When spatial effect was $\rho_\eta > 0$, its estimates were less biased. The coverage rate of 0% for the spatial coefficient of $\rho_\eta = 0$ resulted from the prior specification with a lower boundary of 0 [1]. Structural slope estimates were unbiased under all conditions. The latent interaction had the highest observed bias but was still well within the acceptable range, even in the n = 49 condition.

These findings provide evidence to support the use of the SASEM model with structural en-

[1] Post-hoc small scale simulation results of the endogenous lag model with $\rho_\eta \sim Uniform(-1,1)$ reveal high coverage rates under the population $\rho_\eta = 0$ condition.

dogenous lags. Accurate recovery of the latent interaction effects supported the model specifications and prior distributions set forth by Lee et al. (2007). Accurate ρ_η estimates supported the spatial term and prior specification recommendations by LeSage & Parent (2007) and Stakhovych et al. (2012). However, the prior specification for $\rho \sim Uniform(0, -1)$ made accurately covering an estimate of 0 difficult; a uniform distribution with a lower boundary of -1 presumably would have improved model estimation under this condition.

Further, the results support work by Stakhovych & Bijmolt (2009), who in a regression framework establish that high sample sizes are necessary to accurately recover spatial estimates at or near 0. The SASEM with endogenous lag is the latent variable equivalents of the commonly utilized network auto-correlation model (Palla et al., 2007), also known as SAR in econometrics (Paelinck & Klaassen, 1979). The parameterization of the SASEM model has great potential for application in applied work and is a logical next step for econometrics and social network modeling.

5.1.2.3 Simultaneous Structural Lag Model (A4)

The simultaneous structural lag model exhibited poor performance recovering population coefficients, including both auto-correlation and latent interaction effects.

The spatial parameters ρ_η and ϕ_ζ exhibited strong bias but high coverage rates in population conditions in which $\rho_\eta > 0$. This suggests the estimated posterior distributions for these coefficients have biased mean estimates and they are erroneously wide (high coverage). Structural slope estimates exhibited sensitivity to structural level disturbance dependence, resulting in high bias in the low sample size condition. The structural slopes γ_1, γ_2, and γ_3 were consistently positively biased in n = 49 conditions. However, in the high sample size condition, structural estimates were non-biased even though auto-correlation estimates for ρ_η and ϕ_ζ were.

Structural slope estimates are unbiased; however, the interpretation of them will be inaccurate because ρ_η is biased. This is due to the aforementioned issue associated with the calculation of spillover effects. When an endogenous dependent effect (ρ_η) is biased, the spillover calculation will bias the interpretation of the structural slopes.

Again, this finding supports the model and prior specification set forth by Lee et al. (2007). They also support the work of LeSage (2014), who suggests simultaneous disturbance and endogenous lags may provide *inefficient* estimates. LeSage (2014) uses *inefficiency* in a frequentist context, which refers to a model's inability to estimate test statistics. In a Bayesian context, inefficient estimates are those which have estimated erroneously wide posterior distributions. The results of Study 2 also support the work by Pace & LeSage (2008), who explains simultaneous omitted dependence in both the endogenous variable and disturbance term is the most severe case for inducing bias in intercept and slope estimates. In the majority of conditions model A4 underestimated the auto-regressive effects, thus not controlling for the dependence and in turn resulting in highly biased structural estimates in the low sample size condition.

5.1.2.4 Summary

Study 2 investigated the performance of the three SASEM sub-models (A2, A3, and A4) in their ability to accurately estimate latent interaction and spatial effects. All three models accurately estimated the latent interaction effect and all non-spatial parameters. The measurement lag and simultaneous structural lag models showed an inability to recover the spatial effects ρ_{y2}, ρ_{η}, and ϕ_{ζ}, respectively. The endogenous structural lag model (A3) accurately estimated spatial effects (with the exception of the spatial parameter ρ_{η} when the population level effect was zero [$\rho_{\eta} = 0$]). However, this can be considered a symptom of a misspecified prior specifications $\rho_{\eta} \sim Uniform(0, -1)$; changing the lower bound to -1 presumably alleviates this issue. The successful performance of model A3 aligns with prior research which establishes a single dependent variable (endogenous in SEM) lag to perform similarly in a regression context (Paelinck & Klaassen, 1979; Palla et al., 2007).

5.1.3 Study 3

The goal of Study 3 was to advance research regarding the impact of model performance by varying specifications of W. This work has been conducted in regression contexts. Study 1 showed that the

SASEM should not be assumed to function the same as previous spatial research with regression models. The primary finding of Study 3 is that the properly specified SASEM model is sensitive to W specifications in that higher order connection specifications induce positive bias on spatial effects while non-spatial estimates remain systematically unaffected.

5.1.3.1 Spatial Measurement Lag Model (A2)

The relationship between connection condition and model performance in model A2 remained unclear. In Study 2, model A2 exhibited an inability to accurately estimate ρ_{y2}, resulting in consistently positive bias for its parameter estimate. Distance specifications of W exhibited slightly more bias than other conditions.

This supports literature which has established the need for high sample sizes when using saturated W specifications (Stakhovych & Bijmolt, 2009; LeSage & Pace, 2014a). This finding also suggests the A2 specification of the SASEM model does not provide accurate estimates.

5.1.3.2 Endogenous Lag Model (A3)

In model A3, the relationship between the number of connections in W and model performance was clear. Higher connection conditions were consistently more biased regarding estimates of ρ_{η}. Other parameters were not systematically effected by the condition.

This finding corroborates work which established that as the saturation of W increases, the associated spatial estimates require higher sample sizes to accurately reflect the true value (Stakhovych & Bijmolt, 2009; LeSage & Pace, 2014a). Prior research also suggested that higher order neighbor specifications can be detrimental to both the estimation of spatial effects to an interpretation that becomes redundant (Anselin, 2003; LeSage & Pace, 2014a). Earlier I discussed the process of developing W_C. In addition literature suggests using theory to guide the selection of *Rook* or *Queen* contiguity, as well as *first*, *second*, or higher order neighbor specifications. In section 2.2, I discussed the interpretation of parameter estimates in spatial modeling. Recall that under a simple *Rook First Order Neighbor* specification of W "the spatial lag is expressed as a near infinite process

$I_n + \rho W + \rho^2 W^2 + \rho^3 W^3 + ...$". In practice this means higher order neighbor specifications of W_C imply direct and indirect effects of a case on other cases specified as neighbors. This redundancy can lead to undue complexity in interpretation of parameter estimates and requires higher sample sizes for accurate reflections of the parameters.

5.1.3.3 Simultaneous Structural Lag Model (A4)

Model A4 exhibited biased estimates for ρ_η and ϕ_ζ, regardless of the actual W specification. The W_D^* specification exhibited the strongest bias regarding auto-regressive estimates. Spatial parameters exhibited both strong bias but high coverage rates, again suggesting wide posterior estimates of those parameters (cf. Study 2). Non-spatial parameters did not exhibit variation in bias or coverage as a function of W specification.

This finding supports literature which has established the need for high sample sizes when using saturated W specifications (Stakhovych & Bijmolt, 2009; LeSage & Pace, 2014a). Again, this supports evidence suggesting simultaneous disturbance and endogenous lags are *inefficient* and thus often not of practical use in applied data analysis with smaller sample sizes (LeSage, 2014).

5.1.3.4 Summary

Results of Study 3 suggest parsimonious representations of W result in more accurate and precise estimates. This supports existing work in regression contexts (Stakhovych & Bijmolt, 2009). Study 3 results also suggest increased sample sizes are needed to accurately measure complex spatial processes with many connections. Work by LeSage & Pace (2014a) supports this conclusion.

5.1.4 Study 4

The goal of Study 4 was to investigate the ability of the SASEM sub-models to account for dependence under misspecification of the dependent effect. The primary finding is that non-spatial parameters are robust to misspecification of the spatial effect. The spatial parameters are sensitive to misspecification.

5.1.4.1 Spatial Measurement Lag Model (A2)

The measurement lag model (A2) spatial parameter ρ_{y2} is consistently biased across studies. The non spatial parameters are not and are robust to misspecification.

D3 and D4 population model In population model D3 and D4, analysis model A2 ρ_{y2} estimates consistently deviated from the population value of 0. Results of Study 4 resemble Study 2 and 3, in that the ρ_{y2} estimates were consistently ≈ 0.50. Non-spatial parameters, with one exception, were robust to the structural endogenous spatial misspecification and exhibited consistently unbiased estimates, even under n = 49. In both population models the residual variance σ_{y3} exhibited increased bias as the magnitude of the omitted population ρ_{η} value increased. This suggests the unstable erroneous estimates of ρ_{y2} explain item level error variance, resulting in biased estimates of residual variance in item $y3$.

Routes of dependence differ and thus endogenous lags ρ_{η} are not suited for measuring disturbance dependence ϕ_{ζ}. This supports research by Anselin (1986) and Stakhovych et al. (2012) who posit biased estimates of disturbance variance and spatial parameters when estimating endogenous lags for the analysis of disturbance dependence.

5.1.4.2 Endogenous Lag Model (A3)

The endogenous lag model (A3) spatial parameters are sensitive to omitted structural disturbance dependence, whereas non-spatial parameters are not. Again, we must consider the implication of biased ρ_{η} estimates on the interpretation of structural slopes. While structural slope estimates themselves are unbiased, the presence of biased ρ_{η} estimates bias the interpretation of slope estimates.

D2 Under population model D2, analysis model A3 exhibits unbiased non-spatial parameters but biased ρ_η estimates and decreased coverage of the item level intercept ν_{y2}. Bias for ρ_η was calculated as the estimates deviation from 0, but is likely not a transparent representation of accuracy. D2 includes spatially dependent observed items in the production of η resulting in an unknown magnitude of dependence in η. ρ_η estimates may be a relatively accurate reflection of the spatial dependence in η from the spatially dependent measurement term $\rho_{y2}(Wy_2)$. This consideration is supported by the finding that ρ_η estimates are less biased under the $\rho_{y2} = 0$ condition.

D4 Under population model D4, analysis model A3 exhibits unbiased non-spatial parameters but biased ρ_η estimates. Non-spatial estimates are consistently unbiased under sample sizes of n = 196 and greater, while the ρ_η estimates are consistently biased under population level $\phi_\zeta > 0$.

These finding suggests ρ_η is not robust to omitted disturbance dependence, while non-spatial parameters are. This supports research which asserts endogenous lags are unsuited for measuring and controlling for disturbance dependence (Stakhovych et al., 2012; Anselin, 1986).

5.1.4.3 Simultaneous Structural Lag Model (A4)

D2 and D3 In both population models A4 estimates of the spatial parameters ρ_η and ϕ_ζ were biased and exhibited below average coverage while non-spatial estimates were not. Under population model D2, A4 consistently overestimates ρ_η and ϕ_ζ Following the trends of other studies under population model D3 analysis model A4 exhibits unbiased non-spatial estimates and biased spatial estimates with low coverage. ϕ_ζ estimates vary more from sample size conditions than population level spatial dependence. This results in ϕ_ζ exhibiting below average convergence and coverage rates, as well as high bias. ρ_η shows more sensitivity to population level dependence than ϕ_ζ. Similar to analysis model A3 ρ_η estimates are most accurate when the population dependence parameter is > 0 in the n = 400 condition only exhibiting bias of -5.60%. Non-spatial parameters are consistently unbiased with acceptable coverage rates.

These findings once again support the latent variable specifications established by Lee et al.

(2007) and the cautionary work by LeSage (2014) regarding simultaneous dependent lags.

5.1.4.4 Summary

The primary finding of Study 4 is that the endogenous structural lag model A3 cannot accurately estimate ρ_η in the presence of omitted disturbance dependence. It remains unclear if ρ_η estimates under model A3 are biased when population level dependence is present at the item level (D2). This is due to the confounding effect of the item level dependence on the production of the endogenous factor η. Non-spatial estimates are unbiased with favourable coverage rates, even under biased ρ_η estimates. Models A2 and A4 both exhibit consistently biased estimates of spatial parameters regardless of the population model. Non-spatial parameters are unbiased for all models in all conditions once again supporting the SEM specification set forth by Lee et al. (2007). In situations where slopes are unbiased but ρ_η is biased, we must consider the erroneous interpretations induced from the calculation of *Spillover Effects* (LeSage & Pace, 2014a).

5.2 Empirical Example

Analysis of the homicide data was designed to provide an example of the rich interpretations of coefficients provided by the SASEM. Interpretations of spillover effects are a major advantage of spatial models. The homicide data exhibited a non-zero latent interaction effect, which allows for the computation of marginal slopes. Together marginal slopes and spillover effects provide a deeper understanding of the relationships between latent constructs and dependence processes.

5.3 Practical Recommendations

The SASEM exhibits acceptable model performance under the structural endogenous lag model A3, but some practical considerations should be made. Each Monte-Carlo study contributes practical implications for the use of the SASEM with structural endogenous lags. Model A2 and A4 in the specifications presented here should not be used in applied work due to the inability to recover spatial parameters; thus, their analysis models will not be discussed further in this section.

5.3.1 Omission

Study 1 provides guidance on the impact of omitting dependence of different forms in SEMs. A promising finding is that under solely endogenous structural dependence typical SEM parameters exhibit robustness to omitted endogenous dependence. Researchers who have strong theoretical reasons to believe dependence is only present in the outcome variable may use the endogenous lag model solely for the additional information provided by the framework as opposed to employing it purely for statistical control.

5.3.2 Model and prior specifications

A combination of models and research were integrated to produce the SASEM. Work by Lee et al. (2007) provided the model and prior specification of Bayesian SEM capable of estimating latent interaction effects. Oud & Folmer (2008) provided an approach to estimating latent structural spatial effects with observed exogenous variables. Stakhovych et al. (2012) and LeSage & Parent (2007) provided prior specifications for the spatial parameters. Study 2 provides supporting evidence for each of these specifications with some additional considerations. Regarding the spatial effect of the specifications tested, only model A3 showed consistently accurate estimates on all parameters. Therefore, the SASEM should only be implemented with a single structural endogenous lag without more testing.

One specific condition and prior selection needs further attention. Under model A3 accurate recovery of population $\rho_\eta = 0$ resulted in 0% coverage. This was a symptom of the boundary of the prior distribution specified for ρ_η of $\sim Uniform(0,1)$. Therefore, I recommend using a lower bound of 0 if strong theoretical information suggests a non-zero spatial effect.

5.3.3 Sample size

The appropriate sample size for a traditional latent variable model has been a topic of debate (Westland, 2015). In part, this is a result of the degree of variation in model specification. Each scenario is very different and many factors influence sample size requirements, such as the number of parameters, effect sizes, the number of observed items and latent factors, and the loading structure (Wolf et al., 2013; Westland, 2010; Fan et al., 1999). However, a spatial adaptation such as the SASEM has not been explored fully. Each model should be taken in context; specifically, the number of observed items, factors, and the smallest anticipated effect of interest will all govern sample size considerations. In Study 2 when specified with W_C^* model A3 was consistently unbiased regarding both spatial and non-spatial parameters, including the latent interaction effect at n = 196 and greater. However, W_D^* exhibited more bias and lower coverage under the same circumstances. This corroborates work by Stakhovych & Bijmolt (2009) which posits the need for high sample sizes when using saturated W specifications. Therefore, the following will increase the sample size necessary to accurately estimate the endogenous structural lag SASEM: increased number of observed items; increased number of factors; small structural or measurement level effects of interest; small spatial effects of interest; and higher saturation of W.

5.3.4 W specification

Literature asserts several factors which must be considered when choosing a W specification. Study 3 provides additional considerations (LeSage & Pace, 2014a; Stakhovych & Bijmolt, 2009). First, the choice of W specification will be guided by theory (LeSage & Pace, 2014a). The goal of this choice is to match W with a "population W". That is, the dependence process at the population level has some unknown W which represent the dependence between cases. Study 3 shows that saturated representations of W require higher sample sizes to accurately recover the population value. This is a particular issue for social network modeling. In this setting, cases will have some degree of influence on most other cases, synonymous with the W_D^* condition. To accommodate this, using cutoff criteria W to produce a parsimonious representation might be beneficial. The choice of cutoff value should be selected to align with a low value of "influence", effectively trimming the direct impact of "non-influential" connections. Further research is needed to test if the use of cutoffs improve estimation characteristics of SASEM.

5.3.5 Implications of spillover interpretations of structural estimates

In Section 2.2, I discussed the additional information obtained by interpreting model coefficients via *spillover* effects. Interpretation of structural slopes in the presence of non-zero ρ_η estimates requires calculation of direct and indirect impacts to properly interpret coefficients. This is the primary benefit of dependent modeling in applied situations. However, this means that when ρ_η estimates are biased, interpretation of the structural slopes are biased (LeSage & Pace, 2014a).

Study 4 provides insight on the consequences of model misspecification under different routes of dependence. The endogenous lag model A3 had difficulties accurately recovering ρ_η when the population model had un-modeled disturbance dependence. This resulted in biased ρ_η estimates, which in turn bias the interpretation of structural slope estimates. Therefore, it is important to use substantive theory to account for dependent omissions which would lead to disturbance dependence. If a researcher believes there may be dependent omitted variables the *instrumental variables* approach can be used to diminish the disturbance dependence. This is more practical in network than econometric settings with geographic regions. In short, the *instrumental variables* approach calls for the addition of covariates, which are correlated with the predictors but uncorrelated with the error term (Angrist & Krueger, 2001). For a more in depth discussion of the general instrumental variable approaches, see Angrist & Krueger (2001). For a more detailed explanation of the approaches used to diminish disturbance dependence in a spatial setting, see Stoica et al. (1994).

5.4 Limitations

5.4.1 Monte-Carlo studies

The Monte-Carlo studies were designed to provide a baseline of understanding for the novel SASEM. As a new model, it was important to investigate its performance in practical situations. Therefore, I used applied research to identify reasonable sample size conditions, W specifications, and misspecification conditions which mirror applied situations. Nonetheless, I recognize the implications of the study are limited by the conditions and how they were assessed, as in any simulation study.

Latent variable models have a limitless number of potential specifications. The number of observed and latent variables, the loading structure, the structural relationships, and magnitude of effects all vary between situations. In addition, the SASEM includes spatial effects which also have a vast number of potential specifications. Specifically, the geographic or network structure, location and type of dependent lag, and specification of W will also differ. Together these variations make identifying an "normal" circumstance difficult. Bayesian latent variable modeling approaches are computationally expensive and time consuming; thus, must to be considered when choosing simulation conditions.

As this is a novel model, I attempted to manipulate the conditions which were most important to understanding the baseline SASEM model performance in practical situations. Nonetheless, there were limitations of the studies and the claims made. Sample size impacted computational wall time the most causing higher sample sizes were not explored. Thus, it is unknown whether model A4 is capable of accurately recovering spatial parameters at higher sample sizes. Population level dependence was explored in only three conditions (0, 0.3, and 0.6). Negative spatial dependence is uncommon in applied settings but exists nonetheless. I solely explore a balanced square geographic representation of the dependence process. While less practical, this was chosen to provide control over the W specification. The impact of irregularly shaped geographic (dependence) relationships was not explored. Alternative specifications of the latent variable model were also not considered.

It may be the case that additional observed items may provide the information necessary for model A2 to accurately recover measurement level spatial effects. In addition, latent polynomial effects were not explored. Following the research of Lee et al. (2007), I anticipate that latent polynomial effects such as quadratic effects in Bayesian SEM will exhibit comparable performance to that of the latent interaction effect explored (also see Kelava et al., 2014).

Finally, the measurements of performance employed restrict the conclusions. Biased ρ_η estimates result in biased interpretations of structural slopes. However, I did not explore this in this initial assessment of the SASEM. To measure the impact of biased ρ_η estimates on interpretations of γ_1, γ_2 , and γ_3 fitted values can be computed, compared to population values, and summarized (Stakhovych & Bijmolt, 2009).

5.4.2 Empirical example

The empirical example makes several assumptions about the data which limit confidence in the results. I assume the data are stationary in time regarding all effects in the model. This is a general problem for cross-sectional designs. For predictions to be accurate in cross-sectional work it is important that the relationships measured are reasonably stationary in time. This was not verified in the empirical example and is thus limits generalizations about the conclusions. Only one W specification was implemented. To align with the phenomena, a rook first order neighborhood matrix specification was chosen. This was done to align with the concept that crime is likely to spillover to neighboring counties more which share larger borders. The choice to not explore other potential specifications of W assumes the spatial process follows this logic.

5.4.3 SASEM

The SASEM model is limited in its application in several ways. First and foremost, simultaneous structural lags and measurement level effects did not exhibit acceptable model performance. Thus, these specifications of the SASEM need further investigation.

First, the endogenous structural lag model showed acceptable performance under properly specified situations but failed to estimate ρ_η accurately in the presence of omitted disturbance dependence. This means theory and instrumental variable approaches must be used to diminish the impact of omitted spatially-dependent variables. As discussed above, the valid application of this models using instrumental variable approaches relies on the correct choice of the set of instrumental variables. I did not investigate this aspect of modeling strategy (e.g., if it is necessary to include all relevant instrumental variables or if a subset is sufficient). This would have exceeded what could feasibly be presented in this dissertation.

Second, a limitation of auto-regressive modeling for both econometric and network applications concerns the use of saturated W specifications. Theory may suggest that each case directly impacts all other cases and indirectly impacts all other cases through impacts on other cases. A fully saturated W matrix such as W_D^* reflected this in Simulation Study 3 and requires a much larger sample sizes to accurately recover ρ_η, thus limiting its usefulness.

Third, the SASEM is a cross-sectional model aimed at measuring a situation at a single measurement occasion. As many researchers have pointed out, it is not plausible to believe all phenomena can be accurately predicted from a singular cross-section (Raleigh & Cioffi, 1998; Márquez et al., 2010; Holly et al., 2010). The SASEM model assumes that the spatial process as well as other relationships are *stationary* in time. [2]

Finally, the SASEM and many other spatial models assume the spatial process is uniform across the sample space. This is unreasonable in some circumstances. For example, it is reasonable to consider housing prices may exhibit different spatial properties in rural housing markets compared to metropolitan markets.

5.5 Future Research

The endogenous variant of the SASEM shows promising performance and should be explored further. First the limitations of the Monte-Carlo study should be addressed. For example, addi-

[2]This is an assumption of all cross-sectional analysis. *Stationarity* refers to the relationships between variables (γ, λ, and ρ_η) which do not change in time. However, the mean value of these variables may change without impact. For more information on stationarity see Hadri (2000).

tional sample sizes, negative and small effect sizes for spatial parameters, alternative latent variable specifications, and polynomial effects should be explored in more detail to provide more general recommendations.

The simultaneous structural lag model A4 and measurement lag model A2 should be explored in more detail. It may be the case that sample sizes or other situations may have lead to the inaccurate estimates observed in A2 and A4. Further research should explore these models in more detail.

With some changes the SASEM can be generalized to many more situations than described in this paper. First, in some situations it may be unrealistic to assume that a single scalar summary accurately measures the population level spatial process. The SASEM model could be extended to include a random effect of ρ_η at the structural level (Rabe-Hesketh et al., 2007) which allows variation in the magnitude of the effect of ρ_η across different clusters of spatial units (e.g., counties in states). Second, it could be reasonable to assume that spatial dependence varies across several known or unknown groups (e.g., rural vs metropolitan areas). Latent class analysis is a means to extend the SASEM to account for such heterogeneity in spatial dependency. Third, modern data collection methods (such as mobile phone apps) allow researchers to collect dynamic psychologically relevant data (e.g., attitudes, feelings, and cognitive performances) in field experiments with high external validity. At the same time, dynamic spatial information can be collected where does the participants respond to the questions or tasks. Extensions of the SASEM to account for dynamic changes in spatial dependency are necessary to model such data (e.g., using dynamic latent class structural equation models) (Asparouhov et al., 2017, 2018; Kelava & Brandt, 2019).

Finally, the SASEM model presented here was utilized with normal continuous exogenous and endogenous variables. In theory, the SASEM could be extended to a generalized approach which can accommodate endogenous variables that follow other distributions, like binary or count data. In fact, Song et al. (2013) developed a generalized Bayesian SEM framework which in theory could be extended to accommodate endogenous spatial lags.

5.6 Conclusion

The endogenous structural lag variant of the SASEM is a promising addition to the set of methods for analyzing multivariate spatially or socially dependent data. As network science grows, interest in dependent methodology has increased. Current approaches restrict the types of data that can be analyzed, as methods are typically constrained to univariate outcome variables. The endogenous lag SASEM provides a solution to this problem. Researchers in the social sciences frequently investigate complex relationships between three or more variables. The ability to simultaneously accommodate latent interaction effects with dependent spillover effects provides a means to match methodological approaches and rich interpretations to complex theory.

In applied settings the endogenous lag SASEM should be used with some considerations in mind. Parsimonious specifications of W should be used when possible. Estimates of ρ_η are biased in the presence of structural disturbance dependence, in turn biasing interpretations of γ. Instrumental variable approaches can be incorporated to reduce this threat. This problem plagues all spatial models and is not unique to the SASEM.

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Appendix A

Appendix

A.1 Study 1

Table A.1: Results table for Study 1 measurement lag population model (D2) and non-spatial analysis model (A1) conditions

				$\rho_{y2} = 0$			$\rho_{y2} = 0.3$			$\rho_{y2} = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
W_C^*												
	49											
		α	0.00	-0.01	-0.51	95.50	0.00	0.12	95.60	0.00	-0.07	94.97
		γ_1	0.30	0.32	6.98	96.22	0.32	7.38	96.11	0.32	8.02	96.10
		γ_2	0.30	0.32	6.84	97.03	0.32	6.27	96.63	0.32	7.57	96.62
		γ_3	0.15	0.16	8.24	95.81	0.16	9.16	96.42	0.16	8.65	96.21
		σ_{x_1}	0.50	0.51	1.16	93.66	0.51	1.14	94.99	0.51	1.05	93.95
		σ_{x_2}	0.50	0.51	1.77	95.30	0.50	0.92	94.27	0.50	0.77	95.59
		σ_{x_3}	0.50	0.51	1.68	95.60	0.50	0.03	94.27	0.51	1.33	95.18
		σ_{x_4}	0.50	0.51	2.55	93.46	0.51	1.62	94.79	0.51	1.96	95.90
		σ_{x_5}	0.50	0.50	-0.28	94.99	0.51	1.07	94.68	0.50	0.71	94.67
		σ_{x_6}	0.50	0.51	2.03	94.99	0.50	0.81	94.48	0.51	1.04	95.38
		σ_{y_1}	0.50	0.49	-1.68	95.19	0.49	-1.97	95.81	0.49	-1.87	94.26
		σ_{y_2}	0.50	0.50	0.14	95.09	0.49	-1.01	94.89	0.50	0.82	95.79
		σ_{y_3}	0.50	0.50	0.35	94.79	0.51	1.26	94.68	0.52	4.35	92.10
		σ_{ξ_1}	1.00	1.01	1.36	94.48	1.01	0.96	94.79	1.01	0.84	95.90
		σ_{ξ_2}	1.00	1.01	1.22	93.97	1.00	0.30	96.32	1.01	0.91	94.15
		ν_{x_1}	0.00	0.00	-0.45	96.11	0.00	0.09	94.99	-0.01	-0.93	94.87
		ν_{x_2}	0.00	-0.01	-0.54	95.81	0.00	-0.50	93.56	-0.01	-0.86	94.26
		ν_{x_3}	0.00	0.00	-0.21	96.42	0.00	0.16	95.19	0.00	-0.44	94.87
		ν_{x_4}	0.00	0.01	0.71	94.79	0.01	1.08	95.50	0.00	0.28	94.46
		ν_{x_5}	0.00	0.01	0.76	94.89	0.01	0.87	97.44	0.00	0.01	95.59
		ν_{x_6}	0.00	0.01	0.64	94.89	0.01	1.35	96.01	0.00	0.34	96.21
		ν_{y_2}	0.00	0.00	0.26	94.17	0.00	0.18	95.30	0.00	-0.35	94.36
		ν_{y_3}	0.00	0.01	0.53	93.56	0.00	-0.05	88.75	0.00	-0.30	86.77
		λ_{x_2}	1.00	1.01	1.37	94.27	1.02	2.06	95.81	1.03	2.92	96.31
		λ_{x_3}	1.00	1.02	1.58	94.38	1.02	2.40	95.91	1.03	3.12	95.59
		λ_{x_4}	1.00	1.02	2.12	94.48	1.03	2.78	95.71	1.02	2.41	95.79
		λ_{x_5}	1.00	1.02	1.70	96.52	1.03	2.57	94.79	1.02	2.24	95.18
		λ_{y_2}	1.00	0.92	-8.30	94.17	0.94	-6.49	93.46	0.92	-7.60	93.03
		λ_{y_3}	1.00	0.91	-9.01	91.72	0.92	-7.69	93.15	0.91	-9.00	94.56
	196											
		α	0.00	0.00	0.01	94.39	0.00	-0.24	93.94	0.00	-0.48	96.75
		γ_1	0.30	0.30	1.46	95.69	0.30	1.33	94.91	0.30	1.55	96.75
		γ_2	0.30	0.30	1.63	93.74	0.31	2.00	93.07	0.30	0.72	96.75

Table A.1: Results table for Study 1 Measurement lag population model (D2) and non-spatial analysis model (A1) *(continued)*

				$\rho_{y2} = 0$			$\rho_{y2} = 0.3$			$\rho_{y2} = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
		γ_3	0.15	0.15	2.72	96.44	0.15	2.27	95.02	0.15	1.93	95.56
		σ_{x_1}	0.50	0.50	0.17	95.58	0.50	0.41	94.16	0.50	0.38	93.93
		σ_{x_2}	0.50	0.50	0.45	93.42	0.50	-0.01	94.26	0.50	-0.28	94.58
		σ_{x_3}	0.50	0.50	0.19	94.82	0.50	0.44	95.35	0.50	0.39	92.42
		σ_{x_4}	0.50	0.50	0.77	93.64	0.50	0.62	96.10	0.50	0.33	93.50
		σ_{x_5}	0.50	0.50	0.71	94.39	0.50	0.24	93.18	0.50	0.26	95.77
		σ_{x_6}	0.50	0.50	-0.81	95.15	0.50	0.53	95.02	0.50	0.87	94.26
		σ_{y_1}	0.50	0.50	-0.65	95.04	0.49	-1.25	95.24	0.49	-1.24	95.23
		σ_{y_2}	0.50	0.50	-0.48	96.01	0.50	-0.31	95.13	0.50	-0.69	95.23
		σ_{y_3}	0.50	0.49	-1.13	95.58	0.50	0.51	93.18	0.52	3.92	85.81
		σ_{ξ_1}	1.00	1.00	-0.06	94.93	1.00	0.25	96.32	1.00	0.43	96.21
		σ_{ξ_2}	1.00	1.00	0.23	94.82	1.00	0.50	95.45	1.00	0.33	95.02
		ν_{x_1}	0.00	0.00	0.14	95.58	0.01	0.76	93.72	0.00	-0.32	94.91
		ν_{x_2}	0.00	0.00	0.30	95.36	0.00	0.49	92.32	0.00	-0.27	92.96
		ν_{x_3}	0.00	0.01	0.57	94.61	0.00	0.38	92.75	0.00	-0.43	94.69
		ν_{x_4}	0.00	0.00	-0.19	95.36	-0.01	-0.52	94.16	0.00	-0.06	94.58
		ν_{x_5}	0.00	0.00	-0.28	95.36	0.00	-0.45	92.97	0.00	-0.22	93.82
		ν_{x_6}	0.00	0.00	-0.11	96.12	-0.01	-0.56	94.59	0.00	0.12	94.04
		ν_{y_2}	0.00	0.00	-0.15	94.07	0.00	0.10	94.59	0.00	0.46	95.67
		ν_{y_3}	0.00	0.00	-0.10	94.07	0.00	0.18	92.10	0.01	0.58	88.30
		λ_{x_2}	1.00	1.01	0.57	94.07	1.00	0.31	95.02	1.01	0.71	96.86
		λ_{x_3}	1.00	1.01	0.63	94.93	1.00	0.44	95.13	1.01	0.51	95.67
		λ_{x_4}	1.00	1.01	0.84	94.61	1.00	0.32	95.78	1.00	0.48	94.69
		λ_{x_5}	1.00	1.01	0.94	95.15	1.00	0.32	93.94	1.01	0.77	95.34
		λ_{y_2}	1.00	0.99	-1.03	93.85	0.98	-1.57	94.91	0.99	-1.14	93.07
		λ_{y_3}	1.00	0.99	-1.02	92.99	0.99	-1.36	94.59	0.98	-1.93	95.45
	400											
		α	0.00	0.00	-0.05	94.51	0.00	0.01	94.51	0.00	-0.02	94.98
		γ_1	0.30	0.30	0.09	94.87	0.30	0.06	94.51	0.30	0.35	95.47
		γ_2	0.30	0.30	0.76	95.36	0.30	0.31	95.85	0.30	0.69	94.00
		γ_3	0.15	0.15	-0.82	94.99	0.15	-0.46	93.65	0.15	-0.75	96.20
		σ_{x_1}	0.50	0.50	-0.15	95.85	0.50	-0.06	95.24	0.50	0.49	94.12
		σ_{x_2}	0.50	0.50	0.62	95.60	0.50	0.12	94.14	0.50	0.15	95.47
		σ_{x_3}	0.50	0.50	-0.14	95.36	0.50	0.55	94.26	0.50	0.60	94.49
		σ_{x_4}	0.50	0.50	0.01	93.77	0.50	-0.21	95.60	0.50	0.08	94.61
		σ_{x_5}	0.50	0.50	-0.02	95.24	0.50	-0.22	94.99	0.50	0.12	93.63
		σ_{x_6}	0.50	0.50	0.07	94.26	0.50	0.38	95.36	0.50	0.09	94.85
		σ_{y_1}	0.50	0.50	-0.69	94.51	0.50	-0.61	94.87	0.50	-0.75	93.26
		σ_{y_2}	0.50	0.50	-0.42	95.85	0.50	-0.75	93.16	0.50	-0.53	95.83
		σ_{y_3}	0.50	0.50	-0.47	95.48	0.50	0.28	94.14	0.52	4.02	79.04
		σ_{ξ_1}	1.00	1.00	0.25	94.99	1.00	0.02	95.12	1.00	-0.20	94.36
		σ_{ξ_2}	1.00	1.00	0.26	94.26	1.00	0.30	94.51	1.00	0.12	95.34
		ν_{x_1}	0.00	0.00	0.00	96.21	0.00	0.09	95.97	0.01	0.54	96.32
		ν_{x_2}	0.00	0.00	-0.02	96.58	0.00	0.01	96.09	0.00	0.38	95.71
		ν_{x_3}	0.00	0.00	-0.05	96.58	0.00	0.02	94.87	0.00	0.31	95.47
		ν_{x_4}	0.00	0.00	-0.31	95.85	0.00	0.02	96.58	0.00	0.10	94.61
		ν_{x_5}	0.00	0.00	-0.45	94.26	0.00	0.12	96.58	0.00	0.22	95.10
		ν_{x_6}	0.00	0.00	-0.40	94.26	0.00	0.02	96.83	0.00	0.27	94.36
		ν_{y_2}	0.00	0.00	0.02	96.70	0.00	-0.04	94.99	0.00	0.17	93.75
		ν_{y_3}	0.00	0.00	-0.06	96.21	0.00	-0.05	91.70	0.00	0.11	87.87
		λ_{x_2}	1.00	1.00	-0.05	94.99	1.00	0.10	94.63	1.00	0.07	94.12
		λ_{x_3}	1.00	1.00	0.13	95.48	1.00	0.16	94.75	1.00	0.04	95.10
		λ_{x_4}	1.00	1.00	-0.04	95.97	1.00	0.04	95.36	1.00	-0.09	95.10
		λ_{x_5}	1.00	1.00	-0.39	94.63	1.00	0.04	94.99	1.00	0.02	94.73
		λ_{y_2}	1.00	0.99	-0.52	95.24	0.99	-0.50	94.02	0.99	-0.73	94.73

Table A.1: Results table for Study 1 Measurement lag population model (D2) and non-spatial analysis model (A1) *(continued)*

				$\rho_{y2}=0$			$\rho_{y2}=0.3$			$\rho_{y2}=0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
		λ_{y_3}	1.00	0.99	-0.94	94.51	1.00	0.00	93.04	0.99	-1.13	93.38
W_D^*												
	49											
		α	0.00	0.00	0.24	96.56	-0.01	-0.88	95.82	0.00	0.42	97.18
		γ_1	0.30	0.32	7.61	97.39	0.32	6.80	97.60	0.32	7.68	96.55
		γ_2	0.30	0.32	6.37	95.20	0.32	7.33	95.09	0.32	6.43	96.97
		γ_3	0.15	0.16	9.17	97.70	0.17	11.94	95.72	0.17	10.59	96.76
		σ_{x_1}	0.50	0.51	1.48	93.11	0.51	2.22	93.21	0.51	2.45	94.14
		σ_{x_2}	0.50	0.51	1.59	93.84	0.51	2.03	94.88	0.50	0.79	94.35
		σ_{x_3}	0.50	0.50	0.62	95.20	0.51	1.15	95.61	0.50	0.62	93.93
		σ_{x_4}	0.50	0.52	3.80	93.74	0.51	1.86	94.04	0.51	2.40	93.10
		σ_{x_5}	0.50	0.50	-0.50	94.57	0.50	-0.09	94.15	0.49	-1.38	94.14
		σ_{x_6}	0.50	0.51	1.32	94.47	0.50	0.25	94.57	0.51	1.77	94.77
		σ_{y_1}	0.50	0.49	-2.02	94.57	0.49	-1.36	96.13	0.49	-1.13	94.77
		σ_{y_2}	0.50	0.50	0.90	93.95	0.50	0.72	94.67	0.50	-0.18	96.03
		σ_{y_3}	0.50	0.50	0.48	93.22	0.49	-1.00	94.04	0.49	-1.20	95.40
		σ_{ξ_1}	1.00	1.00	0.43	93.95	1.01	1.33	94.36	1.01	1.41	94.87
		σ_{ξ_2}	1.00	1.00	-0.04	93.11	1.01	1.00	94.67	1.00	0.16	96.03
		ν_{x_1}	0.00	0.00	0.06	94.68	0.00	-0.21	95.92	0.00	0.27	95.61
		ν_{x_2}	0.00	0.00	0.05	96.35	0.00	-0.37	95.72	0.00	0.04	95.82
		ν_{x_3}	0.00	0.00	0.11	94.26	0.00	-0.11	96.45	0.00	0.47	96.55
		ν_{x_4}	0.00	0.01	0.85	95.09	-0.01	-1.10	94.46	0.01	1.04	94.98
		ν_{x_5}	0.00	0.00	0.28	95.72	-0.01	-1.18	95.61	0.01	0.61	96.34
		ν_{x_6}	0.00	0.01	0.51	96.35	-0.01	-1.11	94.46	0.00	0.41	95.19
		ν_{y_2}	0.00	0.00	-0.08	95.30	0.00	0.14	94.98	0.00	-0.50	95.82
		ν_{y_3}	0.00	0.00	0.17	94.78	0.00	0.18	90.91	0.00	0.24	87.66
		λ_{x_2}	1.00	1.02	2.32	96.03	1.03	2.61	93.73	1.02	2.25	94.46
		λ_{x_3}	1.00	1.02	2.04	96.03	1.02	2.39	96.13	1.03	2.55	95.19
		λ_{x_4}	1.00	1.03	3.26	94.89	1.03	2.68	94.15	1.03	2.83	95.71
		λ_{x_5}	1.00	1.03	2.73	95.09	1.02	2.46	94.25	1.02	2.47	96.23
		λ_{y_2}	1.00	0.92	-7.56	92.90	0.92	-7.94	93.63	0.92	-7.66	93.72
		λ_{y_3}	1.00	0.92	-7.81	94.36	0.92	-8.34	93.10	0.90	-9.65	93.93
	196											
		α	0.00	0.00	-0.24	94.96	0.00	0.20	93.96	0.00	0.04	95.47
		γ_1	0.30	0.31	1.81	96.93	0.30	1.66	96.04	0.30	0.69	93.38
		γ_2	0.30	0.30	1.42	95.83	0.31	1.89	94.18	0.30	1.48	96.80
		γ_3	0.15	0.16	3.34	94.41	0.15	2.85	95.38	0.15	1.95	96.14
		σ_{x_1}	0.50	0.50	1.00	94.30	0.50	0.17	94.51	0.50	0.94	94.26
		σ_{x_2}	0.50	0.50	0.24	94.52	0.50	-0.22	94.29	0.50	0.39	93.05
		σ_{x_3}	0.50	0.50	-0.22	94.96	0.50	0.81	94.07	0.50	0.09	95.81
		σ_{x_4}	0.50	0.50	0.52	96.16	0.50	0.84	94.29	0.51	1.06	94.81
		σ_{x_5}	0.50	0.50	0.91	94.85	0.50	0.66	95.05	0.50	0.17	94.59
		σ_{x_6}	0.50	0.50	0.13	93.86	0.50	0.65	94.51	0.50	0.49	94.37
		σ_{y_1}	0.50	0.49	-1.07	94.85	0.50	-0.99	95.60	0.50	-0.62	94.48
		σ_{y_2}	0.50	0.50	-0.80	95.39	0.50	-0.38	95.60	0.50	-0.77	93.93
		σ_{y_3}	0.50	0.50	-0.76	95.07	0.50	-0.89	95.05	0.49	-1.07	95.14
		σ_{ξ_1}	1.00	1.00	0.20	95.07	1.00	0.29	94.62	1.00	0.24	94.81
		σ_{ξ_2}	1.00	1.00	-0.08	94.41	1.00	-0.13	93.52	1.00	-0.12	94.70
		ν_{x_1}	0.00	-0.01	-0.66	96.05	0.00	0.08	94.62	0.00	-0.15	95.25
		ν_{x_2}	0.00	-0.01	-0.61	95.39	0.00	0.17	95.05	0.00	-0.28	95.47
		ν_{x_3}	0.00	-0.01	-0.61	95.50	0.00	0.36	96.15	0.00	-0.42	94.92
		ν_{x_4}	0.00	0.00	-0.32	96.38	0.00	0.16	95.05	0.00	0.19	95.25
		ν_{x_5}	0.00	0.00	0.04	94.85	0.00	0.10	93.74	0.00	0.34	95.92
		ν_{x_6}	0.00	0.00	-0.13	93.64	0.00	-0.07	93.41	0.00	0.22	96.03
		ν_{y_2}	0.00	0.00	-0.27	94.63	0.00	-0.08	94.95	0.00	-0.13	95.25

Table A.1: Results table for Study 1 Measurement lag population model (D2) and non-spatial analysis model (A1) *(continued)*

			$\rho_{y2} = 0$			$\rho_{y2} = 0.3$			$\rho_{y2} = 0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
		ν_{y_3} 0.00	0.00	0.16	96.27	0.00	-0.19	89.89	0.00	-0.33	82.56
		λ_{x_2} 1.00	1.00	0.45	95.29	1.00	0.46	94.18	1.01	0.70	93.82
		λ_{x_3} 1.00	1.01	0.77	94.63	1.01	0.72	95.05	1.01	0.60	93.93
		λ_{x_4} 1.00	1.01	0.80	94.19	1.00	0.44	95.71	1.01	0.81	93.71
		λ_{x_5} 1.00	1.01	0.54	94.41	1.01	0.55	95.16	1.01	0.91	94.70
		λ_{y_2} 1.00	0.99	-1.28	96.71	0.99	-1.05	94.51	0.99	-0.68	94.59
		λ_{y_3} 1.00	0.97	-2.71	94.30	0.99	-1.06	94.18	0.99	-0.59	94.92
	400										
		α 0.00	0.00	-0.06	95.89	0.00	-0.21	94.84	0.00	-0.01	95.43
		γ_1 0.30	0.30	-0.50	93.77	0.30	0.05	95.84	0.30	0.35	95.18
		γ_2 0.30	0.30	0.10	96.26	0.30	0.27	95.21	0.30	0.47	96.57
		γ_3 0.15	0.15	-1.73	96.88	0.15	-1.64	94.96	0.15	-2.11	94.80
		σ_{x_1} 0.50	0.50	0.52	93.52	0.50	0.01	94.58	0.50	0.82	96.45
		σ_{x_2} 0.50	0.50	0.46	95.51	0.50	0.68	94.58	0.50	0.11	94.42
		σ_{x_3} 0.50	0.50	-0.01	94.51	0.50	0.00	94.58	0.50	0.46	93.65
		σ_{x_4} 0.50	0.50	0.55	94.76	0.50	-0.16	93.07	0.50	0.10	94.29
		σ_{x_5} 0.50	0.50	-0.13	94.01	0.50	0.15	93.95	0.50	0.38	96.19
		σ_{x_6} 0.50	0.50	0.07	96.01	0.50	0.37	94.58	0.50	0.58	94.92
		σ_{y_1} 0.50	0.50	-0.70	95.64	0.50	-0.63	93.95	0.50	-0.67	94.42
		σ_{y_2} 0.50	0.50	-0.73	93.27	0.50	-0.62	95.72	0.50	-0.51	95.56
		σ_{y_3} 0.50	0.50	-0.62	95.26	0.50	-0.58	94.71	0.50	-0.84	95.56
		σ_{ξ_1} 1.00	1.00	-0.01	95.39	1.00	0.32	94.58	1.00	0.07	95.30
		σ_{ξ_2} 1.00	1.00	0.14	95.39	1.00	0.21	94.08	1.00	-0.01	95.43
		ν_{x_1} 0.00	0.00	-0.03	94.64	0.00	-0.19	95.72	0.00	0.17	95.30
		ν_{x_2} 0.00	0.00	0.30	95.51	0.00	-0.16	95.59	0.00	0.08	95.56
		ν_{x_3} 0.00	0.00	0.04	95.26	0.00	-0.24	93.70	0.00	0.04	95.56
		ν_{x_4} 0.00	0.00	0.21	94.26	0.00	-0.03	94.84	0.00	-0.17	94.54
		ν_{x_5} 0.00	0.00	0.18	94.39	0.00	-0.20	96.35	0.00	0.06	93.91
		ν_{x_6} 0.00	0.00	0.04	94.51	0.00	0.06	94.96	0.00	-0.20	95.56
		ν_{y_2} 0.00	0.00	0.17	94.51	0.00	-0.21	92.95	0.00	0.00	95.05
		ν_{y_3} 0.00	0.00	0.07	94.89	0.00	0.01	91.06	0.00	0.02	85.15
		λ_{x_2} 1.00	1.00	0.20	94.89	1.00	0.01	94.08	1.00	0.33	93.65
		λ_{x_3} 1.00	1.00	0.18	94.26	1.00	0.23	94.08	1.00	0.40	95.56
		λ_{x_4} 1.00	1.00	0.27	92.39	1.00	0.07	96.35	1.00	0.29	96.32
		λ_{x_5} 1.00	1.00	0.50	93.39	1.00	0.01	95.21	1.00	0.20	95.18
		λ_{y_2} 1.00	0.99	-0.64	95.01	0.99	-1.16	93.45	0.99	-0.74	95.81
		λ_{y_3} 1.00	0.99	-0.61	96.38	0.98	-1.61	92.82	0.99	-1.06	92.51

[1] W^* is the population and analysis W condition.

[2] N is the simulated sample size.

[3] θ is the simulated population value of each parameter.

[4] $\rho_{y2} = ...$ is the simulated spatial autocorrelation of the endogenous lag.

[5] $\bar{\hat{\theta}}$ is the obtained average posterior mean across simulated iterations.

[6] $Bias(\bar{\hat{\theta}})\%$ is the average percent bias across simulation iterations when $\theta \neq 0$, otherwise $Bias(\bar{\hat{\theta}})\%$ provides absolute bias.

[7] Cover% is the percent of simulated trials in which the population value falls within the central 95% density of the posterior estimate.

Table A.2: Results table for Study 1 endogenous structural lag population model (D3) and non-spatial analysis model (A1) conditions

				$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
W_C^*												
	49											
		α	0.00	-0.01	-0.51	95.50	0.00	0.29	91.07	0.00	-0.39	86.93
		γ_1	0.30	0.32	6.98	96.22	0.30	0.42	95.07	0.31	2.53	95.60
		γ_2	0.30	0.32	6.84	97.03	0.31	3.82	96.53	0.30	-1.51	95.60
		γ_3	0.15	0.16	8.24	95.81	0.16	8.22	96.40	0.16	7.79	95.47
		σ_{x_1}	0.50	0.51	1.16	93.66	0.51	1.41	94.93	0.51	2.20	94.00
		σ_{x_2}	0.50	0.51	1.77	95.30	0.50	0.73	94.27	0.50	0.33	94.27
		σ_{x_3}	0.50	0.51	1.68	95.60	0.51	1.02	96.67	0.50	0.84	95.20
		σ_{x_4}	0.50	0.51	2.55	93.46	0.51	1.25	94.27	0.51	1.63	94.40
		σ_{x_5}	0.50	0.50	-0.28	94.99	0.50	0.90	93.87	0.51	1.09	96.53
		σ_{x_6}	0.50	0.51	2.03	94.99	0.50	-0.28	95.73	0.50	0.87	94.67
		σ_{y_1}	0.50	0.49	-1.68	95.19	0.50	-0.69	95.60	0.50	0.94	94.40
		σ_{y_2}	0.50	0.50	0.14	95.09	0.51	2.31	94.53	0.50	0.53	94.13
		σ_{y_3}	0.50	0.50	0.35	94.79	0.51	2.93	95.20	0.51	2.45	95.33
		σ_{ξ_1}	1.00	1.01	1.36	94.48	1.01	1.36	95.87	1.01	0.54	96.27
		σ_{ξ_2}	1.00	1.01	1.22	93.97	1.00	0.42	95.47	1.00	-0.38	96.93
		ν_{x_1}	0.00	0.00	-0.45	96.11	0.00	-0.25	95.60	-0.01	-0.86	96.27
		ν_{x_2}	0.00	-0.01	-0.54	95.81	0.00	-0.06	95.73	0.00	-0.06	96.53
		ν_{x_3}	0.00	0.00	-0.21	96.42	0.00	-0.19	95.73	0.00	-0.13	96.40
		ν_{x_4}	0.00	0.01	0.71	94.79	0.00	-0.05	96.40	0.00	-0.23	95.47
		ν_{x_5}	0.00	0.01	0.76	94.89	0.00	0.24	96.00	0.00	-0.13	95.07
		ν_{x_6}	0.00	0.01	0.64	94.89	0.00	-0.35	96.27	0.00	0.33	94.67
		ν_{y_2}	0.00	0.00	0.26	94.17	0.00	0.00	94.67	0.00	-0.30	93.20
		ν_{y_3}	0.00	0.01	0.53	93.56	0.00	-0.11	96.67	0.00	-0.41	96.27
		λ_{x_2}	1.00	1.01	1.37	94.27	1.02	2.23	95.87	1.02	2.24	95.07
		λ_{x_3}	1.00	1.02	1.58	94.38	1.02	2.38	96.27	1.02	2.10	95.47
		λ_{x_4}	1.00	1.02	2.12	94.48	1.01	1.13	94.40	1.02	2.23	95.20
		λ_{x_5}	1.00	1.02	1.70	96.52	1.02	2.45	94.80	1.02	2.11	95.60
		λ_{y_2}	1.00	0.92	-8.30	94.17	0.98	-1.70	93.60	0.99	-0.92	95.20
		λ_{y_3}	1.00	0.91	-9.01	91.72	0.98	-1.73	94.40	0.99	-0.56	95.47
	196											
		α	0.00	0.00	0.01	94.39	0.00	-0.03	91.95	0.00	0.27	83.05
		γ_1	0.30	0.30	1.46	95.69	0.30	0.36	95.76	0.30	1.36	94.92
		γ_2	0.30	0.30	1.63	93.74	0.30	-1.16	95.06	0.30	0.56	95.48
		γ_3	0.15	0.15	2.72	96.44	0.15	2.75	95.76	0.15	-1.96	95.62
		σ_{x_1}	0.50	0.50	0.17	95.58	0.50	0.91	95.20	0.51	1.00	94.21
		σ_{x_2}	0.50	0.50	0.45	93.42	0.50	0.54	95.06	0.51	1.24	94.35
		σ_{x_3}	0.50	0.50	0.19	94.82	0.50	0.29	94.21	0.50	-0.11	94.21
		σ_{x_4}	0.50	0.50	0.77	93.64	0.50	0.45	95.76	0.50	0.55	95.76
		σ_{x_5}	0.50	0.50	0.71	94.39	0.50	0.46	96.75	0.50	-0.02	96.05
		σ_{x_6}	0.50	0.50	-0.81	95.15	0.50	0.15	97.03	0.51	1.04	94.77
		σ_{y_1}	0.50	0.50	-0.65	95.04	0.50	0.36	94.35	0.50	-0.07	96.05
		σ_{y_2}	0.50	0.50	-0.48	96.01	0.50	0.86	95.76	0.50	0.52	92.94
		σ_{y_3}	0.50	0.49	-1.13	95.58	0.50	0.74	95.34	0.50	0.30	95.76
		σ_{ξ_1}	1.00	1.00	-0.06	94.93	1.00	0.12	95.90	1.00	0.11	94.92
		σ_{ξ_2}	1.00	1.00	0.23	94.82	1.00	0.13	93.08	1.00	0.02	95.62
		ν_{x_1}	0.00	0.00	0.14	95.58	0.00	0.09	96.19	0.00	0.43	95.34
		ν_{x_2}	0.00	0.00	0.30	95.36	0.00	-0.15	96.05	0.00	0.13	95.48
		ν_{x_3}	0.00	0.01	0.57	94.61	0.00	0.11	96.75	0.00	0.27	96.75
		ν_{x_4}	0.00	0.00	-0.19	95.36	0.00	-0.24	94.07	0.00	0.10	94.92
		ν_{x_5}	0.00	0.00	-0.28	95.36	0.00	-0.17	94.63	0.00	-0.22	95.48
		ν_{x_6}	0.00	0.00	-0.11	96.12	0.00	-0.33	94.63	0.00	0.14	92.80
		ν_{y_2}	0.00	0.00	-0.15	94.07	0.00	-0.03	97.46	0.00	0.01	94.77
		ν_{y_3}	0.00	0.00	-0.10	94.07	0.00	-0.08	94.21	0.00	-0.08	94.35

Table A.2: Results table for Study 1 endogenous structural lag population model (D3) and non-spatial analysis model (A1) *(continued)*

W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
				$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
		λ_{x_2}	1.00	1.01	0.57	94.07	1.01	1.06	93.64	1.01	0.51	94.21
		λ_{x_3}	1.00	1.01	0.63	94.93	1.01	0.62	94.63	1.01	1.01	96.05
		λ_{x_4}	1.00	1.01	0.84	94.61	1.00	0.48	95.06	1.01	1.02	93.64
		λ_{x_5}	1.00	1.01	0.94	95.15	1.00	0.37	95.76	1.01	0.75	94.77
		λ_{y_2}	1.00	0.99	-1.03	93.85	1.00	-0.13	95.76	1.00	-0.04	93.50
		λ_{y_3}	1.00	0.99	-1.02	92.99	1.00	-0.33	95.90	1.00	-0.13	95.34
	400											
		α	0.00	0.00	-0.05	94.51	0.00	-0.33	92.38	0.00	0.26	83.69
		γ_1	0.30	0.30	0.09	94.87	0.30	1.08	94.92	0.30	0.99	93.45
		γ_2	0.30	0.30	0.76	95.36	0.30	-1.17	95.05	0.30	-0.75	96.39
		γ_3	0.15	0.15	-0.82	94.99	0.15	0.59	95.45	0.14	-3.73	94.65
		σ_{x_1}	0.50	0.50	-0.15	95.85	0.50	0.22	95.45	0.50	0.02	92.91
		σ_{x_2}	0.50	0.50	0.62	95.60	0.50	0.40	94.65	0.50	0.51	94.39
		σ_{x_3}	0.50	0.50	-0.14	95.36	0.50	0.20	95.45	0.50	0.00	97.19
		σ_{x_4}	0.50	0.50	0.01	93.77	0.50	0.26	93.45	0.50	0.51	95.32
		σ_{x_5}	0.50	0.50	-0.02	95.24	0.50	0.42	94.65	0.50	0.12	95.32
		σ_{x_6}	0.50	0.50	0.07	94.26	0.50	-0.06	93.98	0.50	-0.27	95.32
		σ_{y_1}	0.50	0.50	-0.69	94.51	0.50	0.27	95.45	0.50	0.04	94.12
		σ_{y_2}	0.50	0.50	-0.42	95.85	0.50	0.21	94.39	0.50	0.28	95.86
		σ_{y_3}	0.50	0.50	-0.47	95.48	0.50	-0.06	94.52	0.50	0.43	96.66
		σ_{ξ_1}	1.00	1.00	0.25	94.99	1.00	0.05	93.32	1.00	0.15	96.26
		σ_{ξ_2}	1.00	1.00	0.26	94.26	1.00	0.33	94.79	1.00	0.09	93.85
		ν_{x_1}	0.00	0.00	0.00	96.21	0.00	-0.20	95.05	0.00	0.25	95.45
		ν_{x_2}	0.00	0.00	-0.02	96.58	0.00	-0.04	95.72	0.00	0.02	93.98
		ν_{x_3}	0.00	0.00	-0.05	96.58	0.00	-0.04	95.45	0.00	0.06	95.45
		ν_{x_4}	0.00	0.00	-0.31	95.85	0.00	0.28	94.39	0.00	0.03	94.12
		ν_{x_5}	0.00	0.00	-0.45	94.26	0.00	0.23	94.65	0.00	-0.06	92.38
		ν_{x_6}	0.00	0.00	-0.40	94.26	0.00	0.23	96.12	0.00	0.17	94.92
		ν_{y_2}	0.00	0.00	0.02	96.70	0.00	0.28	95.45	0.00	-0.04	95.45
		ν_{y_3}	0.00	0.00	-0.06	96.21	0.01	0.54	95.99	0.00	-0.25	94.39
		λ_{x_2}	1.00	1.00	-0.05	94.99	1.00	0.43	93.18	1.00	0.37	95.05
		λ_{x_3}	1.00	1.00	0.13	95.48	1.00	0.39	95.72	1.00	0.12	96.39
		λ_{x_4}	1.00	1.00	-0.04	95.97	1.00	0.15	94.39	1.01	0.54	94.12
		λ_{x_5}	1.00	1.00	-0.39	94.63	1.00	0.30	94.12	1.00	0.49	95.45
		λ_{y_2}	1.00	0.99	-0.52	95.24	1.00	-0.14	94.39	1.00	0.22	94.25
		λ_{y_3}	1.00	0.99	-0.94	94.51	1.00	0.05	95.32	1.00	-0.08	93.98
W_D^*												
	49											
		α	0.00	0.00	0.24	96.56	0.00	-0.18	92.24	0.01	0.93	82.46
		γ_1	0.30	0.32	7.61	97.39	0.30	-0.15	96.79	0.30	1.16	96.92
		γ_2	0.30	0.32	6.37	95.20	0.29	-2.52	94.51	0.30	-1.44	95.05
		γ_3	0.15	0.16	9.17	97.70	0.16	6.56	95.98	0.16	3.97	96.65
		σ_{x_1}	0.50	0.51	1.48	93.11	0.51	2.57	95.18	0.52	3.38	93.84
		σ_{x_2}	0.50	0.51	1.59	93.84	0.50	0.32	93.71	0.50	0.65	95.05
		σ_{x_3}	0.50	0.50	0.62	95.20	0.51	1.51	94.24	0.51	1.47	95.05
		σ_{x_4}	0.50	0.52	3.80	93.74	0.51	2.45	94.78	0.51	1.06	95.85
		σ_{x_5}	0.50	0.50	-0.50	94.57	0.51	2.12	94.51	0.51	1.51	94.65
		σ_{x_6}	0.50	0.51	1.32	94.47	0.51	1.30	92.37	0.51	2.33	93.98
		σ_{y_1}	0.50	0.49	-2.02	94.57	0.50	-0.98	94.24	0.50	-0.18	94.11
		σ_{y_2}	0.50	0.50	0.90	93.95	0.51	1.10	95.58	0.51	1.94	95.18
		σ_{y_3}	0.50	0.50	0.48	93.22	0.50	0.59	94.11	0.51	2.05	95.72
		σ_{ξ_1}	1.00	1.00	0.43	93.95	1.01	1.11	94.78	1.01	0.67	94.24
		σ_{ξ_2}	1.00	1.00	-0.04	93.11	1.02	2.44	94.78	1.01	1.12	95.05
		ν_{x_1}	0.00	0.00	0.06	94.68	0.00	-0.09	94.91	-0.01	-0.84	96.25

Table A.2: Results table for Study 1 endogenous structural lag population model (D3) and non-spatial analysis model (A1) *(continued)*

				$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
		ν_{x_2}	0.00	0.00	0.05	96.35	0.00	-0.44	95.31	-0.01	-0.77	95.98
		ν_{x_3}	0.00	0.00	0.11	94.26	0.00	-0.36	94.78	-0.01	-0.89	95.58
		ν_{x_4}	0.00	0.01	0.85	95.09	0.00	0.24	94.78	-0.01	-0.77	96.25
		ν_{x_5}	0.00	0.00	0.28	95.72	-0.01	-0.54	96.25	0.00	-0.25	97.32
		ν_{x_6}	0.00	0.01	0.51	96.35	0.00	0.31	95.58	0.00	0.40	95.31
		ν_{y_2}	0.00	0.00	-0.08	95.30	0.00	-0.08	95.05	0.00	0.46	95.31
		ν_{y_3}	0.00	0.00	0.17	94.78	0.00	-0.23	95.58	0.01	1.03	96.79
		λ_{x_2}	1.00	1.02	2.32	96.03	1.03	2.95	96.92	1.03	2.89	96.65
		λ_{x_3}	1.00	1.02	2.04	96.03	1.02	2.15	95.98	1.03	2.68	94.91
		λ_{x_4}	1.00	1.03	3.26	94.89	1.02	1.96	96.39	1.02	2.22	95.31
		λ_{x_5}	1.00	1.03	2.73	95.09	1.01	1.13	96.39	1.02	2.17	95.72
		λ_{y_2}	1.00	0.92	-7.56	92.90	0.99	-0.97	96.65	0.98	-1.60	95.05
		λ_{y_3}	1.00	0.92	-7.81	94.36	0.99	-0.68	96.39	0.99	-1.22	94.91
	196											
		α	0.00	0.00	-0.24	94.96	0.00	-0.34	89.49	-0.01	-0.95	85.80
		γ_1	0.30	0.31	1.81	96.93	0.30	1.57	95.88	0.29	-2.14	94.60
		γ_2	0.30	0.30	1.42	95.83	0.30	-0.03	94.89	0.30	-0.59	94.46
		γ_3	0.15	0.16	3.34	94.41	0.16	6.86	94.03	0.15	-1.03	95.74
		σ_{x_1}	0.50	0.50	1.00	94.30	0.50	0.66	95.31	0.50	0.46	94.74
		σ_{x_2}	0.50	0.50	0.24	94.52	0.50	-0.07	94.74	0.50	0.25	95.31
		σ_{x_3}	0.50	0.50	-0.22	94.96	0.50	0.03	95.31	0.50	0.45	95.17
		σ_{x_4}	0.50	0.50	0.52	96.16	0.51	1.10	93.32	0.51	1.26	94.60
		σ_{x_5}	0.50	0.50	0.91	94.85	0.50	0.58	94.32	0.50	0.62	94.60
		σ_{x_6}	0.50	0.50	0.13	93.86	0.50	0.47	96.02	0.50	0.57	93.61
		σ_{y_1}	0.50	0.49	-1.07	94.85	0.50	0.35	94.32	0.50	0.34	94.03
		σ_{y_2}	0.50	0.50	-0.80	95.39	0.50	-0.25	96.16	0.50	0.43	94.46
		σ_{y_3}	0.50	0.50	-0.76	95.07	0.51	1.12	94.18	0.50	0.89	94.60
		σ_{ξ_1}	1.00	1.00	0.20	95.07	1.01	0.64	92.76	1.00	-0.19	95.03
		σ_{ξ_2}	1.00	1.00	-0.08	94.41	1.00	0.40	95.31	1.00	-0.27	95.17
		ν_{x_1}	0.00	-0.01	-0.66	96.05	0.00	-0.29	93.32	0.00	-0.33	94.60
		ν_{x_2}	0.00	-0.01	-0.61	95.39	-0.01	-0.61	95.31	-0.01	-0.56	94.32
		ν_{x_3}	0.00	-0.01	-0.61	95.50	0.00	-0.31	95.31	0.00	-0.20	95.03
		ν_{x_4}	0.00	0.00	-0.32	96.38	0.00	0.01	95.17	0.00	-0.25	96.02
		ν_{x_5}	0.00	0.00	0.04	94.85	0.00	-0.03	94.46	0.00	-0.49	94.18
		ν_{x_6}	0.00	0.00	-0.13	93.64	0.00	-0.48	94.60	0.00	-0.40	94.89
		ν_{y_2}	0.00	0.00	-0.27	94.63	0.00	0.14	96.59	0.00	0.07	95.31
		ν_{y_3}	0.00	0.00	0.16	96.27	0.00	0.07	95.74	0.00	0.32	94.89
		λ_{x_2}	1.00	1.00	0.45	95.29	1.01	0.73	95.03	1.00	0.46	94.18
		λ_{x_3}	1.00	1.01	0.77	94.63	1.01	0.74	94.18	1.01	1.02	95.45
		λ_{x_4}	1.00	1.01	0.80	94.19	1.00	0.02	94.74	1.01	0.66	93.47
		λ_{x_5}	1.00	1.01	0.54	94.41	1.00	0.27	94.74	1.01	0.80	93.47
		λ_{y_2}	1.00	0.99	-1.28	96.71	1.00	-0.07	95.60	1.00	-0.14	94.60
		λ_{y_3}	1.00	0.97	-2.71	94.30	1.00	-0.25	95.88	1.00	-0.03	94.46
	400											
		α	0.00	0.00	-0.06	95.89	0.00	0.09	86.83	0.00	0.01	85.62
		γ_1	0.30	0.30	-0.50	93.77	0.30	-0.38	94.76	0.31	2.57	95.43
		γ_2	0.30	0.30	0.10	96.26	0.30	0.87	94.62	0.30	-0.57	95.03
		γ_3	0.15	0.15	-1.73	96.88	0.15	-0.87	96.24	0.15	1.63	94.62
		σ_{x_1}	0.50	0.50	0.52	93.52	0.50	0.43	93.82	0.50	-0.22	94.22
		σ_{x_2}	0.50	0.50	0.46	95.51	0.50	-0.03	94.89	0.50	0.23	94.76
		σ_{x_3}	0.50	0.50	-0.01	94.51	0.50	-0.06	96.37	0.50	0.20	93.55
		σ_{x_4}	0.50	0.50	0.55	94.76	0.50	0.46	96.77	0.50	0.52	94.22
		σ_{x_5}	0.50	0.50	-0.13	94.01	0.50	0.52	96.51	0.50	0.52	95.30
		σ_{x_6}	0.50	0.50	0.07	96.01	0.50	0.13	94.22	0.50	-0.51	93.68
		σ_{y_1}	0.50	0.50	-0.70	95.64	0.50	-0.01	95.83	0.50	-0.15	94.76

Table A.2: Results table for Study 1 endogenous structural lag population model (D3) and non-spatial analysis model (A1) *(continued)*

			$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)$%	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)$%	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)$%	Cover%
		σ_{y_2} 0.50	0.50	-0.73	93.27	0.50	0.39	93.95	0.50	0.67	94.35
		σ_{y_3} 0.50	0.50	-0.62	95.26	0.50	0.01	96.10	0.50	0.27	95.16
		σ_{ξ_1} 1.00	1.00	-0.01	95.39	1.00	0.00	93.68	1.00	0.13	94.76
		σ_{ξ_2} 1.00	1.00	0.14	95.39	1.00	0.13	96.10	1.00	-0.13	96.64
		ν_{x_1} 0.00	0.00	-0.03	94.64	0.00	0.01	95.70	0.00	0.07	94.35
		ν_{x_2} 0.00	0.00	0.30	95.51	0.00	0.04	95.56	0.00	0.19	95.03
		ν_{x_3} 0.00	0.00	0.04	95.26	0.00	-0.02	96.51	0.00	0.06	96.51
		ν_{x_4} 0.00	0.00	0.21	94.26	0.00	0.31	95.83	0.00	-0.26	91.40
		ν_{x_5} 0.00	0.00	0.18	94.39	0.00	0.16	94.89	0.00	-0.20	93.41
		ν_{x_6} 0.00	0.00	0.04	94.51	0.00	0.33	94.22	0.00	-0.14	94.76
		ν_{y_2} 0.00	0.00	0.17	94.51	0.00	-0.06	94.76	0.00	0.03	95.43
		ν_{y_3} 0.00	0.00	0.07	94.89	0.00	-0.10	93.95	0.00	0.08	93.15
		λ_{x_2} 1.00	1.00	0.20	94.89	1.00	0.44	95.70	1.00	0.20	94.09
		λ_{x_3} 1.00	1.00	0.18	94.26	1.00	0.30	93.28	1.00	0.17	95.43
		λ_{x_4} 1.00	1.00	0.27	92.39	1.00	-0.18	94.49	1.00	0.34	94.76
		λ_{x_5} 1.00	1.00	0.50	93.39	1.00	-0.01	94.62	1.00	0.20	95.70
		λ_{y_2} 1.00	0.99	-0.64	95.01	1.00	-0.39	94.62	0.99	-0.61	93.82
		λ_{y_3} 1.00	0.99	-0.61	96.38	1.00	-0.08	93.82	1.00	-0.45	94.76

[1] W^* is the population and analysis W condition.

[2] N is the sample size condition.

[3] θ is the population value of each parameter.

[4] $\rho_\eta = ...$ is the population spatial autocorrelation of the endogenous lag.

[5] $\bar{\hat{\theta}}$ is the obtained average posterior mean across simulated iterations.

[6] $Bias(\bar{\hat{\theta}})\%$ is the average percent bias across simulation iterations when $\theta \neq 0$, otherwise $Bias(\bar{\hat{\theta}})\%$ provides absolute bias.

[7] Cover% is the percent of simulated trials in which the population value falls within the central 95% density of the posterior estimate.

Table A.3: Results table for Study 1 simultaneous structural lag population model (D4) under $\phi_\zeta = 0.3$ condition with non-spatial analysis model (A1)

				$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
W_C^*												
	49											
		α	0.00	0.00	0.22	97.85	0.00	0.42	94.78	0.02	1.83	81.09
		γ_1	0.30	0.32	6.25	96.24	0.32	7.47	96.27	0.34	14.22	92.79
		γ_2	0.30	0.32	6.23	97.31	0.32	8.00	98.01	0.35	16.33	91.79
		γ_3	0.15	0.16	5.58	95.70	0.16	8.65	96.52	0.18	19.49	93.78
		σ_{x_1}	0.50	0.50	0.91	91.94	0.51	1.79	95.52	0.50	0.93	94.53
		σ_{x_2}	0.50	0.50	-0.10	95.70	0.51	1.03	91.79	0.51	1.82	94.53
		σ_{x_3}	0.50	0.51	1.17	95.70	0.51	1.45	94.53	0.51	1.48	94.03
		σ_{x_4}	0.50	0.52	3.28	91.94	0.51	1.88	92.29	0.51	2.52	94.03
		σ_{x_5}	0.50	0.50	0.88	94.62	0.50	0.08	96.77	0.51	1.31	95.77
		σ_{x_6}	0.50	0.50	0.54	93.01	0.51	1.33	93.53	0.50	0.72	94.03
		σ_{y_1}	0.50	0.50	-0.26	97.31	0.50	-0.85	95.02	0.51	1.54	94.78
		σ_{y_2}	0.50	0.50	0.32	94.62	0.50	-0.41	94.78	0.51	1.11	95.02
		σ_{y_3}	0.50	0.50	0.18	93.55	0.50	-0.09	96.02	0.51	1.57	96.02
		σ_{ξ_1}	1.00	1.00	0.06	96.24	1.01	0.67	93.28	1.01	1.29	94.78
		σ_{ξ_2}	1.00	1.02	2.15	93.55	1.01	0.99	95.77	1.01	1.12	94.03
		ν_{x_1}	0.00	0.00	-0.40	97.31	0.00	-0.45	97.01	0.03	3.08	95.27
		ν_{x_2}	0.00	0.00	-0.43	96.24	0.00	-0.31	94.28	0.02	1.78	98.01
		ν_{x_3}	0.00	-0.01	-1.01	96.24	-0.01	-0.71	97.76	0.03	2.82	95.52
		ν_{x_4}	0.00	-0.01	-0.56	96.24	0.00	-0.18	95.77	0.00	-0.31	95.77
		ν_{x_5}	0.00	-0.01	-0.63	97.85	0.00	0.44	97.01	0.00	0.14	96.02
		ν_{x_6}	0.00	0.00	-0.22	95.70	0.00	-0.29	96.52	-0.01	-0.65	95.27
		ν_{y_2}	0.00	0.00	-0.47	96.24	-0.01	-0.77	96.27	0.00	0.40	95.52
		ν_{y_3}	0.00	0.00	0.09	98.39	-0.01	-0.78	94.78	0.01	0.72	94.78
		λ_{x_2}	1.00	1.02	2.33	95.70	1.02	2.10	95.77	1.02	2.17	95.52
		λ_{x_3}	1.00	1.02	2.50	93.55	1.02	2.12	96.52	1.02	2.49	95.52
		λ_{x_4}	1.00	1.02	2.14	94.62	1.02	2.35	94.53	1.02	1.67	96.02
		λ_{x_5}	1.00	1.02	2.00	96.24	1.02	1.69	95.77	1.02	1.60	95.77
		λ_{y_2}	1.00	0.94	-6.30	93.01	0.93	-7.13	93.78	0.93	-7.31	95.52
		λ_{y_3}	1.00	0.91	-9.42	92.47	0.91	-8.75	92.79	0.93	-7.47	95.52
	196											
		α	0.00	0.00	0.11	95.65	0.00	0.12	89.95	0.00	0.48	77.33
		γ_1	0.30	0.31	2.36	95.11	0.31	4.29	93.97	0.33	10.15	88.41
		γ_2	0.30	0.30	0.75	94.57	0.31	3.56	93.97	0.33	9.81	86.15
		γ_3	0.15	0.15	1.68	93.48	0.16	3.59	95.48	0.17	11.79	92.95
		σ_{x_1}	0.50	0.50	0.03	94.02	0.50	0.85	94.47	0.50	0.96	94.71
		σ_{x_2}	0.50	0.51	1.18	95.11	0.50	0.73	95.23	0.50	0.34	94.71
		σ_{x_3}	0.50	0.50	0.83	96.74	0.50	-0.22	92.71	0.50	0.26	94.21
		σ_{x_4}	0.50	0.50	-0.22	94.02	0.50	0.12	93.72	0.50	0.52	94.21
		σ_{x_5}	0.50	0.51	1.04	96.74	0.51	1.02	94.72	0.50	0.10	95.47
		σ_{x_6}	0.50	0.51	1.22	94.57	0.50	-0.17	95.23	0.50	0.75	96.22
		σ_{y_1}	0.50	0.50	-0.23	97.83	0.50	-0.35	95.23	0.50	-0.57	96.47
		σ_{y_2}	0.50	0.49	-1.33	95.65	0.50	-0.07	96.48	0.50	0.55	94.96
		σ_{y_3}	0.50	0.50	-0.32	95.65	0.50	-0.22	93.72	0.50	0.48	94.96
		σ_{ξ_1}	1.00	1.00	0.24	95.11	1.00	-0.45	93.47	1.01	0.67	94.46
		σ_{ξ_2}	1.00	1.01	1.44	92.93	1.00	-0.17	95.73	1.01	0.82	96.22
		ν_{x_1}	0.00	0.00	0.08	95.11	0.00	0.06	92.96	0.00	0.05	96.22
		ν_{x_2}	0.00	0.00	0.20	96.74	0.00	-0.06	93.22	0.00	-0.05	95.72
		ν_{x_3}	0.00	0.00	-0.11	96.20	0.00	0.12	93.97	0.00	0.10	95.47
		ν_{x_4}	0.00	0.00	0.12	97.28	0.00	0.17	94.97	0.01	0.96	95.72
		ν_{x_5}	0.00	0.00	-0.11	95.11	0.00	0.11	95.23	0.01	0.72	95.21
		ν_{x_6}	0.00	0.00	0.23	97.83	0.00	0.19	94.22	0.00	0.32	95.47
		ν_{y_2}	0.00	0.00	-0.01	91.85	0.00	-0.06	94.47	0.00	-0.21	95.21
		ν_{y_3}	0.00	0.00	-0.38	94.57	0.00	-0.02	92.21	0.00	-0.24	95.97

Table A.3: Results table for Study 1 simultaneous structural lag population model (D4) under $\phi_\zeta = 0.3$ condition with non-spatial analysis model (A1) *(continued)*

				$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	$Cover\%$	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	$Cover\%$	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	$Cover\%$
		λ_{x_2}	1.00	1.01	0.70	94.57	1.00	0.43	95.73	1.00	0.11	92.95
		λ_{x_3}	1.00	1.01	0.72	95.11	1.01	0.85	95.98	1.00	-0.14	95.72
		λ_{x_4}	1.00	1.00	-0.34	95.11	1.00	0.04	92.21	1.00	0.08	96.22
		λ_{x_5}	1.00	1.00	-0.49	95.65	1.01	0.55	94.47	1.00	0.07	93.45
		λ_{y_2}	1.00	0.98	-1.96	94.02	0.99	-1.43	94.97	0.98	-1.96	95.47
		λ_{y_3}	1.00	0.98	-2.03	94.57	0.98	-2.20	94.47	0.98	-1.72	94.46
	400											
		α	0.00	-0.01	-1.11	90.91	0.00	-0.21	91.86	0.00	-0.07	72.09
		γ_1	0.30	0.30	-0.43	95.45	0.30	-0.71	95.35	0.32	7.81	79.84
		γ_2	0.30	0.30	-0.63	100.00	0.31	2.37	94.19	0.32	5.99	82.95
		γ_3	0.15	0.15	-0.06	93.18	0.14	-4.79	92.64	0.16	7.50	91.09
		σ_{x_1}	0.50	0.51	1.86	93.18	0.50	0.51	94.57	0.50	0.34	95.74
		σ_{x_2}	0.50	0.50	0.28	93.18	0.50	0.30	94.19	0.50	0.10	94.57
		σ_{x_3}	0.50	0.50	0.63	97.73	0.50	0.41	93.02	0.50	0.20	93.41
		σ_{x_4}	0.50	0.50	-0.18	84.09	0.50	0.00	96.12	0.51	1.02	98.45
		σ_{x_5}	0.50	0.50	0.14	97.73	0.50	0.05	94.19	0.50	0.37	93.80
		σ_{x_6}	0.50	0.50	0.50	93.18	0.50	0.76	92.25	0.50	-0.30	94.19
		σ_{y_1}	0.50	0.50	-0.42	95.45	0.49	-1.05	95.74	0.50	0.03	95.74
		σ_{y_2}	0.50	0.50	-0.13	97.73	0.50	0.04	94.57	0.50	-0.61	97.67
		σ_{y_3}	0.50	0.50	-0.04	93.18	0.50	-0.55	94.96	0.50	-0.57	93.41
		σ_{ξ_1}	1.00	0.99	-0.80	93.18	1.00	0.16	95.74	1.00	0.30	93.02
		σ_{ξ_2}	1.00	1.00	0.15	90.91	1.00	-0.42	96.90	1.01	0.89	96.90
		ν_{x_1}	0.00	-0.02	-2.01	86.36	0.00	0.39	94.96	0.00	-0.24	93.80
		ν_{x_2}	0.00	-0.02	-1.61	90.91	0.00	0.14	93.41	0.00	-0.48	90.31
		ν_{x_3}	0.00	-0.02	-1.77	86.36	0.00	0.09	94.19	0.00	-0.27	94.19
		ν_{x_4}	0.00	0.00	-0.39	100.00	0.00	-0.29	95.35	0.00	0.08	94.96
		ν_{x_5}	0.00	-0.01	-0.67	100.00	0.00	-0.19	94.96	0.00	-0.12	94.19
		ν_{x_6}	0.00	0.00	-0.29	100.00	0.00	-0.38	95.35	0.00	-0.11	93.80
		ν_{y_2}	0.00	0.00	-0.39	100.00	0.00	0.39	92.64	0.00	0.01	93.02
		ν_{y_3}	0.00	0.01	1.05	93.18	0.00	-0.05	93.80	0.00	0.00	92.64
		λ_{x_2}	1.00	1.00	-0.06	90.91	1.00	0.00	95.74	1.00	0.45	94.57
		λ_{x_3}	1.00	1.00	-0.10	97.73	1.00	0.00	96.90	1.00	0.29	95.74
		λ_{x_4}	1.00	1.01	1.49	97.73	1.00	0.44	97.29	1.00	-0.13	96.51
		λ_{x_5}	1.00	1.00	0.24	90.91	1.01	0.54	94.19	1.00	0.20	94.96
		λ_{y_2}	1.00	1.02	1.51	95.45	0.99	-0.57	94.19	1.00	0.24	93.80
		λ_{y_3}	1.00	1.01	1.27	90.91	1.00	-0.30	96.51	1.00	0.00	94.57
W_D^*												
	49											
		α	0.00	0.00	-0.46	95.21	0.01	0.99	96.41	-0.01	-0.53	83.23
		γ_1	0.30	0.32	6.69	96.41	0.31	4.69	96.41	0.32	5.71	97.01
		γ_2	0.30	0.33	10.79	94.61	0.32	6.67	95.81	0.33	10.39	94.61
		γ_3	0.15	0.16	7.98	95.81	0.16	7.55	95.81	0.17	12.04	95.81
		σ_{x_1}	0.50	0.51	1.60	97.60	0.51	1.84	97.60	0.52	4.79	94.01
		σ_{x_2}	0.50	0.51	2.38	90.42	0.51	2.41	98.20	0.50	-0.40	93.41
		σ_{x_3}	0.50	0.51	2.05	91.02	0.50	-0.57	95.81	0.50	-0.47	96.41
		σ_{x_4}	0.50	0.51	2.47	95.81	0.50	0.51	96.41	0.50	-0.41	94.01
		σ_{x_5}	0.50	0.51	1.35	92.22	0.51	1.45	92.81	0.50	0.63	92.22
		σ_{x_6}	0.50	0.51	1.00	91.02	0.50	-0.57	95.21	0.52	3.01	94.01
		σ_{y_1}	0.50	0.48	-3.96	94.61	0.49	-2.24	95.81	0.50	-0.94	96.41
		σ_{y_2}	0.50	0.51	1.03	94.01	0.51	1.03	94.01	0.50	0.23	93.41
		σ_{y_3}	0.50	0.50	-0.38	94.61	0.50	0.45	94.61	0.51	1.45	95.21
		σ_{ξ_1}	1.00	1.03	3.13	94.61	0.99	-0.80	92.81	1.00	-0.34	95.81
		σ_{ξ_2}	1.00	1.03	2.97	94.01	1.00	0.15	94.61	1.01	1.45	95.21
		ν_{x_1}	0.00	0.00	0.40	94.61	0.02	1.90	98.20	0.01	1.31	95.21

Table A.3: Results table for Study 1 simultaneous structural lag population model (D4) under $\phi_\zeta = 0.3$ condition with non-spatial analysis model (A1) *(continued)*

			$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	$Cover\%$	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	$Cover\%$	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	$Cover\%$
		ν_{x_2} 0.00	0.01	0.58	95.81	0.02	2.00	97.01	0.01	1.37	97.60
		ν_{x_3} 0.00	0.01	0.85	95.81	0.01	1.28	96.41	0.01	0.85	97.01
		ν_{x_4} 0.00	0.00	0.29	96.41	-0.01	-1.49	97.60	-0.02	-2.03	94.61
		ν_{x_5} 0.00	0.00	-0.45	94.01	-0.02	-1.51	98.20	-0.02	-2.09	93.41
		ν_{x_6} 0.00	0.00	-0.11	95.21	-0.02	-1.77	98.20	-0.03	-2.77	95.21
		ν_{y_2} 0.00	0.01	0.80	91.62	0.00	-0.39	94.61	0.00	-0.36	95.21
		ν_{y_3} 0.00	0.00	0.42	95.81	0.00	-0.50	98.20	0.00	0.00	95.21
		λ_{x_2} 1.00	1.02	2.04	94.61	1.01	1.39	98.80	1.04	3.90	95.81
		λ_{x_3} 1.00	1.01	0.94	95.81	1.03	2.60	98.80	1.05	4.97	95.21
		λ_{x_4} 1.00	1.02	1.86	94.61	1.02	1.71	96.41	1.01	0.95	95.81
		λ_{x_5} 1.00	1.00	0.20	92.81	1.02	2.12	94.61	1.01	0.83	96.41
		λ_{y_2} 1.00	0.91	-9.26	90.42	0.95	-5.35	92.81	0.89	-10.95	84.43
		λ_{y_3} 1.00	0.90	-9.73	92.22	0.96	-4.43	89.22	0.87	-12.87	88.02
	196										
		α 0.00	0.00	0.12	97.58	0.00	0.10	96.93	0.01	0.95	67.28
		γ_1 0.30	0.30	0.76	96.36	0.31	2.24	98.16	0.30	0.56	94.44
		γ_2 0.30	0.30	0.80	96.97	0.31	3.03	96.32	0.30	-1.08	94.44
		γ_3 0.15	0.15	2.38	97.58	0.15	0.69	94.48	0.15	0.16	93.83
		σ_{x_1} 0.50	0.50	0.97	94.55	0.50	0.34	93.87	0.51	1.38	95.68
		σ_{x_2} 0.50	0.50	0.40	96.36	0.50	-0.10	95.09	0.50	0.04	96.91
		σ_{x_3} 0.50	0.50	-0.05	96.36	0.50	0.57	95.71	0.50	0.55	95.06
		σ_{x_4} 0.50	0.50	0.10	94.55	0.50	-0.35	98.16	0.50	0.32	95.68
		σ_{x_5} 0.50	0.50	-0.58	95.15	0.50	0.51	92.02	0.50	0.50	93.21
		σ_{x_6} 0.50	0.51	1.03	91.52	0.50	0.78	94.48	0.50	-0.29	95.68
		σ_{y_1} 0.50	0.50	-0.37	93.94	0.50	-0.21	95.71	0.49	-1.06	95.06
		σ_{y_2} 0.50	0.50	-0.56	94.55	0.50	-0.88	95.71	0.50	-0.60	96.30
		σ_{y_3} 0.50	0.50	-0.33	96.36	0.49	-1.01	93.87	0.50	-0.87	95.06
		σ_{ξ_1} 1.00	1.01	0.57	95.76	0.99	-0.56	96.32	1.00	0.16	95.68
		σ_{ξ_2} 1.00	1.00	0.31	96.36	1.01	0.60	98.77	1.01	0.53	97.53
		ν_{x_1} 0.00	0.01	0.79	93.33	0.00	0.36	95.71	-0.01	-0.56	91.36
		ν_{x_2} 0.00	0.00	0.02	94.55	0.00	-0.34	98.77	0.00	-0.23	93.83
		ν_{x_3} 0.00	0.00	0.03	96.97	0.00	0.43	98.16	0.00	-0.25	95.06
		ν_{x_4} 0.00	-0.01	-0.86	92.12	0.00	0.46	95.71	0.01	1.22	93.21
		ν_{x_5} 0.00	-0.01	-0.58	92.73	0.01	0.85	96.32	0.01	1.29	96.91
		ν_{x_6} 0.00	-0.01	-0.78	92.73	0.00	0.44	95.09	0.01	0.93	93.21
		ν_{y_2} 0.00	0.00	0.31	98.18	0.01	0.59	97.55	0.00	0.23	96.91
		ν_{y_3} 0.00	0.01	0.64	95.76	0.00	0.30	95.71	0.01	0.54	94.44
		λ_{x_2} 1.00	1.00	0.45	92.12	1.00	0.29	95.09	1.00	0.35	97.53
		λ_{x_3} 1.00	1.01	0.61	95.15	1.01	1.14	96.32	1.01	0.96	95.68
		λ_{x_4} 1.00	1.01	0.56	98.18	1.00	0.43	95.09	1.00	-0.01	95.06
		λ_{x_5} 1.00	1.01	0.96	93.94	1.01	0.56	93.87	1.00	0.04	95.06
		λ_{y_2} 1.00	0.99	-1.34	96.36	0.95	-4.62	90.80	0.99	-0.54	94.44
		λ_{y_3} 1.00	0.99	-1.29	93.94	0.98	-1.82	96.93	1.00	-0.16	93.21
	400										
		α 0.00	0.00	0.45	100.00	0.01	0.63	87.88	-0.01	-0.50	75.76
		γ_1 0.30	0.30	-1.04	97.06	0.31	3.40	87.88	0.31	2.72	96.97
		γ_2 0.30	0.30	0.59	100.00	0.30	-0.73	93.94	0.30	1.46	93.94
		γ_3 0.15	0.15	2.36	88.24	0.16	6.97	96.97	0.15	1.21	93.94
		σ_{x_1} 0.50	0.50	-0.64	94.12	0.50	0.34	96.97	0.51	1.10	93.94
		σ_{x_2} 0.50	0.50	0.15	97.06	0.50	-0.06	96.97	0.50	-0.45	96.97
		σ_{x_3} 0.50	0.50	0.18	94.12	0.50	-0.06	100.00	0.50	-0.10	100.00
		σ_{x_4} 0.50	0.50	-0.99	97.06	0.50	-0.98	90.91	0.50	0.83	90.91
		σ_{x_5} 0.50	0.50	-0.61	97.06	0.50	0.20	87.88	0.51	1.41	87.88
		σ_{x_6} 0.50	0.51	1.33	94.12	0.50	-0.55	96.97	0.51	2.09	87.88
		σ_{y_1} 0.50	0.50	-0.87	88.24	0.50	-0.27	96.97	0.50	-0.72	96.97

Table A.3: Results table for Study 1 simultaneous structural lag population model (D4) under $\phi_\zeta = 0.3$ condition with non-spatial analysis model (A1) *(continued)*

			$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	$Cover\%$	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	$Cover\%$	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	$Cover\%$
		σ_{y_2} 0.50	0.50	-0.88	94.12	0.50	0.10	93.94	0.50	-0.62	93.94
		σ_{y_3} 0.50	0.50	-0.77	88.24	0.50	-0.69	93.94	0.50	-0.09	96.97
		σ_{ξ_1} 1.00	0.99	-1.06	100.00	1.00	0.17	100.00	0.99	-0.80	93.94
		σ_{ξ_2} 1.00	1.00	-0.03	91.18	1.00	0.49	93.94	0.99	-0.75	90.91
		ν_{x_1} 0.00	0.00	-0.48	91.18	0.00	0.05	96.97	0.00	-0.33	90.91
		ν_{x_2} 0.00	0.00	0.17	94.12	0.00	0.09	93.94	-0.01	-0.69	90.91
		ν_{x_3} 0.00	-0.01	-0.53	91.18	0.00	0.39	96.97	-0.01	-1.27	90.91
		ν_{x_4} 0.00	0.01	0.95	97.06	0.02	1.58	90.91	0.01	1.12	96.97
		ν_{x_5} 0.00	0.00	0.31	94.12	0.00	0.40	96.97	0.00	0.45	96.97
		ν_{x_6} 0.00	0.01	0.84	94.12	0.02	1.98	93.94	0.00	-0.37	100.00
		ν_{y_2} 0.00	-0.01	-0.65	97.06	0.01	0.54	96.97	0.00	0.31	87.88
		ν_{y_3} 0.00	0.00	-0.11	97.06	0.00	0.00	100.00	-0.01	-0.52	93.94
		λ_{x_2} 1.00	1.01	0.55	94.12	1.01	0.83	96.97	1.00	0.31	96.97
		λ_{x_3} 1.00	1.00	0.01	97.06	1.00	0.02	93.94	1.01	0.69	96.97
		λ_{x_4} 1.00	0.99	-0.67	91.18	1.00	-0.38	96.97	1.01	0.55	93.94
		λ_{x_5} 1.00	1.01	0.54	97.06	1.01	0.60	87.88	1.00	0.39	96.97
		λ_{y_2} 1.00	1.01	1.28	97.06	1.01	0.76	96.97	0.98	-2.32	100.00
		λ_{y_3} 1.00	1.00	-0.32	97.06	0.98	-1.96	93.94	0.99	-0.68	96.97

[1] W^* is the population and analysis W condition.

[2] N is the simulated sample size.

[3] θ is the simulated population value of each parameter.

[4] $\rho_{y2} = ...$ is the simulated spatial autocorrelation of the endogenous lag.

[5] $\bar{\hat{\theta}}$ is the obtained average posterior mean across simulated iterations.

[6] $Bias(\bar{\hat{\theta}})\%$ is the average percent bias across simulation iterations when $\theta \neq 0$, otherwise $Bias(\bar{\hat{\theta}})\%$ provides absolute bias.

[7] Cover% is the percent of simulated trials in which the population value falls within the central 95% density of the posterior estimate.

[8] $\phi_\zeta = 0.3$ at the population level.

Table A.4: Results table for Study 1 simultaneous structural lag population model (D4) under $\phi_\zeta = 0.6$ condition with non-spatial analysis model (A1)

				$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
W_C^*												
	49											
		α	0.00	-0.01	-0.55	96.77	-0.01	-1.44	93.52	0.00	0.17	79.15
		γ_1	0.30	0.32	6.86	97.31	0.32	5.81	97.51	0.35	15.55	91.96
		γ_2	0.30	0.31	3.10	97.85	0.33	8.50	97.26	0.34	13.56	92.96
		γ_3	0.15	0.16	3.97	96.77	0.17	13.36	96.51	0.18	18.31	95.23
		σ_{x_1}	0.50	0.52	3.90	95.16	0.51	1.82	96.01	0.50	0.39	94.97
		σ_{x_2}	0.50	0.51	2.20	93.01	0.51	1.00	95.01	0.51	2.60	94.22
		σ_{x_3}	0.50	0.50	-0.39	94.62	0.49	-1.09	93.27	0.50	0.74	94.97
		σ_{x_4}	0.50	0.49	-1.55	95.16	0.51	2.83	92.77	0.51	2.55	94.47
		σ_{x_5}	0.50	0.50	0.72	94.62	0.51	1.18	95.76	0.50	0.84	93.72
		σ_{x_6}	0.50	0.50	0.90	93.55	0.50	0.31	96.26	0.51	1.70	95.73
		σ_{y_1}	0.50	0.48	-3.52	91.40	0.50	-0.52	96.26	0.50	-0.66	94.97
		σ_{y_2}	0.50	0.51	1.25	94.62	0.50	-0.06	96.51	0.50	0.68	94.72
		σ_{y_3}	0.50	0.50	-0.47	96.77	0.50	0.01	95.01	0.51	1.37	96.48
		σ_{ξ_1}	1.00	1.00	0.11	94.09	1.00	-0.15	95.51	1.02	2.16	94.47
		σ_{ξ_2}	1.00	1.02	2.47	93.55	1.00	0.37	95.51	1.01	0.71	95.23
		ν_{x_1}	0.00	-0.02	-2.43	96.24	0.00	-0.36	97.26	0.01	0.58	95.48
		ν_{x_2}	0.00	-0.02	-1.62	94.09	-0.01	-1.19	96.51	0.00	0.19	96.48
		ν_{x_3}	0.00	-0.02	-1.87	97.31	-0.01	-0.83	96.01	0.00	0.09	95.48
		ν_{x_4}	0.00	0.00	0.06	95.70	-0.01	-1.05	95.76	-0.01	-0.82	96.23
		ν_{x_5}	0.00	0.01	0.73	96.24	-0.01	-1.08	96.76	0.00	-0.30	95.23
		ν_{x_6}	0.00	0.00	0.14	93.55	-0.01	-1.16	96.26	0.00	-0.29	95.23
		ν_{y_2}	0.00	0.00	-0.32	96.77	0.01	0.80	94.51	-0.01	-0.77	94.72
		ν_{y_3}	0.00	0.00	-0.16	94.09	0.01	0.95	95.01	0.00	-0.17	96.73
		λ_{x_2}	1.00	1.02	1.99	96.77	1.03	2.73	96.01	1.01	1.21	95.23
		λ_{x_3}	1.00	1.03	2.96	95.16	1.03	3.45	95.26	1.02	1.95	96.23
		λ_{x_4}	1.00	1.01	0.64	95.70	1.03	3.17	95.51	1.03	2.62	96.73
		λ_{x_5}	1.00	1.01	0.68	93.55	1.03	2.95	95.76	1.03	2.88	95.73
		λ_{y_2}	1.00	0.93	-6.80	92.47	0.94	-5.83	95.51	0.96	-4.33	94.22
		λ_{y_3}	1.00	0.95	-5.04	90.32	0.91	-8.93	93.77	0.95	-5.42	92.21
	196											
		α	0.00	0.00	-0.48	90.61	0.00	0.27	91.92	0.00	-0.23	77.75
		γ_1	0.30	0.30	1.46	96.69	0.31	4.59	95.71	0.33	10.73	85.42
		γ_2	0.30	0.31	1.85	94.48	0.31	3.20	94.19	0.33	10.54	83.38
		γ_3	0.15	0.15	1.38	94.48	0.15	2.96	95.71	0.17	11.27	91.05
		σ_{x_1}	0.50	0.50	0.99	93.92	0.50	0.48	94.70	0.50	-0.19	94.12
		σ_{x_2}	0.50	0.50	-0.59	97.24	0.50	0.20	93.43	0.50	0.30	94.63
		σ_{x_3}	0.50	0.50	0.05	91.16	0.50	0.03	92.93	0.50	0.44	95.14
		σ_{x_4}	0.50	0.50	0.58	94.48	0.50	0.53	91.41	0.50	0.58	96.93
		σ_{x_5}	0.50	0.50	0.24	95.58	0.50	0.32	94.19	0.50	0.20	95.91
		σ_{x_6}	0.50	0.50	0.17	93.37	0.50	-0.17	94.44	0.50	0.31	95.91
		σ_{y_1}	0.50	0.50	-0.89	94.48	0.50	-0.78	93.18	0.50	0.38	95.14
		σ_{y_2}	0.50	0.50	-0.21	95.58	0.50	-0.51	94.19	0.50	-0.31	95.91
		σ_{y_3}	0.50	0.49	-1.18	96.13	0.50	-0.06	93.43	0.50	0.33	95.91
		σ_{ξ_1}	1.00	1.01	0.84	94.48	1.00	0.15	94.44	1.01	0.84	94.63
		σ_{ξ_2}	1.00	1.01	0.85	92.27	1.00	0.20	96.21	1.00	0.29	93.35
		ν_{x_1}	0.00	0.00	-0.17	93.37	0.00	0.32	94.95	-0.01	-0.79	96.16
		ν_{x_2}	0.00	0.01	0.51	88.95	0.00	0.49	93.94	0.00	-0.49	94.63
		ν_{x_3}	0.00	0.00	0.45	92.27	0.00	0.37	94.95	0.00	-0.45	94.37
		ν_{x_4}	0.00	0.00	-0.29	94.48	0.00	-0.23	93.18	0.00	-0.08	96.16
		ν_{x_5}	0.00	0.00	-0.19	97.79	0.00	-0.09	94.19	0.00	0.29	94.12
		ν_{x_6}	0.00	0.00	0.00	95.58	0.00	-0.19	93.69	0.00	0.23	96.42
		ν_{y_2}	0.00	0.00	0.41	97.24	0.00	0.10	95.45	0.00	0.01	97.44
		ν_{y_3}	0.00	0.01	0.53	93.92	0.00	-0.07	95.20	0.00	0.03	95.65

Table A.4: Results table for Study 1 simultaneous structural lag population model (D4) under $\phi_\zeta = 0.6$ condition with non-spatial analysis model (A1) *(continued)*

			$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
		λ_{x_2} 1.00	1.01	0.88	92.82	1.00	0.32	95.20	1.00	0.48	94.63
		λ_{x_3} 1.00	1.00	0.36	95.03	1.01	0.78	95.20	1.00	0.11	95.65
		λ_{x_4} 1.00	1.01	0.80	96.13	1.00	0.29	95.71	1.01	0.81	96.68
		λ_{x_5} 1.00	1.01	0.83	94.48	1.01	0.69	95.20	1.01	0.56	95.40
		λ_{y_2} 1.00	0.99	-1.27	92.27	0.98	-2.37	95.96	0.99	-0.64	93.09
		λ_{y_3} 1.00	0.98	-1.64	92.27	0.98	-1.78	92.42	0.99	-1.18	93.09
	400										
		α 0.00	0.00	-0.10	95.12	0.00	-0.14	88.98	0.00	0.02	77.47
		γ_1 0.30	0.31	1.79	90.24	0.31	2.07	94.88	0.32	7.78	82.61
		γ_2 0.30	0.31	1.80	97.56	0.31	2.75	94.09	0.32	7.54	83.00
		γ_3 0.15	0.16	6.14	90.24	0.15	2.77	96.46	0.16	8.33	88.93
		σ_{x_1} 0.50	0.50	-0.06	95.12	0.50	0.67	94.88	0.50	-0.22	94.07
		σ_{x_2} 0.50	0.51	1.65	90.24	0.50	0.29	94.09	0.50	-0.19	96.84
		σ_{x_3} 0.50	0.50	-0.10	90.24	0.50	-0.27	96.46	0.50	0.46	96.44
		σ_{x_4} 0.50	0.50	0.98	97.56	0.50	0.07	93.31	0.50	0.48	96.44
		σ_{x_5} 0.50	0.51	1.16	97.56	0.50	-0.19	95.28	0.50	0.56	91.70
		σ_{x_6} 0.50	0.50	-0.60	97.56	0.50	0.32	97.64	0.50	-0.48	96.05
		σ_{y_1} 0.50	0.50	0.07	97.56	0.50	-0.77	93.70	0.50	0.31	95.65
		σ_{y_2} 0.50	0.49	-1.31	97.56	0.50	-0.44	93.31	0.50	0.12	97.63
		σ_{y_3} 0.50	0.50	-0.48	95.12	0.50	-0.28	96.85	0.50	-0.36	93.28
		σ_{ξ_1} 1.00	1.01	0.80	97.56	1.00	0.31	92.91	0.99	-0.58	91.30
		σ_{ξ_2} 1.00	1.00	0.19	92.68	1.00	0.07	94.49	0.99	-0.60	92.89
		ν_{x_1} 0.00	0.00	0.27	92.68	0.00	-0.31	92.13	0.00	-0.18	95.26
		ν_{x_2} 0.00	0.00	-0.21	95.12	0.00	-0.18	93.31	0.00	0.28	92.89
		ν_{x_3} 0.00	-0.01	-0.72	95.12	-0.01	-0.55	91.73	0.00	0.06	94.07
		ν_{x_4} 0.00	-0.01	-0.95	82.93	0.00	-0.13	94.49	0.01	0.75	94.47
		ν_{x_5} 0.00	-0.01	-0.60	90.24	0.00	-0.47	95.28	0.00	0.30	94.47
		ν_{x_6} 0.00	0.00	-0.27	87.80	0.00	-0.23	95.28	0.00	0.34	95.26
		ν_{y_2} 0.00	0.00	0.46	92.68	0.00	-0.37	94.09	0.00	0.33	96.44
		ν_{y_3} 0.00	0.00	0.28	92.68	0.00	-0.03	93.31	0.00	0.37	93.68
		λ_{x_2} 1.00	1.00	-0.21	92.68	1.00	0.12	98.03	1.01	0.76	96.84
		λ_{x_3} 1.00	0.99	-0.73	87.80	1.00	0.27	96.46	1.00	0.20	95.26
		λ_{x_4} 1.00	1.01	0.75	95.12	1.01	0.66	93.31	1.01	0.62	95.26
		λ_{x_5} 1.00	1.01	0.50	100.00	1.00	0.34	95.28	1.00	0.28	96.05
		λ_{y_2} 1.00	0.98	-1.79	90.24	0.98	-1.57	94.49	0.99	-1.18	92.09
		λ_{y_3} 1.00	0.96	-4.34	92.68	0.99	-1.41	94.49	0.99	-0.94	94.47
W_D^*											
	49										
		α 0.00	-0.01	-0.97	97.01	0.01	0.93	91.02	0.02	2.02	75.45
		γ_1 0.30	0.32	6.48	98.80	0.32	7.39	93.41	0.32	5.39	98.20
		γ_2 0.30	0.33	8.45	95.81	0.33	9.18	97.01	0.32	7.13	95.21
		γ_3 0.15	0.17	10.55	95.21	0.16	6.41	95.81	0.16	5.45	97.60
		σ_{x_1} 0.50	0.50	-0.28	96.41	0.50	0.10	92.22	0.51	1.66	95.21
		σ_{x_2} 0.50	0.51	1.44	91.62	0.51	1.71	94.61	0.50	-0.15	91.62
		σ_{x_3} 0.50	0.50	0.82	95.21	0.52	3.62	94.61	0.53	5.08	95.81
		σ_{x_4} 0.50	0.51	1.57	93.41	0.52	4.29	96.41	0.51	1.01	91.02
		σ_{x_5} 0.50	0.51	2.41	93.41	0.50	-0.54	95.81	0.50	0.82	97.01
		σ_{x_6} 0.50	0.50	-0.06	97.60	0.51	2.82	92.81	0.50	0.67	95.81
		σ_{y_1} 0.50	0.49	-1.51	94.01	0.50	0.00	94.61	0.50	-0.20	98.20
		σ_{y_2} 0.50	0.51	1.36	95.81	0.50	-0.15	95.81	0.51	2.72	95.21
		σ_{y_3} 0.50	0.50	0.85	92.81	0.51	1.01	96.41	0.50	-0.73	94.61
		σ_{ξ_1} 1.00	1.00	0.20	94.61	1.01	1.00	96.41	1.00	0.44	95.81
		σ_{ξ_2} 1.00	1.00	-0.28	96.41	0.98	-1.97	95.81	1.01	0.77	95.81
		ν_{x_1} 0.00	-0.01	-0.92	96.41	0.02	2.17	95.81	0.01	1.43	97.60

Table A.4: Results table for Study 1 simultaneous structural lag population model (D4) under $\phi_\zeta = 0.6$ condition with non-spatial analysis model (A1) *(continued)*

			$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
		ν_{x_2} 0.00	0.00	0.04	97.01	0.02	2.48	94.61	0.00	0.47	94.61
		ν_{x_3} 0.00	0.00	-0.39	95.21	0.02	2.17	98.80	0.01	1.01	97.60
		ν_{x_4} 0.00	-0.02	-2.23	96.41	0.00	0.50	94.01	0.02	1.55	97.01
		ν_{x_5} 0.00	-0.02	-1.82	97.60	0.01	1.16	95.21	0.02	2.36	94.01
		ν_{x_6} 0.00	-0.01	-1.38	96.41	0.01	1.14	95.81	0.01	1.05	96.41
		ν_{y_2} 0.00	0.01	1.15	94.01	0.00	0.24	95.21	-0.01	-1.17	96.41
		ν_{y_3} 0.00	0.00	-0.25	96.41	0.01	1.32	96.41	0.00	-0.35	95.21
		λ_{x_2} 1.00	1.03	2.69	96.41	1.03	2.72	95.81	1.04	3.56	94.01
		λ_{x_3} 1.00	1.02	2.15	96.41	1.02	2.43	95.21	1.03	3.32	95.21
		λ_{x_4} 1.00	1.02	2.30	95.81	1.04	4.07	96.41	1.02	1.98	96.41
		λ_{x_5} 1.00	1.05	4.51	95.21	1.02	2.18	96.41	1.01	1.01	95.21
		λ_{y_2} 1.00	0.90	-9.60	88.02	0.93	-7.03	94.01	0.92	-7.60	92.22
		λ_{y_3} 1.00	0.93	-6.59	92.22	0.91	-9.44	92.22	0.92	-7.59	92.22
	196										
		α 0.00	0.01	0.78	97.52	-0.01	-0.61	93.75	0.01	1.17	72.15
		γ_1 0.30	0.31	2.60	96.27	0.31	2.11	93.75	0.31	2.84	95.57
		γ_2 0.30	0.31	2.57	94.41	0.29	-2.49	95.62	0.31	1.76	94.94
		γ_3 0.15	0.15	-0.50	94.41	0.15	-1.36	98.12	0.16	4.43	93.67
		σ_{x_1} 0.50	0.50	0.25	95.03	0.50	0.90	96.88	0.50	0.25	94.94
		σ_{x_2} 0.50	0.50	0.70	95.03	0.50	0.47	96.25	0.50	0.61	95.57
		σ_{x_3} 0.50	0.50	-0.26	93.17	0.50	-0.37	96.25	0.50	0.10	95.57
		σ_{x_4} 0.50	0.50	0.06	95.65	0.51	1.15	93.12	0.51	1.33	93.67
		σ_{x_5} 0.50	0.51	1.10	93.17	0.51	1.32	91.25	0.50	-0.29	97.47
		σ_{x_6} 0.50	0.50	0.97	93.79	0.50	0.11	96.88	0.50	0.78	96.84
		σ_{y_1} 0.50	0.50	-0.87	93.79	0.50	-0.89	93.75	0.50	-0.65	93.04
		σ_{y_2} 0.50	0.50	-0.85	98.14	0.49	-1.15	91.25	0.50	0.10	95.57
		σ_{y_3} 0.50	0.50	-0.41	93.17	0.50	-0.50	94.38	0.50	0.41	93.67
		σ_{ξ_1} 1.00	1.00	0.33	95.03	0.99	-0.54	95.00	1.01	0.79	94.94
		σ_{ξ_2} 1.00	1.00	0.33	95.03	1.00	0.07	95.00	1.01	0.83	93.67
		ν_{x_1} 0.00	0.00	0.22	96.27	-0.01	-0.93	95.62	0.00	-0.15	96.84
		ν_{x_2} 0.00	0.00	0.40	94.41	0.00	-0.39	93.75	0.00	0.09	92.41
		ν_{x_3} 0.00	0.00	-0.19	92.55	-0.01	-1.33	91.25	0.00	0.35	95.57
		ν_{x_4} 0.00	0.00	0.32	98.14	0.00	-0.23	97.50	0.00	0.38	93.67
		ν_{x_5} 0.00	0.00	0.13	98.14	0.00	0.21	96.25	0.00	0.17	94.94
		ν_{x_6} 0.00	0.00	0.50	98.76	0.00	-0.01	95.62	0.00	0.43	93.67
		ν_{y_2} 0.00	-0.01	-0.50	96.89	0.01	0.56	93.75	-0.01	-0.53	93.04
		ν_{y_3} 0.00	-0.01	-0.73	94.41	0.00	0.08	93.75	-0.01	-1.23	94.94
		λ_{x_2} 1.00	1.00	-0.40	96.89	1.01	0.84	94.38	1.01	0.82	94.94
		λ_{x_3} 1.00	0.99	-0.62	94.41	1.01	0.87	93.12	1.01	1.36	93.67
		λ_{x_4} 1.00	1.01	0.61	95.65	1.00	0.12	97.50	1.01	0.94	97.47
		λ_{x_5} 1.00	1.01	0.89	93.79	1.01	0.90	95.62	1.00	0.27	96.84
		λ_{y_2} 1.00	0.98	-1.72	93.79	0.99	-1.14	92.50	0.97	-2.79	91.77
		λ_{y_3} 1.00	0.97	-2.80	92.55	1.00	0.13	95.00	0.99	-1.38	95.57
	400										
		α 0.00	0.01	0.61	96.97	0.00	-0.10	90.62	0.01	0.72	75.00
		γ_1 0.30	0.30	0.95	87.88	0.30	0.43	90.62	0.29	-1.74	92.86
		γ_2 0.30	0.31	2.50	96.97	0.31	3.54	90.62	0.30	-0.69	89.29
		γ_3 0.15	0.15	3.02	100.00	0.15	-0.34	93.75	0.15	1.92	100.00
		σ_{x_1} 0.50	0.50	-0.24	90.91	0.50	-0.51	100.00	0.50	-0.52	96.43
		σ_{x_2} 0.50	0.50	-0.07	100.00	0.49	-1.26	90.62	0.50	0.17	96.43
		σ_{x_3} 0.50	0.51	2.12	96.97	0.49	-2.44	100.00	0.51	1.95	96.43
		σ_{x_4} 0.50	0.50	0.20	96.97	0.50	0.68	90.62	0.50	0.55	96.43
		σ_{x_5} 0.50	0.50	0.24	93.94	0.51	1.17	93.75	0.51	1.09	100.00
		σ_{x_6} 0.50	0.50	0.78	100.00	0.50	-0.34	100.00	0.50	-0.74	92.86
		σ_{y_1} 0.50	0.50	-0.37	96.97	0.50	-0.32	90.62	0.49	-1.16	92.86

Table A.4: Results table for Study 1 simultaneous structural lag population model (D4) under $\phi_\zeta = 0.6$ condition with non-spatial analysis model (A1) *(continued)*

			$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
	σ_{y_2}	0.50	0.49	-1.17	96.97	0.50	-0.33	96.88	0.49	-1.57	100.00
	σ_{y_3}	0.50	0.50	-0.80	93.94	0.50	0.81	100.00	0.50	-0.80	100.00
	σ_{ξ_1}	1.00	1.00	0.21	100.00	1.01	1.01	93.75	0.99	-0.62	96.43
	σ_{ξ_2}	1.00	1.00	0.00	93.94	1.00	0.38	90.62	1.01	1.13	89.29
	ν_{x_1}	0.00	0.01	1.14	93.94	0.00	-0.18	93.75	0.02	1.97	100.00
	ν_{x_2}	0.00	0.01	0.85	100.00	0.00	0.14	96.88	0.02	2.17	92.86
	ν_{x_3}	0.00	0.00	0.11	100.00	0.00	0.30	87.50	0.01	1.24	92.86
	ν_{x_4}	0.00	-0.01	-1.31	100.00	-0.01	-0.58	100.00	-0.01	-0.66	89.29
	ν_{x_5}	0.00	0.00	0.02	100.00	0.01	0.53	93.75	-0.02	-1.68	92.86
	ν_{x_6}	0.00	0.00	0.10	100.00	0.00	0.03	96.88	-0.02	-1.62	92.86
	ν_{y_2}	0.00	-0.01	-0.61	93.94	0.01	0.55	93.75	0.00	-0.25	100.00
	ν_{y_3}	0.00	-0.01	-1.14	90.91	0.01	0.66	96.88	0.01	0.50	100.00
	λ_{x_2}	1.00	1.00	-0.36	96.97	1.00	-0.32	96.88	1.00	0.19	96.43
	λ_{x_3}	1.00	1.00	0.38	93.94	1.00	0.01	96.88	1.00	0.18	85.71
	λ_{x_4}	1.00	1.01	1.13	93.94	1.00	0.33	96.88	0.99	-1.23	96.43
	λ_{x_5}	1.00	1.01	0.53	100.00	1.00	0.17	90.62	1.00	-0.11	92.86
	λ_{y_2}	1.00	1.00	-0.24	96.97	0.98	-1.53	93.75	1.02	1.64	100.00
	λ_{y_3}	1.00	0.99	-0.94	93.94	1.00	-0.41	87.50	1.01	0.97	100.00

[1] W^* is the population and analysis W condition.

[2] N is the simulated sample size.

[3] θ is the simulated population value of each parameter.

[4] $\rho_{y2} = ...$ is the simulated spatial autocorrelation of the endogenous lag.

[5] $\bar{\hat{\theta}}$ is the obtained average posterior mean across simulated iterations.

[6] $Bias(\bar{\hat{\theta}})\%$ is the average percent bias across simulation iterations when $\theta \neq 0$, otherwise $Bias(\bar{\hat{\theta}})\%$ provides absolute bias.

[7] Cover% is the percent of simulated trials in which the population value falls within the central 95% density of the posterior estimate.

[8] $\phi_\zeta = 0.6$ at the population level.

A.2 Study 2

Table A.5: Results table for Study 2 measurement lag population model (D2) and measurement lag analysis model (A2)

				$\rho_{y2} = 0$			$\rho_{y2} = 0.3$			$\rho_{y2} = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
W_C^*												
	49											
		ρ_{y2}	-	0.41	41.43	0.00	0.51	70.54	98.26	0.65	8.12	98.56
		α	0.00	-0.01	-0.52	95.91	0.00	0.10	94.68	0.00	-0.08	95.28
		γ_1	0.30	0.32	6.76	96.52	0.32	7.69	96.42	0.32	8.31	96.31
		γ_2	0.30	0.32	6.83	97.14	0.32	6.11	96.83	0.32	7.87	96.72
		γ_3	0.15	0.16	8.24	95.40	0.16	9.65	96.52	0.16	9.00	95.69
		σ_{x_1}	0.50	0.51	1.12	92.84	0.51	1.21	94.89	0.50	0.98	93.95
		σ_{x_2}	0.50	0.51	1.69	94.48	0.51	1.02	93.87	0.50	0.89	96.21
		σ_{x_3}	0.50	0.51	1.70	95.30	0.50	-0.09	93.87	0.51	1.39	95.18
		σ_{x_4}	0.50	0.51	2.58	93.46	0.51	1.38	94.89	0.51	1.77	95.69
		σ_{x_5}	0.50	0.50	-0.22	94.89	0.51	1.11	94.89	0.50	0.84	94.87
		σ_{x_6}	0.50	0.51	2.12	95.09	0.50	0.83	94.48	0.50	0.98	94.77
		σ_{y_1}	0.50	0.49	-1.61	94.79	0.49	-1.84	96.22	0.49	-1.41	94.46
		σ_{y_2}	0.50	0.50	0.10	94.99	0.49	-1.01	95.40	0.50	0.82	96.10
		σ_{y_3}	0.50	0.50	0.11	90.70	0.31	-37.29	79.24	0.22	-56.29	37.95
		σ_{ξ_1}	1.00	1.01	1.41	93.76	1.01	0.85	94.58	1.01	0.79	94.97
		σ_{ξ_2}	1.00	1.01	1.32	93.76	1.00	0.49	96.22	1.01	0.99	93.74
		ν_{x_1}	0.00	0.00	-0.48	95.50	0.00	0.14	95.19	-0.01	-0.92	94.97
		ν_{x_2}	0.00	-0.01	-0.60	96.01	0.00	-0.42	93.35	-0.01	-0.87	93.85
		ν_{x_3}	0.00	0.00	-0.29	96.22	0.00	0.16	95.30	0.00	-0.42	94.26
		ν_{x_4}	0.00	0.01	0.65	94.58	0.01	1.15	95.40	0.00	0.27	94.46
		ν_{x_5}	0.00	0.01	0.75	95.40	0.01	0.89	96.73	0.00	-0.01	94.97
		ν_{x_6}	0.00	0.01	0.61	94.07	0.01	1.37	96.11	0.00	0.35	96.10
		ν_{y_2}	0.00	0.00	0.24	94.17	0.00	0.20	95.40	0.00	-0.40	95.08
		ν_{y_3}	0.00	0.00	0.49	96.22	0.00	0.02	94.27	0.00	-0.18	96.82
		λ_{x_2}	1.00	1.01	1.41	94.07	1.02	2.08	95.81	1.03	2.87	96.41
		λ_{x_3}	1.00	1.01	1.48	94.58	1.03	2.51	96.22	1.03	3.08	95.18
		λ_{x_4}	1.00	1.02	2.02	94.48	1.03	2.64	95.71	1.02	2.33	95.28
		λ_{x_5}	1.00	1.02	1.60	97.03	1.02	2.47	95.19	1.02	2.21	94.46
		λ_{y_2}	1.00	0.92	-8.17	93.87	0.94	-6.26	93.56	0.93	-7.19	92.62
		λ_{y_3}	1.00	0.91	-9.11	91.62	0.92	-7.83	92.43	0.92	-8.44	93.85
	196											
		ρ_{y2}	-	0.37	37.37	0.00	0.51	71.45	89.83	0.72	19.63	87.22
		α	0.00	0.00	0.00	94.17	0.00	-0.25	94.59	0.00	-0.49	96.64
		γ_1	0.30	0.30	1.58	95.58	0.30	1.37	94.81	0.30	1.61	96.53
		γ_2	0.30	0.31	1.71	93.96	0.31	1.98	94.05	0.30	1.01	96.53
		γ_3	0.15	0.15	2.90	96.55	0.15	2.26	95.02	0.15	1.73	95.56
		σ_{x_1}	0.50	0.50	0.17	95.47	0.50	0.44	94.05	0.50	0.40	94.15
		σ_{x_2}	0.50	0.50	0.45	94.07	0.50	-0.05	94.37	0.50	-0.29	94.58
		σ_{x_3}	0.50	0.50	0.19	94.71	0.50	0.44	95.02	0.50	0.33	92.20
		σ_{x_4}	0.50	0.50	0.82	93.20	0.50	0.59	95.78	0.50	0.29	93.93
		σ_{x_5}	0.50	0.50	0.64	94.93	0.50	0.23	92.97	0.50	0.27	95.45
		σ_{x_6}	0.50	0.50	-0.84	95.04	0.50	0.56	94.91	0.50	0.86	93.93
		σ_{y_1}	0.50	0.50	-0.62	94.61	0.49	-1.19	95.24	0.50	-1.00	95.99
		σ_{y_2}	0.50	0.50	-0.46	95.69	0.50	-0.27	95.24	0.50	-0.45	95.56
		σ_{y_3}	0.50	0.49	-0.30	82.63	0.29	-42.33	24.89	0.16	-67.86	0.00
		σ_{ξ_1}	1.00	1.00	-0.08	94.39	1.00	0.24	95.89	1.00	0.43	96.10
		σ_{ξ_2}	1.00	1.00	0.18	95.04	1.01	0.53	94.81	1.00	0.35	94.69
		ν_{x_1}	0.00	0.00	0.11	95.25	0.01	0.68	92.97	0.00	-0.34	95.02
		ν_{x_2}	0.00	0.00	0.28	94.93	0.00	0.44	92.97	0.00	-0.26	92.74
		ν_{x_3}	0.00	0.01	0.55	94.61	0.00	0.34	92.32	0.00	-0.44	94.58

Table A.5: Results table for Study 2 Measurement lag population model (D2) and measurement lag analysis model (A2) *(continued)*

			$\rho_{y2} = 0$			$\rho_{y2} = 0.3$			$\rho_{y2} = 0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
	ν_{x_4}	0.00	0.00	-0.19	94.93	-0.01	-0.53	94.81	0.00	-0.07	94.80
	ν_{x_5}	0.00	0.00	-0.29	95.69	0.00	-0.46	93.29	0.00	-0.22	94.47
	ν_{x_6}	0.00	0.00	-0.12	95.47	-0.01	-0.54	94.81	0.00	0.13	93.93
	ν_{y_2}	0.00	0.00	-0.16	93.74	0.00	0.09	94.81	0.00	0.45	95.56
	ν_{y_3}	0.00	0.00	-0.09	94.39	0.00	0.20	95.35	0.01	0.55	95.99
	λ_{x_2}	1.00	1.01	0.58	94.50	1.00	0.36	95.13	1.01	0.71	96.53
	λ_{x_3}	1.00	1.01	0.66	95.25	1.00	0.45	94.91	1.01	0.53	95.99
	λ_{x_4}	1.00	1.01	0.88	94.82	1.00	0.31	95.67	1.00	0.46	94.58
	λ_{x_5}	1.00	1.01	0.98	94.28	1.00	0.28	93.83	1.01	0.77	94.15
	λ_{y_2}	1.00	0.99	-1.12	94.28	0.98	-1.60	95.56	0.99	-1.13	93.61
	λ_{y_3}	1.00	0.99	-1.07	92.99	0.99	-1.39	93.40	0.99	-1.47	94.91
	400										
	ρ_{y2}	-	0.35	35.28	0.00	0.53	76.56	85.71	0.73	20.88	84.19
	α	0.00	0.00	-0.03	94.99	0.00	0.02	94.26	0.00	-0.03	95.22
	γ_1	0.30	0.30	-1.02	95.24	0.30	0.34	94.26	0.30	0.02	95.71
	γ_2	0.30	0.30	0.84	95.36	0.30	0.31	95.97	0.30	0.05	93.75
	γ_3	0.15	0.15	-2.91	94.75	0.15	0.11	93.77	0.15	-1.94	96.20
	σ_{x_1}	0.50	0.50	0.01	95.48	0.50	-0.10	95.48	0.50	0.53	94.24
	σ_{x_2}	0.50	0.50	0.75	95.60	0.50	0.05	95.36	0.50	0.18	95.83
	σ_{x_3}	0.50	0.50	0.03	94.63	0.50	0.51	94.75	0.50	0.61	94.73
	σ_{x_4}	0.50	0.50	0.00	94.02	0.50	-0.17	95.36	0.50	0.13	95.10
	σ_{x_5}	0.50	0.50	-0.06	94.99	0.50	-0.26	95.36	0.50	0.19	93.87
	σ_{x_6}	0.50	0.50	0.04	94.14	0.50	0.41	94.87	0.50	0.15	93.50
	σ_{y_1}	0.50	0.50	-0.69	94.26	0.50	-0.56	95.12	0.50	-0.53	93.75
	σ_{y_2}	0.50	0.50	-0.41	95.60	0.50	-0.71	94.26	0.50	-0.32	96.20
	σ_{y_3}	0.50	0.49	-1.03	80.83	0.28	-44.87	3.30	0.15	-69.78	0.00
	σ_{ξ_1}	1.00	1.00	0.14	94.87	1.00	0.05	94.75	1.00	-0.24	94.36
	σ_{ξ_2}	1.00	1.00	0.29	93.53	1.00	0.30	94.14	1.00	0.07	95.22
	ν_{x_1}	0.00	0.00	-0.01	96.46	0.00	0.13	95.73	0.01	0.52	95.71
	ν_{x_2}	0.00	0.00	-0.03	96.46	0.00	0.05	95.97	0.00	0.38	95.47
	ν_{x_3}	0.00	0.00	-0.05	95.36	0.00	0.04	95.60	0.00	0.30	95.47
	ν_{x_4}	0.00	0.00	-0.28	95.85	0.00	0.03	96.83	0.00	0.06	94.85
	ν_{x_5}	0.00	0.00	-0.41	93.89	0.00	0.13	96.46	0.00	0.18	94.49
	ν_{x_6}	0.00	0.00	-0.37	94.51	0.00	0.03	96.46	0.00	0.23	93.50
	ν_{y_2}	0.00	0.00	0.02	96.34	0.00	-0.04	94.87	0.00	0.15	94.61
	ν_{y_3}	0.00	0.00	-0.07	96.46	0.00	-0.06	95.36	0.00	0.12	96.20
	λ_{x_2}	1.00	1.00	-0.14	94.99	1.00	0.14	94.99	1.00	0.04	94.24
	λ_{x_3}	1.00	1.00	0.05	94.87	1.00	0.20	94.75	1.00	0.03	94.24
	λ_{x_4}	1.00	1.00	-0.03	95.85	1.00	0.04	95.24	1.00	-0.16	95.47
	λ_{x_5}	1.00	1.00	-0.39	94.38	1.00	0.05	94.87	1.00	-0.05	95.10
	λ_{y_2}	1.00	1.00	-0.48	94.75	0.99	-0.52	94.26	0.99	-0.73	94.61
	λ_{y_3}	1.00	0.99	-0.93	94.63	1.00	0.04	93.41	0.99	-0.93	92.89
W_D^*											
	49										
	ρ_{y2}	-	0.49	49.29	0.00	0.50	66.34	100.00	0.51	-15.64	100.00
	α	0.00	0.00	0.26	96.24	-0.01	-0.91	95.61	0.00	0.37	96.55
	γ_1	0.30	0.32	7.71	96.97	0.32	6.95	97.91	0.32	7.67	96.23
	γ_2	0.30	0.32	6.49	94.99	0.32	7.45	96.34	0.32	6.67	97.07
	γ_3	0.15	0.16	9.38	97.39	0.17	11.94	96.24	0.17	10.85	96.65
	σ_{x_1}	0.50	0.51	1.57	93.11	0.51	2.29	93.83	0.51	2.44	93.51
	σ_{x_2}	0.50	0.51	1.54	93.95	0.51	1.91	95.72	0.50	0.67	94.67
	σ_{x_3}	0.50	0.50	0.53	95.30	0.51	1.24	96.66	0.50	0.70	93.62
	σ_{x_4}	0.50	0.52	3.77	94.36	0.51	1.85	94.25	0.51	2.37	93.10
	σ_{x_5}	0.50	0.50	-0.38	94.78	0.50	-0.09	94.36	0.49	-1.34	94.35

Table A.5: Results table for Study 2 Measurement lag population model (D2) and measurement lag analysis model (A2) *(continued)*

				$\rho_{y2} = 0$			$\rho_{y2} = 0.3$			$\rho_{y2} = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
		σ_{x_6}	0.50	0.51	1.26	94.57	0.50	0.08	94.36	0.51	1.65	94.77
		σ_{y_1}	0.50	0.49	-2.03	95.30	0.49	-1.37	96.03	0.49	-1.15	95.19
		σ_{y_2}	0.50	0.50	0.90	93.74	0.50	0.78	94.36	0.50	-0.20	96.13
		σ_{y_3}	0.50	0.49	-2.04	94.94	0.33	-34.10	86.00	0.32	-35.43	86.61
		σ_{ξ_1}	1.00	1.00	0.42	94.36	1.01	1.28	94.25	1.01	1.49	94.77
		σ_{ξ_2}	1.00	1.00	-0.11	93.42	1.01	0.99	93.73	1.00	0.19	95.92
		ν_{x_1}	0.00	0.00	0.12	95.09	0.00	-0.25	96.24	0.00	0.23	95.92
		ν_{x_2}	0.00	0.00	0.13	96.35	0.00	-0.42	96.55	0.00	0.01	96.34
		ν_{x_3}	0.00	0.00	0.17	93.95	0.00	-0.18	96.66	0.00	0.43	96.55
		ν_{x_4}	0.00	0.01	0.81	95.41	-0.01	-1.16	94.88	0.01	0.92	95.19
		ν_{x_5}	0.00	0.00	0.23	95.62	-0.01	-1.22	96.45	0.01	0.51	96.44
		ν_{x_6}	0.00	0.00	0.47	96.03	-0.01	-1.16	94.67	0.00	0.33	95.29
		ν_{y_2}	0.00	0.00	-0.10	94.89	0.00	0.13	94.67	-0.01	-0.52	96.03
		ν_{y_3}	0.00	0.00	0.09	97.91	0.00	0.20	95.09	0.00	0.28	93.31
		λ_{x_2}	1.00	1.02	2.39	96.24	1.03	2.66	94.67	1.02	2.17	94.77
		λ_{x_3}	1.00	1.02	2.11	95.72	1.02	2.42	96.24	1.02	2.44	95.19
		λ_{x_4}	1.00	1.03	3.25	95.51	1.03	2.73	94.25	1.03	2.78	96.34
		λ_{x_5}	1.00	1.03	2.74	94.47	1.02	2.48	93.73	1.02	2.48	96.23
		λ_{y_2}	1.00	0.92	-7.51	92.69	0.92	-7.88	93.83	0.92	-7.79	93.62
		λ_{y_3}	1.00	0.92	-8.36	93.74	0.91	-8.81	93.21	0.90	-10.19	93.51
	196											
		ρ_{y2}	-	0.48	48.06	0.00	0.50	65.35	100.00	0.51	-14.27	100.00
		α	0.00	0.00	-0.26	95.61	0.00	0.21	94.62	0.00	0.03	95.36
		γ_1	0.30	0.30	1.13	96.38	0.31	1.71	95.82	0.30	0.69	93.71
		γ_2	0.30	0.30	1.42	96.93	0.31	1.81	94.51	0.30	1.56	97.02
		γ_3	0.15	0.15	1.90	94.08	0.15	2.93	96.59	0.15	2.01	96.14
		σ_{x_1}	0.50	0.51	1.15	94.08	0.50	0.18	94.40	0.50	0.91	94.70
		σ_{x_2}	0.50	0.50	0.42	93.86	0.50	-0.22	95.05	0.50	0.40	92.72
		σ_{x_3}	0.50	0.50	-0.10	95.39	0.50	0.83	94.40	0.50	0.12	95.70
		σ_{x_4}	0.50	0.50	0.56	96.38	0.50	0.84	94.84	0.51	1.07	94.48
		σ_{x_5}	0.50	0.50	0.89	94.63	0.50	0.65	94.84	0.50	0.17	95.03
		σ_{x_6}	0.50	0.50	0.11	94.19	0.50	0.66	94.73	0.50	0.50	94.81
		σ_{y_1}	0.50	0.49	-1.03	95.18	0.50	-0.94	96.04	0.50	-0.60	94.70
		σ_{y_2}	0.50	0.50	-0.79	94.63	0.50	-0.36	95.71	0.50	-0.74	93.93
		σ_{y_3}	0.50	0.51	1.06	72.92	0.32	-35.26	69.89	0.31	-37.45	65.34
		σ_{ξ_1}	1.00	1.00	0.11	94.41	1.00	0.30	94.62	1.00	0.23	94.59
		σ_{ξ_2}	1.00	1.00	-0.10	94.74	1.00	-0.14	94.18	1.00	-0.11	94.81
		ν_{x_1}	0.00	-0.01	-0.66	96.49	0.00	0.06	95.27	0.00	-0.19	95.36
		ν_{x_2}	0.00	-0.01	-0.61	95.61	0.00	0.14	95.49	0.00	-0.31	95.58
		ν_{x_3}	0.00	-0.01	-0.60	95.50	0.00	0.33	96.81	0.00	-0.45	95.14
		ν_{x_4}	0.00	0.00	-0.36	95.83	0.00	0.20	95.16	0.00	0.16	94.81
		ν_{x_5}	0.00	0.00	0.00	94.41	0.00	0.17	93.30	0.00	0.32	95.81
		ν_{x_6}	0.00	0.00	-0.17	93.42	0.00	-0.03	93.19	0.00	0.18	96.14
		ν_{y_2}	0.00	0.00	-0.26	95.50	0.00	-0.10	94.62	0.00	-0.13	95.03
		ν_{y_3}	0.00	0.00	0.13	97.81	0.00	-0.23	94.07	0.00	-0.33	89.85
		λ_{x_2}	1.00	1.00	0.36	95.07	1.00	0.43	94.40	1.01	0.66	93.82
		λ_{x_3}	1.00	1.01	0.71	94.52	1.01	0.71	94.73	1.01	0.59	93.93
		λ_{x_4}	1.00	1.01	0.84	93.86	1.00	0.42	95.60	1.01	0.82	93.49
		λ_{x_5}	1.00	1.01	0.55	94.30	1.01	0.54	95.05	1.01	0.93	94.59
		λ_{y_2}	1.00	0.99	-1.26	96.16	0.99	-1.02	94.62	0.99	-0.70	94.37
		λ_{y_3}	1.00	0.97	-2.71	94.30	0.99	-1.13	94.18	0.99	-0.70	94.15
	400											
		ρ_{y2}	-	0.49	48.75	0.00	0.50	67.37	100.00	0.52	-13.78	100.00
		α	0.00	0.00	-0.04	96.01	0.00	-0.24	94.58	0.00	-0.03	95.69
		γ_1	0.30	0.30	0.17	93.39	0.30	0.35	96.22	0.30	0.28	95.30

Table A.5: Results table for Study 2 Measurement lag population model (D2) and measurement lag analysis model (A2) *(continued)*

			$\rho_{y2} = 0$			$\rho_{y2} = 0.3$			$\rho_{y2} = 0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
	γ_2	0.30	0.30	0.39	96.63	0.30	0.24	95.21	0.30	1.39	96.45
	γ_3	0.15	0.15	-0.28	97.13	0.15	-0.98	95.47	0.15	-0.84	94.67
	σ_{x_1}	0.50	0.50	0.44	93.64	0.50	-0.04	94.33	0.50	0.87	95.69
	σ_{x_2}	0.50	0.50	0.43	95.64	0.50	0.62	94.84	0.50	0.19	95.30
	σ_{x_3}	0.50	0.50	-0.09	95.26	0.50	-0.03	94.71	0.50	0.54	94.04
	σ_{x_4}	0.50	0.50	0.49	94.14	0.50	-0.17	93.20	0.50	-0.05	94.92
	σ_{x_5}	0.50	0.50	-0.16	94.26	0.50	0.17	94.58	0.50	0.26	95.69
	σ_{x_6}	0.50	0.50	0.06	95.01	0.50	0.40	94.71	0.50	0.46	95.18
	σ_{y_1}	0.50	0.50	-0.67	95.14	0.50	-0.60	94.33	0.50	-0.64	94.80
	σ_{y_2}	0.50	0.50	-0.71	93.52	0.50	-0.60	95.84	0.50	-0.48	95.05
	σ_{y_3}	0.50	0.50	0.60	59.48	0.32	-36.14	54.91	0.31	-37.72	50.38
	σ_{ξ_1}	1.00	1.00	0.06	95.51	1.00	0.34	94.58	1.00	0.05	95.43
	σ_{ξ_2}	1.00	1.00	0.16	95.14	1.00	0.20	94.21	1.00	0.06	96.32
	ν_{x_1}	0.00	0.00	-0.04	95.14	0.00	-0.20	95.59	0.00	0.14	94.80
	ν_{x_2}	0.00	0.00	0.30	96.26	0.00	-0.18	95.09	0.00	0.07	95.30
	ν_{x_3}	0.00	0.00	0.04	94.89	0.00	-0.25	93.07	0.00	0.02	95.69
	ν_{x_4}	0.00	0.00	0.23	94.51	0.00	-0.06	93.95	0.00	-0.20	95.05
	ν_{x_5}	0.00	0.00	0.22	94.76	0.00	-0.23	96.60	0.00	0.02	93.91
	ν_{x_6}	0.00	0.00	0.07	94.89	0.00	0.04	95.21	0.00	-0.24	96.45
	ν_{y_2}	0.00	0.00	0.16	93.89	0.00	-0.20	93.20	0.00	0.01	95.05
	ν_{y_3}	0.00	0.00	0.06	97.01	0.00	0.03	94.58	0.00	0.01	90.86
	λ_{x_2}	1.00	1.00	0.26	95.26	1.00	0.05	93.32	1.00	0.34	93.27
	λ_{x_3}	1.00	1.00	0.24	94.51	1.00	0.27	94.21	1.00	0.41	95.05
	λ_{x_4}	1.00	1.00	0.30	92.52	1.00	0.06	96.47	1.00	0.38	96.19
	λ_{x_5}	1.00	1.01	0.53	93.14	1.00	0.01	94.71	1.00	0.31	94.80
	λ_{y_2}	1.00	0.99	-0.67	95.26	0.99	-1.15	93.45	0.99	-0.75	96.19
	λ_{y_3}	1.00	0.99	-0.65	96.63	0.98	-1.65	92.70	0.99	-1.06	92.64

[1] W^* is the population and analysis W condition.

[2] N is the simulated sample size.

[3] θ is the simulated population value of each parameter.

[4] $\rho_{y2} = ...$ is the simulated spatial autocorrelation of the endogenous lag.

[5] $\bar{\hat{\theta}}$ is the obtained average posterior mean across simulated iterations.

[6] $Bias(\bar{\hat{\theta}})\%$ is the average percent bias across simulation iterations when $\theta \neq 0$, otherwise $Bias(\bar{\hat{\theta}})\%$ provides absolute bias.

[7] Cover% is the percent of simulated trials in which the population value falls within the central 95% density of the posterior estimate.

Table A.6: Results table for Study 2 endogenous structural lag population model (D3) and endogenous structural lag analysis model (A3)

				$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
W_C^*												
	49											
		ρ_η	-	0.20	19.99	0.00	0.35	15.45	98.27	0.58	-3.21	97.33
		α	0.00	0.00	-0.14	97.73	0.00	0.28	95.73	0.00	-0.39	95.73
		γ_1	0.30	0.31	1.70	94.80	0.30	0.82	96.53	0.31	2.93	95.20
		γ_2	0.30	0.30	0.21	95.73	0.31	3.66	96.00	0.30	-0.90	95.73
		γ_3	0.15	0.14	-6.12	95.47	0.16	9.00	95.07	0.16	4.05	97.33
		σ_{x_1}	0.50	0.51	1.88	96.27	0.51	1.58	94.53	0.51	1.78	93.87
		σ_{x_2}	0.50	0.50	0.75	95.60	0.50	0.66	94.40	0.50	0.61	93.47
		σ_{x_3}	0.50	0.50	0.33	92.80	0.50	0.85	96.67	0.50	0.93	95.20
		σ_{x_4}	0.50	0.51	1.48	94.00	0.51	1.26	94.80	0.51	1.86	94.27
		σ_{x_5}	0.50	0.51	2.11	94.00	0.50	0.81	93.87	0.50	0.89	96.27
		σ_{x_6}	0.50	0.51	1.11	95.87	0.50	-0.12	96.27	0.50	0.78	95.47
		σ_{y_1}	0.50	0.50	0.33	92.53	0.50	-0.46	95.87	0.51	1.35	96.00
		σ_{y_2}	0.50	0.52	3.33	96.40	0.51	2.73	93.73	0.50	0.96	92.93
		σ_{y_3}	0.50	0.51	2.20	94.93	0.52	3.12	95.73	0.51	2.57	93.73
		σ_{ξ_1}	1.00	1.01	0.53	96.13	1.01	1.23	96.00	1.01	0.55	95.87
		σ_{ξ_2}	1.00	1.00	0.05	96.40	1.00	0.42	95.20	1.00	-0.46	96.40
		ν_{x_1}	0.00	-0.01	-0.75	95.20	0.00	-0.26	95.07	-0.01	-0.87	96.27
		ν_{x_2}	0.00	-0.01	-0.65	95.07	0.00	-0.10	96.13	0.00	-0.07	96.67
		ν_{x_3}	0.00	-0.01	-0.74	93.47	0.00	-0.22	95.47	0.00	-0.15	95.87
		ν_{x_4}	0.00	-0.01	-1.20	95.47	0.00	-0.15	96.67	0.00	-0.20	96.13
		ν_{x_5}	0.00	-0.01	-1.10	95.73	0.00	0.12	96.00	0.00	-0.08	94.67
		ν_{x_6}	0.00	-0.01	-0.97	96.00	0.00	-0.46	96.13	0.00	0.36	95.73
		ν_{y_2}	0.00	0.00	-0.22	95.07	0.00	0.05	95.07	0.00	-0.40	93.07
		ν_{y_3}	0.00	0.00	-0.30	95.73	0.00	-0.06	96.93	0.00	-0.45	96.00
		λ_{x_2}	1.00	1.03	2.56	96.40	1.02	2.32	95.60	1.02	2.17	95.07
		λ_{x_3}	1.00	1.02	2.34	96.80	1.02	2.50	96.53	1.02	2.09	96.00
		λ_{x_4}	1.00	1.03	2.53	95.47	1.01	1.15	93.87	1.02	2.41	95.07
		λ_{x_5}	1.00	1.03	2.80	95.47	1.02	2.38	94.13	1.02	2.19	95.87
		λ_{y_2}	1.00	0.98	-1.58	94.67	0.98	-1.76	93.87	0.99	-0.97	94.40
		λ_{y_3}	1.00	0.99	-0.94	95.07	0.98	-1.75	95.33	0.99	-0.59	96.13
	196											
		ρ_η	-	0.10	10.04	0.00	0.31	2.34	96.75	0.60	-0.37	93.93
		α	0.00	0.00	-0.16	97.04	0.00	0.00	95.90	0.00	0.24	94.92
		γ_1	0.30	0.29	-1.84	93.65	0.30	0.56	95.90	0.30	1.59	94.49
		γ_2	0.30	0.29	-1.75	96.33	0.30	-1.12	93.36	0.30	1.16	92.80
		γ_3	0.15	0.15	-1.13	94.22	0.16	4.43	95.06	0.15	0.16	94.77
		σ_{x_1}	0.50	0.50	0.60	94.08	0.50	0.91	94.92	0.51	1.04	94.63
		σ_{x_2}	0.50	0.50	0.34	95.06	0.50	0.55	95.06	0.51	1.21	95.06
		σ_{x_3}	0.50	0.50	0.47	96.05	0.50	0.29	94.92	0.50	-0.08	94.49
		σ_{x_4}	0.50	0.50	0.66	95.20	0.50	0.47	95.62	0.50	0.56	95.62
		σ_{x_5}	0.50	0.50	0.28	93.23	0.50	0.47	96.33	0.50	-0.02	95.48
		σ_{x_6}	0.50	0.50	0.53	92.81	0.50	0.16	97.18	0.51	1.05	94.92
		σ_{y_1}	0.50	0.50	0.42	94.64	0.50	0.43	94.21	0.50	-0.02	96.47
		σ_{y_2}	0.50	0.50	0.29	93.23	0.50	0.90	95.48	0.50	0.40	93.22
		σ_{y_3}	0.50	0.50	0.26	93.94	0.50	0.72	95.48	0.50	0.57	94.21
		σ_{ξ_1}	1.00	1.00	0.11	95.49	1.00	0.13	95.62	1.00	0.11	94.77
		σ_{ξ_2}	1.00	1.01	0.70	95.91	1.00	0.11	93.36	1.00	0.02	94.92
		ν_{x_1}	0.00	0.00	0.37	96.90	0.00	0.08	95.90	0.00	0.44	95.62
		ν_{x_2}	0.00	0.00	0.21	95.35	0.00	-0.16	96.19	0.00	0.16	95.20
		ν_{x_3}	0.00	0.00	0.47	97.32	0.00	0.10	96.89	0.00	0.29	96.89
		ν_{x_4}	0.00	0.00	-0.42	95.49	0.00	-0.20	94.21	0.00	0.04	95.20
		ν_{x_5}	0.00	0.00	-0.37	94.78	0.00	-0.12	95.06	0.00	-0.28	95.48
		ν_{x_6}	0.00	0.00	-0.32	95.49	0.00	-0.29	94.49	0.00	0.08	93.22

Table A.6: Results table for Study 2 endogenous structural lag population model (D3) and endogenous structural lag analysis model (A3) *(continued)*

				$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
		ν_{y_2}	0.00	0.00	-0.38	96.33	0.00	-0.03	97.60	0.00	0.01	94.92
		ν_{y_3}	0.00	0.00	0.10	95.35	0.00	-0.08	95.06	0.00	-0.08	94.07
		λ_{x_2}	1.00	1.01	0.74	96.33	1.01	1.06	94.63	1.01	0.52	94.21
		λ_{x_3}	1.00	1.00	0.25	94.08	1.01	0.62	93.93	1.01	1.00	96.19
		λ_{x_4}	1.00	1.00	0.45	95.77	1.00	0.49	95.20	1.01	1.03	94.49
		λ_{x_5}	1.00	1.01	0.61	94.22	1.00	0.38	96.19	1.01	0.76	94.49
		λ_{y_2}	1.00	1.00	-0.01	96.05	1.00	-0.14	95.48	1.00	-0.02	94.21
		λ_{y_3}	1.00	1.00	0.35	96.33	1.00	-0.31	95.76	1.00	-0.18	94.92
	400											
		ρ_η	-	0.07	6.82	0.00	0.30	0.33	94.39	0.60	-0.20	95.99
		α	0.00	0.00	0.22	95.73	0.00	-0.34	97.06	0.00	0.26	93.58
		γ_1	0.30	0.30	0.59	95.46	0.30	0.73	94.65	0.30	0.45	93.45
		γ_2	0.30	0.30	1.09	94.53	0.30	-0.88	95.59	0.30	-0.37	95.45
		γ_3	0.15	0.15	1.28	94.53	0.15	-0.19	94.65	0.15	-2.87	94.52
		σ_{x_1}	0.50	0.50	0.42	94.13	0.50	0.22	95.59	0.50	0.03	93.18
		σ_{x_2}	0.50	0.50	-0.04	95.19	0.50	0.39	94.65	0.50	0.48	93.98
		σ_{x_3}	0.50	0.50	0.17	94.53	0.50	0.21	95.32	0.50	0.03	95.86
		σ_{x_4}	0.50	0.50	0.36	96.80	0.50	0.27	93.58	0.50	0.54	95.19
		σ_{x_5}	0.50	0.50	0.12	97.20	0.50	0.41	94.39	0.50	0.12	95.19
		σ_{x_6}	0.50	0.50	0.08	94.26	0.50	-0.05	94.12	0.50	-0.29	94.92
		σ_{y_1}	0.50	0.50	-0.29	95.99	0.50	0.32	95.05	0.50	0.07	93.45
		σ_{y_2}	0.50	0.50	0.63	96.26	0.50	0.19	93.85	0.50	0.40	96.39
		σ_{y_3}	0.50	0.50	0.81	94.13	0.50	-0.05	95.05	0.50	0.37	97.06
		σ_{ξ_1}	1.00	1.00	0.26	93.86	1.00	0.05	93.32	1.00	0.14	95.59
		σ_{ξ_2}	1.00	1.00	0.44	97.06	1.00	0.32	94.92	1.00	0.09	94.12
		ν_{x_1}	0.00	0.00	-0.19	94.53	0.00	-0.21	94.65	0.00	0.26	95.99
		ν_{x_2}	0.00	0.00	-0.24	94.39	0.00	-0.05	95.72	0.00	0.02	95.32
		ν_{x_3}	0.00	0.00	-0.08	94.13	0.00	-0.04	95.59	0.00	0.07	95.19
		ν_{x_4}	0.00	0.00	-0.15	94.53	0.00	0.28	94.92	0.00	0.02	94.52
		ν_{x_5}	0.00	0.00	-0.14	95.19	0.00	0.24	95.32	0.00	-0.05	92.65
		ν_{x_6}	0.00	0.00	-0.21	95.59	0.00	0.24	95.45	0.00	0.17	94.92
		ν_{y_2}	0.00	0.00	-0.12	95.19	0.00	0.28	95.32	0.00	-0.04	95.45
		ν_{y_3}	0.00	0.00	-0.01	95.06	0.01	0.54	95.86	0.00	-0.25	94.25
		λ_{x_2}	1.00	1.00	0.15	95.73	1.00	0.43	94.12	1.00	0.38	95.32
		λ_{x_3}	1.00	1.00	0.19	93.59	1.00	0.39	95.59	1.00	0.12	96.12
		λ_{x_4}	1.00	1.00	0.22	95.99	1.00	0.16	93.72	1.01	0.55	94.12
		λ_{x_5}	1.00	1.00	0.26	95.33	1.00	0.29	94.12	1.00	0.50	95.32
		λ_{y_2}	1.00	1.00	-0.18	94.79	1.00	-0.13	94.12	1.00	0.19	93.32
		λ_{y_3}	1.00	1.00	-0.12	93.86	1.00	0.05	94.65	1.00	-0.07	93.32
W_D^*												
	49											
		ρ_η	-	0.48	48.20	0.00	0.49	64.03	100.00	0.50	-16.46	100.00
		α	0.00	0.00	-0.25	99.20	0.00	-0.04	98.26	0.01	0.82	94.78
		γ_1	0.30	0.30	-1.65	94.65	0.30	0.01	96.65	0.30	0.69	96.52
		γ_2	0.30	0.29	-2.73	95.85	0.29	-2.65	94.78	0.29	-1.79	95.05
		γ_3	0.15	0.14	-6.19	95.85	0.16	5.54	96.12	0.15	2.91	95.98
		σ_{x_1}	0.50	0.51	1.70	93.31	0.51	2.53	94.78	0.52	3.18	94.38
		σ_{x_2}	0.50	0.51	2.12	93.57	0.50	0.28	93.84	0.50	0.68	94.11
		σ_{x_3}	0.50	0.50	0.94	94.65	0.51	1.42	93.17	0.51	1.48	95.05
		σ_{x_4}	0.50	0.51	2.01	94.38	0.51	2.51	95.45	0.51	1.02	95.58
		σ_{x_5}	0.50	0.50	-0.96	93.84	0.51	1.95	94.51	0.51	1.48	95.05
		σ_{x_6}	0.50	0.51	2.33	94.91	0.51	1.25	92.10	0.51	2.24	92.50
		σ_{y_1}	0.50	0.49	-1.62	94.65	0.50	-0.90	94.24	0.50	0.03	93.98
		σ_{y_2}	0.50	0.52	3.04	95.45	0.51	1.17	95.05	0.51	1.94	95.31

Table A.6: Results table for Study 2 endogenous structural lag population model (D3) and endogenous structural lag analysis model (A3) *(continued)*

				$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
		σ_{y_3}	0.50	0.51	1.18	93.31	0.50	0.73	94.78	0.51	2.04	95.05
		σ_{ξ_1}	1.00	1.01	0.65	94.65	1.01	1.07	94.91	1.01	0.69	94.24
		σ_{ξ_2}	1.00	1.01	1.33	94.24	1.02	2.41	94.91	1.01	1.12	95.18
		ν_{x_1}	0.00	0.00	0.06	94.91	0.00	-0.01	94.91	-0.01	-0.89	95.58
		ν_{x_2}	0.00	0.00	-0.31	94.11	0.00	-0.32	95.85	-0.01	-0.77	95.98
		ν_{x_3}	0.00	0.00	-0.29	95.58	0.00	-0.29	94.78	-0.01	-0.92	95.58
		ν_{x_4}	0.00	0.00	0.40	96.12	0.00	0.18	94.38	-0.01	-0.73	96.39
		ν_{x_5}	0.00	0.00	-0.37	94.78	-0.01	-0.57	96.25	0.00	-0.25	97.05
		ν_{x_6}	0.00	0.00	0.33	94.91	0.00	0.28	95.72	0.00	0.44	95.18
		ν_{y_2}	0.00	0.01	0.75	96.79	0.00	-0.08	94.24	0.00	0.43	95.18
		ν_{y_3}	0.00	0.00	0.08	97.46	0.00	-0.23	95.85	0.01	1.00	96.52
		λ_{x_2}	1.00	1.02	1.97	95.18	1.03	3.05	96.39	1.03	2.86	96.12
		λ_{x_3}	1.00	1.02	2.25	96.92	1.02	2.22	95.85	1.03	2.67	95.05
		λ_{x_4}	1.00	1.03	2.78	95.98	1.02	2.03	95.85	1.02	2.17	95.45
		λ_{x_5}	1.00	1.01	1.48	95.85	1.01	1.14	96.65	1.02	2.17	95.45
		λ_{y_2}	1.00	0.98	-1.73	95.31	0.99	-0.96	96.39	0.98	-1.56	95.58
		λ_{y_3}	1.00	0.99	-1.40	96.25	0.99	-0.68	96.39	0.99	-1.17	94.51
	196											
		ρ_η	-	0.47	46.62	0.00	0.48	60.12	100.00	0.51	-14.68	100.00
		α	0.00	0.00	0.07	99.86	0.00	-0.34	97.44	-0.01	-0.95	96.73
		γ_1	0.30	0.31	2.13	95.45	0.30	1.53	95.45	0.29	-2.03	95.31
		γ_2	0.30	0.30	-0.26	95.31	0.30	0.05	94.60	0.30	-0.66	94.32
		γ_3	0.15	0.15	-0.56	96.45	0.16	6.82	93.18	0.15	-1.40	95.74
		σ_{x_1}	0.50	0.51	1.18	94.60	0.50	0.65	94.89	0.50	0.44	94.46
		σ_{x_2}	0.50	0.50	-0.25	96.16	0.50	-0.05	95.03	0.50	0.23	95.17
		σ_{x_3}	0.50	0.50	-0.29	94.03	0.50	0.02	95.45	0.50	0.48	94.74
		σ_{x_4}	0.50	0.50	-0.01	93.75	0.51	1.11	93.61	0.51	1.25	95.17
		σ_{x_5}	0.50	0.50	0.42	94.32	0.50	0.58	94.32	0.50	0.62	94.60
		σ_{x_6}	0.50	0.50	0.73	94.60	0.50	0.46	95.74	0.50	0.56	93.18
		σ_{y_1}	0.50	0.50	0.41	94.03	0.50	0.37	94.60	0.50	0.34	94.46
		σ_{y_2}	0.50	0.50	0.63	93.47	0.50	-0.24	96.31	0.50	0.43	94.03
		σ_{y_3}	0.50	0.50	0.63	94.32	0.51	1.12	94.60	0.50	0.87	94.32
		σ_{ξ_1}	1.00	1.00	0.30	94.89	1.01	0.65	93.18	1.00	-0.18	95.17
		σ_{ξ_2}	1.00	1.00	0.15	96.16	1.00	0.40	95.45	1.00	-0.24	95.74
		ν_{x_1}	0.00	0.00	-0.17	95.88	0.00	-0.26	93.75	0.00	-0.31	94.46
		ν_{x_2}	0.00	0.00	0.08	94.60	-0.01	-0.59	96.16	-0.01	-0.55	95.03
		ν_{x_3}	0.00	0.00	-0.04	94.32	0.00	-0.29	95.60	0.00	-0.18	95.74
		ν_{x_4}	0.00	0.00	-0.05	93.18	0.00	0.01	96.02	0.00	-0.25	96.31
		ν_{x_5}	0.00	0.00	0.11	94.74	0.00	-0.02	94.74	0.00	-0.50	94.89
		ν_{x_6}	0.00	0.00	-0.03	94.46	0.00	-0.48	95.17	0.00	-0.41	95.45
		ν_{y_2}	0.00	0.01	0.70	95.74	0.00	0.12	96.59	0.00	0.07	95.31
		ν_{y_3}	0.00	0.01	0.63	96.02	0.00	0.05	95.31	0.00	0.32	95.17
		λ_{x_2}	1.00	1.01	0.62	95.17	1.01	0.72	95.03	1.00	0.46	94.03
		λ_{x_3}	1.00	1.01	0.69	95.31	1.01	0.74	94.32	1.01	1.01	95.60
		λ_{x_4}	1.00	1.01	0.61	95.60	1.00	0.03	94.60	1.01	0.65	93.89
		λ_{x_5}	1.00	1.00	0.22	93.89	1.00	0.28	95.45	1.01	0.79	93.18
		λ_{y_2}	1.00	1.00	-0.02	95.31	1.00	-0.07	95.17	1.00	-0.12	94.46
		λ_{y_3}	1.00	1.00	-0.02	95.45	1.00	-0.24	95.88	1.00	-0.01	94.32
	400											
		ρ_η	-	0.45	44.90	0.00	0.47	58.26	99.73	0.51	-14.72	100.00
		α	0.00	0.00	-0.03	98.39	0.00	0.10	96.37	0.00	0.00	95.56
		γ_1	0.30	0.30	-0.29	95.56	0.30	-0.39	94.76	0.31	2.62	94.89
		γ_2	0.30	0.30	0.18	96.24	0.30	0.87	94.76	0.30	-0.53	95.56
		γ_3	0.15	0.16	5.49	93.01	0.15	-1.05	95.56	0.15	1.66	94.76
		σ_{x_1}	0.50	0.50	0.19	95.83	0.50	0.44	93.68	0.50	-0.23	94.22

Table A.6: Results table for Study 2 endogenous structural lag population model (D3) and endogenous structural lag analysis model (A3) *(continued)*

			$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
	σ_{x_2}	0.50	0.50	0.17	96.51	0.50	-0.05	94.89	0.50	0.23	94.76
	σ_{x_3}	0.50	0.50	-0.23	95.03	0.50	-0.06	95.83	0.50	0.21	93.41
	σ_{x_4}	0.50	0.50	0.51	93.28	0.50	0.45	97.18	0.50	0.52	94.09
	σ_{x_5}	0.50	0.50	0.10	94.35	0.50	0.51	95.97	0.50	0.52	95.43
	σ_{x_6}	0.50	0.50	0.24	97.18	0.50	0.14	94.35	0.50	-0.50	93.68
	σ_{y_1}	0.50	0.50	0.31	94.62	0.50	-0.01	95.56	0.50	-0.17	94.76
	σ_{y_2}	0.50	0.50	0.54	95.30	0.50	0.40	94.09	0.50	0.66	94.62
	σ_{y_3}	0.50	0.50	0.13	94.49	0.50	0.00	96.24	0.50	0.29	95.03
	σ_{ξ_1}	1.00	1.00	0.23	92.74	1.00	0.01	93.41	1.00	0.13	95.30
	σ_{ξ_2}	1.00	1.00	0.19	96.10	1.00	0.13	95.30	1.00	-0.13	96.37
	ν_{x_1}	0.00	0.00	0.04	93.95	0.00	-0.01	95.83	0.00	0.06	94.22
	ν_{x_2}	0.00	0.00	0.01	96.91	0.00	0.02	95.56	0.00	0.17	95.56
	ν_{x_3}	0.00	0.00	-0.05	95.83	0.00	-0.03	96.24	0.00	0.05	96.51
	ν_{x_4}	0.00	0.00	0.14	94.22	0.00	0.30	95.30	0.00	-0.25	91.80
	ν_{x_5}	0.00	0.00	0.19	95.16	0.00	0.15	95.03	0.00	-0.19	93.55
	ν_{x_6}	0.00	0.00	0.23	93.95	0.00	0.32	94.89	0.00	-0.13	95.03
	ν_{y_2}	0.00	0.00	-0.15	94.35	0.00	-0.06	95.30	0.00	0.03	95.03
	ν_{y_3}	0.00	0.00	0.04	94.62	0.00	-0.11	94.22	0.00	0.08	93.28
	λ_{x_2}	1.00	1.00	0.17	94.62	1.00	0.45	95.97	1.00	0.20	94.09
	λ_{x_3}	1.00	1.00	0.33	94.09	1.00	0.30	93.41	1.00	0.17	95.83
	λ_{x_4}	1.00	1.00	0.27	97.18	1.00	-0.18	94.89	1.00	0.35	95.03
	λ_{x_5}	1.00	1.00	0.17	95.43	1.00	-0.01	94.89	1.00	0.20	96.10
	λ_{y_2}	1.00	1.00	0.00	94.62	1.00	-0.39	94.35	0.99	-0.60	93.41
	λ_{y_3}	1.00	1.00	0.05	93.68	1.00	-0.07	93.82	1.00	-0.45	95.16

[1] W^* is the population and analysis W condition.

[2] N is the simulated sample size.

[3] θ is the simulated population value of each parameter.

[4] $\rho_\eta = ...$ is the simulated spatial autocorrelation of the endogenous lag.

[5] $\bar{\hat{\theta}}$ is the obtained average posterior mean across simulated iterations.

[6] $Bias(\bar{\hat{\theta}})\%$ is the average percent bias across simulation iterations when $\theta \neq 0$, otherwise $Bias(\bar{\hat{\theta}})\%$ provides absolute bias.

[7] Cover% is the percent of simulated trials in which the population value falls within the central 95% density of the posterior estimate.

Table A.7: Results table for Study 2 simultaneous structural lag population model (D4) and simultaneous structural lag analysis model (A4) under population level $\phi_\zeta = 0.3$

			$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
W_C^*											
	49										
	ρ_η	-	0.18	17.61	0.00	0.24	-19.56	98.92	0.42	-29.35	89.78
	ϕ_ζ	0.30	0.36	18.64	100.00	0.37	23.20	98.39	0.37	22.45	98.92
	α	0.00	0.00	-0.05	100.00	0.00	-0.07	98.92	0.01	0.78	96.77
	γ_1	0.30	0.32	5.22	96.77	0.32	7.95	96.77	0.32	7.13	97.31
	γ_2	0.30	0.31	4.73	97.31	0.32	5.42	99.46	0.33	9.62	96.77
	γ_3	0.15	0.16	4.95	96.77	0.17	10.36	95.16	0.17	11.36	97.31
	σ_{x_1}	0.50	0.50	0.65	92.47	0.52	3.51	96.77	0.50	0.68	95.70
	σ_{x_2}	0.50	0.50	-0.24	95.16	0.49	-1.31	91.94	0.51	1.14	96.24
	σ_{x_3}	0.50	0.51	1.25	95.70	0.51	2.54	92.47	0.51	1.10	94.62
	σ_{x_4}	0.50	0.52	3.36	93.01	0.51	2.69	93.55	0.51	2.20	96.24
	σ_{x_5}	0.50	0.50	0.75	95.16	0.49	-1.21	97.31	0.50	-0.01	96.24
	σ_{x_6}	0.50	0.50	0.28	94.09	0.50	0.61	97.85	0.50	0.12	93.01
	σ_{y_1}	0.50	0.50	0.04	97.85	0.50	0.00	93.55	0.51	2.56	93.55
	σ_{y_2}	0.50	0.51	1.18	95.16	0.50	0.30	93.55	0.50	-0.28	94.62
	σ_{y_3}	0.50	0.50	0.78	94.62	0.49	-1.06	94.09	0.50	-0.20	94.09
	σ_{ξ_1}	1.00	1.00	0.12	94.62	0.99	-1.48	93.01	1.02	2.44	96.77
	σ_{ξ_2}	1.00	1.02	2.24	94.09	1.00	-0.04	95.16	1.01	0.87	90.32
	ν_{x_1}	0.00	0.00	-0.39	97.85	0.00	-0.39	96.24	0.04	4.11	95.70
	ν_{x_2}	0.00	0.00	-0.49	95.16	0.00	-0.34	92.47	0.02	2.22	96.77
	ν_{x_3}	0.00	-0.01	-0.99	96.77	-0.01	-0.73	97.31	0.04	3.83	95.70
	ν_{x_4}	0.00	-0.01	-0.68	95.70	0.00	0.44	97.31	-0.01	-0.77	95.16
	ν_{x_5}	0.00	-0.01	-0.71	97.31	0.00	0.15	98.92	-0.01	-1.15	95.70
	ν_{x_6}	0.00	0.00	-0.36	95.70	0.00	0.03	98.39	-0.01	-1.13	93.01
	ν_{y_2}	0.00	0.00	-0.44	96.24	-0.01	-1.14	96.77	0.01	1.05	98.39
	ν_{y_3}	0.00	0.00	0.08	98.39	0.00	-0.23	93.01	0.01	1.19	97.31
	λ_{x_2}	1.00	1.02	2.30	95.70	1.03	3.30	97.31	1.02	1.65	95.16
	λ_{x_3}	1.00	1.03	2.53	95.16	1.03	2.91	96.77	1.02	1.65	95.16
	λ_{x_4}	1.00	1.02	2.07	95.16	1.03	3.28	94.09	1.01	1.37	96.77
	λ_{x_5}	1.00	1.02	2.02	96.24	1.02	2.25	96.24	1.02	2.03	95.16
	λ_{y_2}	1.00	0.91	-9.11	92.47	0.90	-9.87	94.62	0.93	-7.33	94.62
	λ_{y_3}	1.00	0.88	-11.60	92.47	0.88	-11.76	95.70	0.92	-8.13	95.16
	196										
	ρ_η	-	0.09	9.46	0.00	0.26	-12.37	91.76	0.53	-11.75	86.19
	ϕ_ζ	0.30	0.31	2.20	100.00	0.29	-2.70	99.45	0.27	-10.14	98.90
	α	0.00	0.00	-0.01	96.20	0.00	-0.20	95.05	0.00	0.33	93.92
	γ_1	0.30	0.30	0.52	95.65	0.31	1.76	93.96	0.31	1.73	97.24
	γ_2	0.30	0.30	-0.11	96.20	0.30	1.55	95.05	0.31	1.70	94.48
	γ_3	0.15	0.15	-1.17	95.11	0.15	1.21	97.25	0.15	2.71	98.34
	σ_{x_1}	0.50	0.50	0.20	95.11	0.50	0.84	94.51	0.51	1.30	93.92
	σ_{x_2}	0.50	0.51	1.35	95.65	0.50	0.85	94.51	0.50	0.68	93.92
	σ_{x_3}	0.50	0.50	0.97	96.20	0.50	0.32	93.41	0.50	-0.92	95.58
	σ_{x_4}	0.50	0.50	-0.22	93.48	0.50	-0.41	94.51	0.50	0.48	95.58
	σ_{x_5}	0.50	0.51	1.04	97.28	0.51	1.29	94.51	0.50	-0.19	97.24
	σ_{x_6}	0.50	0.51	1.13	92.93	0.50	-0.71	96.15	0.50	0.62	95.03
	σ_{y_1}	0.50	0.50	-0.34	95.65	0.50	-0.57	95.60	0.50	-0.95	97.79
	σ_{y_2}	0.50	0.49	-1.15	92.93	0.50	-0.53	97.80	0.50	-0.24	96.13
	σ_{y_3}	0.50	0.50	-0.14	95.65	0.50	-0.53	94.51	0.50	0.03	94.48
	σ_{ξ_1}	1.00	1.00	0.15	94.57	0.99	-0.70	97.80	1.00	0.46	96.13
	σ_{ξ_2}	1.00	1.01	1.44	92.93	1.00	-0.12	96.15	1.01	0.90	95.58
	ν_{x_1}	0.00	0.00	0.07	94.57	0.01	0.72	91.76	0.00	-0.14	96.13
	ν_{x_2}	0.00	0.00	0.17	95.65	0.00	0.07	92.86	-0.01	-0.67	96.13
	ν_{x_3}	0.00	0.00	-0.11	95.11	0.00	0.39	94.51	0.00	-0.25	95.03
	ν_{x_4}	0.00	0.00	0.15	95.65	-0.01	-0.66	93.96	0.02	1.50	93.37

Table A.7: Results table for Study 2 simultaneous structural lag population model (D4) and Simultaneous structural lag analysis model (A4) $\phi_\zeta = 0.3$ *(continued)*

				$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
		ν_{x_5}	0.00	0.00	-0.10	94.02	0.00	-0.22	93.41	0.01	0.98	92.27
		ν_{x_6}	0.00	0.00	0.25	97.83	0.00	-0.36	95.60	0.01	0.98	93.92
		ν_{y_2}	0.00	0.00	-0.01	91.85	0.00	-0.22	94.51	-0.01	-0.67	96.13
		ν_{y_3}	0.00	0.00	-0.38	95.11	0.00	0.09	91.21	-0.01	-0.56	95.03
		λ_{x_2}	1.00	1.01	0.57	95.11	1.01	1.00	95.60	1.00	-0.11	87.85
		λ_{x_3}	1.00	1.01	0.60	95.65	1.01	0.76	96.15	1.00	-0.29	95.03
		λ_{x_4}	1.00	1.00	-0.35	93.48	1.00	0.06	91.21	1.00	-0.17	97.79
		λ_{x_5}	1.00	1.00	-0.47	96.20	1.01	1.11	93.96	1.00	-0.31	93.92
		λ_{y_2}	1.00	0.97	-3.26	92.93	0.98	-2.18	95.60	0.97	-2.72	95.58
		λ_{y_3}	1.00	0.97	-3.29	95.11	0.97	-3.00	90.11	0.97	-2.98	95.58
	400											
		ρ_η	-	0.14	14.01	16.33	0.32	-11.17	93.62	0.57	-4.83	93.48
		ϕ_ζ	0.30	0.29	-11.90	98.98	0.28	-14.28	98.94	0.25	-26.18	98.91
		α	0.00	0.00	0.22	96.94	-0.06	-5.99	89.36	0.00	0.15	92.39
		γ_1	0.30	0.29	-2.46	94.90	0.31	1.89	94.68	0.30	0.76	98.91
		γ_2	0.30	0.30	0.05	98.98	0.30	1.24	96.81	0.30	0.30	95.65
		γ_3	0.15	0.14	-5.17	95.92	0.15	2.40	96.81	0.15	1.07	93.48
		σ_{x_1}	0.50	0.51	1.31	98.98	0.50	0.95	94.68	0.50	0.49	96.74
		σ_{x_2}	0.50	0.50	0.67	94.90	0.50	0.64	94.68	0.50	-0.26	96.74
		σ_{x_3}	0.50	0.50	-0.16	97.96	0.50	0.76	89.36	0.50	0.16	92.39
		σ_{x_4}	0.50	0.49	-1.54	92.86	0.50	0.24	95.74	0.50	0.43	93.48
		σ_{x_5}	0.50	0.50	0.21	95.92	0.50	0.27	93.62	0.50	-0.49	92.39
		σ_{x_6}	0.50	0.51	1.42	95.92	0.50	0.72	89.36	0.50	-0.40	91.30
		σ_{y_1}	0.50	0.50	-0.60	92.86	0.49	-1.70	91.49	0.50	-0.90	97.83
		σ_{y_2}	0.50	0.49	-1.72	92.86	0.50	-0.65	95.74	0.50	-0.84	95.65
		σ_{y_3}	0.50	0.50	-0.56	96.94	0.50	-0.84	94.68	0.50	-0.47	95.65
		σ_{ξ_1}	1.00	1.01	0.65	97.96	1.00	0.05	94.68	1.01	0.51	98.91
		σ_{ξ_2}	1.00	1.01	0.88	97.96	1.01	0.51	98.94	1.01	0.57	97.83
		ν_{x_1}	0.00	-0.01	-0.56	90.82	0.00	-0.28	98.94	-0.01	-0.54	92.39
		ν_{x_2}	0.00	0.00	-0.23	92.86	-0.01	-0.74	96.81	-0.01	-0.61	88.04
		ν_{x_3}	0.00	0.00	-0.07	91.84	0.00	-0.29	95.74	-0.01	-0.80	92.39
		ν_{x_4}	0.00	0.00	-0.25	97.96	0.00	0.02	97.87	0.01	0.63	97.83
		ν_{x_5}	0.00	0.01	0.73	100.00	0.00	0.30	95.74	0.01	0.99	94.57
		ν_{x_6}	0.00	0.01	0.91	98.98	0.00	0.37	93.62	0.01	1.05	95.65
		ν_{y_2}	0.00	-0.01	-0.86	96.94	0.00	-0.24	95.74	-0.01	-0.65	95.65
		ν_{y_3}	0.00	0.00	0.30	97.96	0.00	-0.14	96.81	-0.01	-0.59	90.22
		λ_{x_2}	1.00	0.99	-0.87	94.90	1.01	0.54	92.55	1.00	0.46	98.91
		λ_{x_3}	1.00	0.99	-0.65	95.92	1.00	-0.17	96.81	1.00	0.30	94.57
		λ_{x_4}	1.00	1.00	0.10	96.94	1.00	-0.27	94.68	1.00	0.16	93.48
		λ_{x_5}	1.00	0.99	-0.94	92.86	0.99	-0.50	98.94	1.00	0.34	95.65
		λ_{y_2}	1.00	1.00	0.33	95.92	0.97	-2.59	87.23	0.99	-1.01	95.65
		λ_{y_3}	1.00	1.01	0.76	95.92	0.99	-1.10	94.68	0.99	-0.76	89.13
W_D^*												
	49											
		ρ_η	-	0.32	32.05	0.00	0.33	9.41	100.00	0.35	-42.14	98.80
		ϕ_ζ	0.30	0.48	58.46	99.40	0.48	60.18	99.40	0.48	60.09	100.00
		α	0.00	0.00	-0.21	100.00	0.00	0.05	99.40	0.01	0.65	100.00
		γ_1	0.30	0.32	8.23	95.81	0.32	6.40	97.60	0.32	7.91	96.41
		γ_2	0.30	0.34	12.12	95.21	0.33	8.53	97.01	0.34	12.17	93.41
		γ_3	0.15	0.17	10.42	95.81	0.16	9.37	95.81	0.17	14.61	96.41
		σ_{x_1}	0.50	0.51	1.84	97.01	0.51	1.83	97.01	0.53	5.11	93.41
		σ_{x_2}	0.50	0.51	2.26	90.42	0.51	2.26	98.80	0.50	-0.47	91.62
		σ_{x_3}	0.50	0.51	2.04	90.42	0.50	-0.72	96.41	0.50	-0.61	97.01
		σ_{x_4}	0.50	0.51	2.23	95.21	0.50	0.70	95.81	0.50	-0.48	94.61

Table A.7: Results table for Study 2 simultaneous structural lag population model (D4) and Simultaneous structural lag analysis model (A4) $\phi_\zeta = 0.3$ *(continued)*

				$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
		σ_{x_5}	0.50	0.51	1.41	94.01	0.51	1.14	92.81	0.50	0.77	91.62
		σ_{x_6}	0.50	0.51	1.11	91.62	0.50	-0.68	96.41	0.51	2.91	94.61
		σ_{y_1}	0.50	0.48	-4.07	94.61	0.49	-2.22	95.81	0.50	-0.74	96.41
		σ_{y_2}	0.50	0.50	0.75	97.01	0.50	0.76	95.21	0.50	-0.05	92.81
		σ_{y_3}	0.50	0.50	-0.43	95.21	0.50	0.29	94.01	0.51	1.47	94.01
		σ_{ξ_1}	1.00	1.03	3.11	95.81	0.99	-0.92	93.41	1.00	-0.36	95.81
		σ_{ξ_2}	1.00	1.03	2.87	93.41	1.00	0.02	95.21	1.01	1.39	94.01
		ν_{x_1}	0.00	0.00	0.44	94.01	0.02	1.90	98.20	0.01	1.22	95.21
		ν_{x_2}	0.00	0.01	0.67	95.81	0.02	1.99	96.41	0.01	1.29	97.01
		ν_{x_3}	0.00	0.01	0.80	95.21	0.01	1.32	96.41	0.01	0.70	97.01
		ν_{x_4}	0.00	0.00	0.19	95.21	-0.02	-1.51	98.20	-0.02	-1.84	95.21
		ν_{x_5}	0.00	0.00	-0.48	94.61	-0.02	-1.59	98.80	-0.02	-1.85	94.01
		ν_{x_6}	0.00	0.00	-0.23	95.81	-0.02	-1.75	97.60	-0.03	-2.51	94.61
		ν_{y_2}	0.00	0.01	0.93	92.22	0.00	-0.40	94.61	0.00	-0.44	95.21
		ν_{y_3}	0.00	0.00	0.31	95.21	0.00	-0.49	98.20	0.00	-0.22	94.61
		λ_{x_2}	1.00	1.02	2.00	95.81	1.02	1.66	98.20	1.04	3.98	97.01
		λ_{x_3}	1.00	1.01	0.92	95.81	1.03	2.79	98.80	1.05	4.89	95.81
		λ_{x_4}	1.00	1.02	1.95	94.61	1.02	1.89	95.81	1.01	0.99	97.01
		λ_{x_5}	1.00	1.00	0.27	94.01	1.02	2.15	95.81	1.01	0.81	96.41
		λ_{y_2}	1.00	0.91	-9.42	89.22	0.94	-6.45	91.62	0.88	-11.50	86.23
		λ_{y_3}	1.00	0.90	-10.26	92.22	0.95	-5.26	89.22	0.86	-13.83	86.23
	196											
		ρ_η	-	0.31	31.04	0.00	0.32	8.18	98.16	0.35	-41.23	97.53
		ϕ_ζ	0.30	0.48	59.74	98.79	0.48	60.28	98.16	0.48	60.27	98.15
		α	0.00	0.00	-0.32	99.39	0.00	0.02	100.00	0.01	0.50	99.38
		γ_1	0.30	0.30	-0.01	96.97	0.30	-0.41	96.32	0.30	1.22	94.44
		γ_2	0.30	0.30	1.43	96.36	0.30	1.32	96.93	0.29	-2.26	93.21
		γ_3	0.15	0.15	-0.06	97.58	0.14	-5.80	95.71	0.15	0.33	94.44
		σ_{x_1}	0.50	0.51	1.23	94.55	0.51	1.11	93.25	0.51	1.37	96.30
		σ_{x_2}	0.50	0.50	0.89	95.76	0.50	0.77	93.87	0.50	-0.07	96.30
		σ_{x_3}	0.50	0.50	0.39	97.58	0.51	1.21	95.71	0.50	0.54	95.68
		σ_{x_4}	0.50	0.50	0.32	96.36	0.50	-0.17	96.93	0.50	0.41	95.68
		σ_{x_5}	0.50	0.50	-0.36	95.76	0.50	0.67	91.41	0.50	0.68	93.21
		σ_{x_6}	0.50	0.51	1.38	90.91	0.50	0.95	94.48	0.50	-0.05	94.44
		σ_{y_1}	0.50	0.50	-0.67	93.33	0.50	-0.50	94.48	0.49	-1.35	93.83
		σ_{y_2}	0.50	0.50	-0.64	94.55	0.49	-1.07	95.71	0.50	-0.89	95.06
		σ_{y_3}	0.50	0.50	-0.61	96.36	0.49	-1.33	94.48	0.49	-1.22	95.06
		σ_{ξ_1}	1.00	1.00	0.39	95.76	0.99	-1.06	96.32	1.00	0.18	95.68
		σ_{ξ_2}	1.00	1.00	0.25	95.76	1.01	0.50	98.77	1.00	0.45	97.53
		ν_{x_1}	0.00	0.01	0.84	95.76	0.00	0.30	95.09	-0.01	-0.58	92.59
		ν_{x_2}	0.00	0.00	0.03	93.94	0.00	-0.42	98.16	0.00	-0.25	92.59
		ν_{x_3}	0.00	0.00	0.11	95.15	0.00	0.36	98.16	0.00	-0.28	95.06
		ν_{x_4}	0.00	-0.01	-0.91	92.73	0.00	0.40	96.93	0.01	1.16	94.44
		ν_{x_5}	0.00	0.00	-0.49	94.55	0.01	0.84	96.32	0.01	1.20	97.53
		ν_{x_6}	0.00	-0.01	-0.69	93.94	0.00	0.36	95.71	0.01	0.88	93.21
		ν_{y_2}	0.00	0.00	0.29	97.58	0.01	0.66	98.16	0.00	0.26	96.91
		ν_{y_3}	0.00	0.01	0.69	95.15	0.00	0.34	95.09	0.01	0.58	92.59
		λ_{x_2}	1.00	1.00	0.16	92.12	1.00	0.10	95.09	1.00	0.40	96.91
		λ_{x_3}	1.00	1.00	0.26	95.15	1.01	1.02	96.93	1.01	0.95	95.06
		λ_{x_4}	1.00	1.00	0.36	98.18	1.01	0.52	95.71	1.00	-0.13	95.68
		λ_{x_5}	1.00	1.01	0.95	93.33	1.00	0.39	94.48	1.00	-0.10	95.68
		λ_{y_2}	1.00	0.98	-2.41	94.55	0.95	-5.43	90.80	0.99	-1.29	95.06
		λ_{y_3}	1.00	0.98	-2.21	93.94	0.97	-2.58	95.71	0.99	-0.89	93.83
	400											
		ρ_η	-	0.30	30.56	22.00	0.32	-5.28	100.00	0.37	-38.69	100.00

Table A.7: Results table for Study 2 simultaneous structural lag population model (D4) and Simultaneous structural lag analysis model (A4) $\phi_\zeta = 0.3$ *(continued)*

				$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$	
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
		ϕ_ζ	0.30	0.49	46.57	100.00	0.49	44.37	100.00	0.50	49.18	100.00
		α	0.00	0.00	0.34	100.00	0.00	0.04	100.00	-0.01	-0.65	100.00
		γ_1	0.30	0.30	-0.05	98.23	0.31	4.68	87.76	0.31	3.35	93.88
		γ_2	0.30	0.31	1.78	100.00	0.31	2.32	95.92	0.31	2.67	93.88
		γ_3	0.15	0.15	-0.60	94.45	0.15	1.52	97.96	0.15	-0.53	100.00
		σ_{x_1}	0.50	0.49	-1.88	96.36	0.50	0.15	95.92	0.50	-0.13	95.91
		σ_{x_2}	0.50	0.50	0.89	98.12	0.50	-0.93	95.92	0.49	-1.80	89.81
		σ_{x_3}	0.50	0.50	0.46	94.31	0.50	-0.41	97.96	0.51	1.79	93.65
		σ_{x_4}	0.50	0.49	-1.25	92.34	0.50	-0.71	91.84	0.51	1.27	91.84
		σ_{x_5}	0.50	0.49	-2.43	96.10	0.50	0.57	85.71	0.50	-0.91	91.81
		σ_{x_6}	0.50	0.51	1.58	90.01	0.49	-1.48	97.96	0.51	1.23	95.92
		σ_{y_1}	0.50	0.49	-2.74	92.10	0.49	-1.77	93.88	0.49	-1.92	91.81
		σ_{y_2}	0.50	0.50	-0.36	92.35	0.50	0.29	100.00	0.50	-0.49	97.96
		σ_{y_3}	0.50	0.49	-2.10	92.11	0.49	-1.51	89.80	0.49	-1.19	93.83
		σ_{ξ_1}	1.00	1.01	0.88	98.76	1.01	1.08	97.96	1.00	0.09	95.92
		σ_{ξ_2}	1.00	1.00	-0.06	94.00	0.98	-1.51	87.76	0.98	-1.80	89.80
		ν_{x_1}	0.00	0.00	-0.05	96.00	-0.01	-1.04	93.88	-0.01	-0.96	95.92
		ν_{x_2}	0.00	0.00	0.50	96.34	-0.01	-1.08	95.92	-0.01	-0.85	89.80
		ν_{x_3}	0.00	0.00	0.35	91.87	0.00	0.37	93.88	0.00	-0.30	93.88
		ν_{x_4}	0.00	0.00	0.15	96.45	0.01	1.00	95.92	0.00	0.33	97.96
		ν_{x_5}	0.00	0.01	0.52	98.12	0.01	0.74	95.92	0.00	-0.19	93.88
		ν_{x_6}	0.00	0.00	0.21	92.12	0.02	1.85	93.88	-0.01	-0.78	100.00
		ν_{y_2}	0.00	-0.01	-0.57	98.23	0.01	0.81	100.00	0.01	1.02	93.88
		ν_{y_3}	0.00	-0.01	-1.01	90.14	0.00	-0.35	100.00	0.00	-0.20	97.96
		λ_{x_2}	1.00	1.00	0.33	96.33	1.01	0.68	100.00	1.02	1.73	93.88
		λ_{x_3}	1.00	0.99	-0.67	100.00	1.00	0.48	97.96	1.01	0.56	97.96
		λ_{x_4}	1.00	1.00	0.34	95.58	1.01	1.02	100.00	1.02	1.85	89.80
		λ_{x_5}	1.00	1.01	0.74	100.00	1.02	2.02	83.67	1.02	1.51	95.92
		λ_{y_2}	1.00	1.00	0.24	94.72	0.96	-3.56	93.88	0.97	-3.14	91.84
		λ_{y_3}	1.00	0.99	-0.99	94.36	0.98	-2.08	97.96	0.98	-1.55	91.84

[1] W^* is the population and analysis W condition.

[2] N is the simulated sample size.

[3] θ is the simulated population value of each parameter.

[4] $\rho_\eta = ...$ is the simulated spatial autocorrelation of the endogenous lag.

[5] $\bar{\hat{\theta}}$ is the obtained average posterior mean across simulated iterations.

[6] $Bias(\bar{\hat{\theta}})\%$ is the average percent bias across simulation iterations when $\theta \neq 0$, otherwise $Bias(\bar{\hat{\theta}})\%$ provides absolute bias.

[7] Cover% is the percent of simulated trials in which the population value falls within the central 95% density of the posterior estimate.

[8] $\phi_\zeta = 0.3$ at the population level.

Table A.8: Results table for Study 2 simultaneous structural lag population model (D4) and simultaneous structural lag analysis model (A4) under population level $\phi_\zeta = 0.6$

			$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
W_C^*											
	49										
	ρ_η	-	0.16	15.80	0.00	0.25	-17.87	97.85	0.42	-29.44	81.72
	ϕ_ζ	0.60	0.36	-39.79	98.39	0.36	-40.04	97.85	0.36	-39.43	97.31
	α	0.00	0.00	-0.36	99.46	-0.02	-1.73	99.46	0.01	0.92	96.77
	γ_1	0.30	0.32	6.82	97.31	0.31	1.88	96.24	0.33	9.33	95.70
	γ_2	0.30	0.31	2.68	98.39	0.31	3.32	100.00	0.32	7.20	93.55
	γ_3	0.15	0.16	3.57	96.24	0.17	14.60	97.85	0.17	11.01	97.85
	σ_{x_1}	0.50	0.52	3.84	95.70	0.51	1.93	94.09	0.50	0.87	94.09
	σ_{x_2}	0.50	0.51	2.01	93.55	0.50	0.92	94.09	0.51	2.71	96.24
	σ_{x_3}	0.50	0.50	0.06	95.16	0.49	-1.63	94.09	0.49	-1.19	93.01
	σ_{x_4}	0.50	0.49	-1.65	96.77	0.51	2.28	93.55	0.51	2.87	96.24
	σ_{x_5}	0.50	0.50	0.91	94.62	0.51	1.73	96.24	0.50	0.14	93.55
	σ_{x_6}	0.50	0.50	0.97	93.01	0.50	0.39	96.77	0.51	2.47	96.24
	σ_{y_1}	0.50	0.48	-3.36	92.47	0.50	-0.08	96.24	0.50	0.42	97.31
	σ_{y_2}	0.50	0.51	1.46	96.24	0.51	1.24	94.62	0.50	0.38	90.32
	σ_{y_3}	0.50	0.50	0.40	97.31	0.49	-1.03	95.16	0.50	-0.44	93.55
	σ_{ξ_1}	1.00	1.00	0.14	94.09	1.00	0.33	95.70	1.01	1.29	94.09
	σ_{ξ_2}	1.00	1.03	2.52	93.55	1.02	1.60	95.16	1.00	-0.01	95.16
	ν_{x_1}	0.00	-0.03	-2.52	95.70	-0.01	-0.88	97.85	0.02	1.88	97.31
	ν_{x_2}	0.00	-0.02	-1.68	93.55	-0.02	-1.66	98.39	0.02	1.80	97.85
	ν_{x_3}	0.00	-0.02	-2.01	96.77	-0.01	-1.16	96.24	0.02	1.85	97.31
	ν_{x_4}	0.00	0.00	-0.12	97.31	-0.02	-1.91	95.70	-0.01	-1.40	96.24
	ν_{x_5}	0.00	0.01	0.51	95.70	-0.02	-1.73	96.77	-0.01	-0.86	96.24
	ν_{x_6}	0.00	0.00	-0.03	93.55	-0.01	-1.08	98.39	-0.01	-0.87	94.09
	ν_{y_2}	0.00	0.00	-0.21	96.77	0.01	1.40	95.70	-0.02	-1.52	94.09
	ν_{y_3}	0.00	0.00	-0.11	94.09	0.01	0.91	91.94	0.00	-0.43	95.70
	λ_{x_2}	1.00	1.02	2.05	97.31	1.03	2.92	93.55	1.02	2.02	95.16
	λ_{x_3}	1.00	1.03	2.86	95.70	1.04	4.05	93.01	1.02	2.41	96.24
	λ_{x_4}	1.00	1.01	0.57	96.77	1.02	2.44	97.31	1.03	3.36	97.31
	λ_{x_5}	1.00	1.01	0.59	94.62	1.03	3.14	97.31	1.04	3.76	95.16
	λ_{y_2}	1.00	0.91	-8.50	91.94	0.93	-7.16	94.09	0.96	-3.53	98.39
	λ_{y_3}	1.00	0.93	-7.47	90.86	0.90	-10.07	90.86	0.94	-5.60	94.09
	196										
	ρ_η	-	0.10	9.82	0.00	0.27	-11.40	96.13	0.53	-12.45	82.12
	ϕ_ζ	0.60	0.31	-48.42	96.69	0.30	-49.79	92.27	0.27	-54.88	91.06
	α	0.00	0.00	-0.44	93.37	0.00	0.50	96.13	0.00	-0.24	96.09
	γ_1	0.30	0.30	0.49	96.69	0.30	0.50	97.24	0.31	2.45	96.09
	γ_2	0.30	0.30	0.91	93.92	0.30	0.65	95.03	0.31	3.16	97.21
	γ_3	0.15	0.15	0.28	94.48	0.15	-2.42	96.13	0.15	3.18	98.32
	σ_{x_1}	0.50	0.50	0.96	94.48	0.51	1.36	92.82	0.50	-0.09	96.09
	σ_{x_2}	0.50	0.50	-0.65	95.58	0.50	0.04	94.48	0.50	0.32	91.62
	σ_{x_3}	0.50	0.50	0.00	92.27	0.50	-0.44	91.16	0.50	-0.03	94.41
	σ_{x_4}	0.50	0.50	0.53	95.03	0.50	0.52	93.37	0.50	0.30	93.85
	σ_{x_5}	0.50	0.50	0.17	96.69	0.50	0.31	95.58	0.50	-0.35	97.77
	σ_{x_6}	0.50	0.50	0.16	93.92	0.50	0.14	92.82	0.50	0.07	94.97
	σ_{y_1}	0.50	0.50	-0.97	95.58	0.49	-1.14	91.71	0.50	-0.48	93.85
	σ_{y_2}	0.50	0.50	-0.21	94.48	0.49	-1.04	93.37	0.49	-1.40	93.85
	σ_{y_3}	0.50	0.50	-0.97	95.58	0.50	-0.53	90.06	0.50	-0.28	98.32
	σ_{ξ_1}	1.00	1.01	0.89	95.58	1.00	-0.44	91.71	1.01	0.98	92.18
	σ_{ξ_2}	1.00	1.01	0.92	92.82	1.00	0.15	96.13	1.00	0.18	94.97
	ν_{x_1}	0.00	0.00	-0.13	92.27	0.01	0.90	96.13	-0.01	-1.07	96.09
	ν_{x_2}	0.00	0.01	0.50	91.71	0.01	1.18	93.92	0.00	-0.34	93.85
	ν_{x_3}	0.00	0.00	0.48	92.27	0.02	1.52	96.13	0.00	-0.46	93.30
	ν_{x_4}	0.00	0.00	-0.31	93.37	0.00	-0.32	91.16	0.00	0.43	97.21

Table A.8: Results table for Study 2 simultaneous structural lag population model (D4) and Simultaneous structural lag analysis model (A4) $\phi_\zeta = 0.6$ *(continued)*

				$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
		ν_{x_5}	0.00	0.00	-0.15	97.24	0.01	0.67	93.37	0.00	0.42	96.09
		ν_{x_6}	0.00	0.00	0.01	96.13	0.00	-0.08	90.61	0.01	0.71	97.77
		ν_{y_2}	0.00	0.00	0.36	96.13	0.00	0.18	93.37	0.00	0.00	97.77
		ν_{y_3}	0.00	0.01	0.51	93.37	0.00	-0.22	92.82	0.00	-0.26	94.41
		λ_{x_2}	1.00	1.01	0.89	92.82	1.00	0.34	94.48	1.01	0.85	92.74
		λ_{x_3}	1.00	1.00	0.32	96.13	1.01	0.83	96.69	1.00	-0.02	97.21
		λ_{x_4}	1.00	1.01	0.79	97.24	1.00	0.01	93.92	1.01	1.02	96.65
		λ_{x_5}	1.00	1.01	0.81	95.03	1.01	0.85	95.58	1.01	0.78	92.18
		λ_{y_2}	1.00	0.98	-2.36	92.82	0.97	-2.93	96.13	0.99	-1.09	95.53
		λ_{y_3}	1.00	0.97	-2.95	92.82	0.97	-2.74	94.48	0.98	-1.55	92.74
	400											
		ρ_η	-	0.16	15.99	18.18	0.32	-11.19	94.25	0.56	-6.05	96.47
		ϕ_ζ	0.60	0.29	-51.55	95.45	0.28	-53.38	81.61	0.25	-59.09	57.65
		α	0.00	0.00	-0.26	94.32	0.01	0.54	94.25	0.00	0.13	85.88
		γ_1	0.30	0.30	1.22	93.18	0.30	1.14	98.85	0.29	-4.24	100.00
		γ_2	0.30	0.31	2.02	96.59	0.31	1.84	94.25	0.31	2.56	97.65
		γ_3	0.15	0.16	3.65	96.59	0.16	4.08	96.55	0.14	-8.24	97.65
		σ_{x_1}	0.50	0.50	0.73	92.05	0.50	0.41	98.85	0.50	0.41	97.65
		σ_{x_2}	0.50	0.51	1.05	88.64	0.50	0.46	96.55	0.51	1.01	97.65
		σ_{x_3}	0.50	0.50	-0.93	96.59	0.50	-0.32	96.55	0.50	0.99	98.82
		σ_{x_4}	0.50	0.50	0.11	88.64	0.50	-0.35	93.10	0.49	-1.13	97.65
		σ_{x_5}	0.50	0.50	-0.10	95.45	0.50	-0.59	94.25	0.50	-0.14	88.24
		σ_{x_6}	0.50	0.50	-0.07	95.45	0.50	0.71	90.80	0.50	0.45	95.29
		σ_{y_1}	0.50	0.49	-1.33	92.05	0.49	-2.99	88.51	0.50	-0.90	96.47
		σ_{y_2}	0.50	0.49	-1.33	95.45	0.50	-0.99	93.10	0.49	-1.64	97.65
		σ_{y_3}	0.50	0.49	-1.29	95.45	0.50	-0.71	100.00	0.50	-0.75	95.29
		σ_{ξ_1}	1.00	1.00	0.49	96.59	1.00	-0.15	94.25	0.99	-1.43	91.76
		σ_{ξ_2}	1.00	1.00	-0.08	94.32	1.00	-0.40	93.10	1.00	-0.06	92.94
		ν_{x_1}	0.00	0.00	-0.36	92.05	0.01	0.93	96.55	0.00	0.49	97.65
		ν_{x_2}	0.00	0.00	-0.34	95.45	0.01	0.59	94.25	0.00	0.37	97.65
		ν_{x_3}	0.00	0.00	-0.48	92.05	0.01	0.89	94.25	0.01	0.52	97.65
		ν_{x_4}	0.00	-0.01	-0.78	90.91	-0.01	-0.75	97.70	-0.01	-0.69	92.94
		ν_{x_5}	0.00	0.00	-0.49	97.73	0.01	0.56	97.70	0.00	0.01	95.29
		ν_{x_6}	0.00	0.00	0.22	94.32	0.00	0.25	100.00	0.00	-0.50	94.12
		ν_{y_2}	0.00	0.01	0.56	92.05	-0.01	-0.72	97.70	0.00	0.22	90.59
		ν_{y_3}	0.00	0.00	-0.22	95.45	0.00	-0.34	90.80	0.00	0.06	92.94
		λ_{x_2}	1.00	1.00	0.35	97.73	1.01	0.87	97.70	1.00	0.39	95.29
		λ_{x_3}	1.00	1.00	0.10	92.05	1.00	0.42	91.95	0.99	-0.65	97.65
		λ_{x_4}	1.00	1.01	1.06	89.77	1.01	0.91	93.10	1.00	0.29	92.94
		λ_{x_5}	1.00	1.01	0.87	94.32	1.00	0.27	97.70	1.00	0.20	95.29
		λ_{y_2}	1.00	0.97	-2.57	95.45	0.98	-1.76	96.55	1.00	-0.34	95.29
		λ_{y_3}	1.00	0.98	-2.45	92.05	0.99	-0.54	90.80	0.99	-1.26	94.12
W_D^*												
	49											
		ρ_η	-	0.32	32.41	0.00	0.32	8.07	99.40	0.35	-42.33	98.80
		ϕ_ζ	0.60	0.48	-20.58	98.20	0.48	-20.25	97.60	0.48	-20.11	97.60
		α	0.00	-0.01	-1.30	99.40	0.02	1.53	99.40	0.01	1.36	100.00
		γ_1	0.30	0.33	8.56	98.20	0.33	8.76	94.61	0.32	6.66	97.60
		γ_2	0.30	0.33	10.12	95.81	0.33	10.93	97.60	0.33	8.64	95.21
		γ_3	0.15	0.17	10.92	96.41	0.16	9.86	95.21	0.16	6.07	97.01
		σ_{x_1}	0.50	0.50	-0.20	96.41	0.50	0.15	93.41	0.51	1.65	95.81
		σ_{x_2}	0.50	0.51	1.69	93.41	0.51	1.73	94.61	0.50	-0.56	91.02
		σ_{x_3}	0.50	0.50	0.58	96.41	0.52	3.48	95.21	0.53	5.15	95.21
		σ_{x_4}	0.50	0.51	1.46	94.01	0.52	4.10	97.01	0.51	1.05	92.22

Table A.8: Results table for Study 2 simultaneous structural lag population model (D4) and Simultaneous structural lag analysis model (A4) $\phi_\zeta = 0.6$ *(continued)*

			$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
	σ_{x_5}	0.50	0.51	2.21	92.22	0.50	-0.68	96.41	0.50	0.93	98.20
	σ_{x_6}	0.50	0.50	-0.05	97.01	0.51	2.86	92.22	0.50	0.60	94.61
	σ_{y_1}	0.50	0.49	-1.46	94.61	0.50	0.00	94.61	0.50	-0.21	98.80
	σ_{y_2}	0.50	0.51	1.40	96.41	0.50	-0.29	96.41	0.51	2.48	94.61
	σ_{y_3}	0.50	0.50	0.73	92.81	0.50	0.94	96.41	0.50	-0.92	95.21
	σ_{ξ_1}	1.00	1.00	0.29	95.21	1.01	1.21	97.01	1.00	0.39	95.21
	σ_{ξ_2}	1.00	1.00	-0.25	96.41	0.98	-1.96	96.41	1.01	0.71	95.81
	ν_{x_1}	0.00	-0.01	-0.89	96.41	0.02	1.77	95.21	0.02	1.52	97.60
	ν_{x_2}	0.00	0.00	0.06	95.81	0.02	2.15	96.41	0.01	0.55	92.81
	ν_{x_3}	0.00	0.00	-0.35	95.81	0.02	1.75	98.80	0.01	1.06	98.20
	ν_{x_4}	0.00	-0.02	-2.15	95.81	0.00	0.41	94.61	0.01	1.49	97.01
	ν_{x_5}	0.00	-0.02	-1.79	97.01	0.01	1.15	96.41	0.02	2.32	94.01
	ν_{x_6}	0.00	-0.01	-1.42	96.41	0.01	1.04	97.60	0.01	1.01	95.21
	ν_{y_2}	0.00	0.01	1.07	94.61	0.00	0.02	95.81	-0.01	-1.08	95.81
	ν_{y_3}	0.00	0.00	-0.44	94.01	0.01	1.14	96.41	0.00	-0.36	94.61
	λ_{x_2}	1.00	1.03	2.63	96.41	1.03	2.57	95.81	1.04	3.63	94.01
	λ_{x_3}	1.00	1.02	2.20	95.81	1.02	2.45	95.81	1.03	3.30	95.21
	λ_{x_4}	1.00	1.02	2.26	95.81	1.04	4.22	95.81	1.02	2.03	96.41
	λ_{x_5}	1.00	1.05	4.53	95.21	1.02	2.38	96.41	1.01	0.99	94.61
	λ_{y_2}	1.00	0.90	-10.41	88.62	0.92	-7.60	93.41	0.92	-8.06	94.01
	λ_{y_3}	1.00	0.93	-7.21	91.62	0.90	-10.34	91.62	0.92	-7.82	92.22
	196										
	ρ_η	-	0.31	30.71	0.00	0.33	10.39	98.75	0.35	-41.94	97.47
	ϕ_ζ	0.60	0.48	-19.64	97.52	0.48	-19.51	97.50	0.48	-20.41	96.84
	α	0.00	0.00	0.28	100.00	0.00	-0.14	100.00	0.02	1.78	98.73
	γ_1	0.30	0.31	3.80	95.03	0.31	3.09	95.00	0.31	3.51	96.20
	γ_2	0.30	0.31	3.96	93.17	0.30	-0.32	96.25	0.31	2.52	95.57
	γ_3	0.15	0.15	1.05	93.17	0.15	1.87	98.75	0.16	4.62	94.94
	σ_{x_1}	0.50	0.50	0.19	95.03	0.50	0.87	96.88	0.50	0.20	94.30
	σ_{x_2}	0.50	0.50	0.62	95.65	0.50	0.53	96.88	0.50	0.66	96.20
	σ_{x_3}	0.50	0.50	-0.24	93.79	0.50	-0.51	96.25	0.50	0.08	95.57
	σ_{x_4}	0.50	0.50	0.09	95.65	0.50	0.85	93.12	0.51	1.28	94.30
	σ_{x_5}	0.50	0.50	1.00	94.41	0.51	1.06	91.88	0.50	-0.31	96.84
	σ_{x_6}	0.50	0.50	1.00	93.79	0.50	-0.08	96.88	0.50	0.71	97.47
	σ_{y_1}	0.50	0.49	-1.25	94.41	0.49	-1.23	93.75	0.49	-1.07	93.04
	σ_{y_2}	0.50	0.49	-1.10	98.14	0.49	-1.48	90.62	0.50	-0.21	95.57
	σ_{y_3}	0.50	0.50	-0.68	95.03	0.50	-0.70	93.75	0.50	0.11	94.94
	σ_{ξ_1}	1.00	1.00	0.34	96.27	1.00	-0.47	95.62	1.01	0.76	94.94
	σ_{ξ_2}	1.00	1.00	0.30	95.65	1.00	0.21	95.00	1.01	0.84	93.04
	ν_{x_1}	0.00	0.00	0.26	96.27	-0.01	-0.90	95.62	0.00	-0.14	96.84
	ν_{x_2}	0.00	0.00	0.48	95.65	0.00	-0.32	92.50	0.00	0.10	91.77
	ν_{x_3}	0.00	0.00	-0.19	94.41	-0.01	-1.29	90.00	0.00	0.34	94.30
	ν_{x_4}	0.00	0.00	0.28	98.14	0.00	-0.26	97.50	0.00	0.41	91.77
	ν_{x_5}	0.00	0.00	0.12	97.52	0.00	0.17	96.25	0.00	0.22	94.30
	ν_{x_6}	0.00	0.00	0.44	98.14	0.00	-0.01	96.88	0.00	0.48	94.30
	ν_{y_2}	0.00	-0.01	-0.53	95.65	0.00	0.45	94.38	-0.01	-0.68	92.41
	ν_{y_3}	0.00	-0.01	-0.78	95.03	0.00	-0.04	94.38	-0.01	-1.39	95.57
	λ_{x_2}	1.00	1.00	-0.41	96.27	1.01	0.83	93.75	1.01	0.79	94.94
	λ_{x_3}	1.00	0.99	-0.63	94.41	1.01	0.88	93.75	1.01	1.36	93.67
	λ_{x_4}	1.00	1.01	0.64	95.65	1.00	0.30	97.50	1.01	0.97	97.47
	λ_{x_5}	1.00	1.01	0.92	91.93	1.01	1.02	95.62	1.00	0.28	96.84
	λ_{y_2}	1.00	0.97	-2.59	95.03	0.98	-1.83	93.75	0.96	-3.57	90.51
	λ_{y_3}	1.00	0.96	-3.55	92.55	0.99	-0.72	95.00	0.98	-2.23	93.04
	400										
	ρ_η	-	0.29	28.87	22.45	0.32	-4.74	100.00	0.34	-43.25	100.00

Table A.8: Results table for Study 2 simultaneous structural lag population model (D4) and Simultaneous structural lag analysis model (A4) $\phi_\zeta = 0.6$ *(continued)*

			$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
	ϕ_ζ	0.60	0.49	-17.63	100.00	0.49	-18.48	100.00	0.49	-17.86	100.00
	α	0.00	0.00	-0.04	100.00	0.00	-0.05	100.00	0.01	0.74	100.00
	γ_1	0.30	0.31	3.02	89.80	0.30	0.79	95.65	0.31	1.83	95.00
	γ_2	0.30	0.31	2.11	93.88	0.31	2.89	97.83	0.30	1.03	95.00
	γ_3	0.15	0.15	-1.47	100.00	0.14	-3.93	95.65	0.15	1.19	95.00
	σ_{x_1}	0.50	0.50	0.68	93.88	0.50	0.28	91.30	0.50	-0.40	97.50
	σ_{x_2}	0.50	0.50	-0.03	97.96	0.49	-1.26	93.48	0.50	0.01	95.00
	σ_{x_3}	0.50	0.50	0.10	89.80	0.49	-1.82	93.48	0.50	0.28	97.50
	σ_{x_4}	0.50	0.50	-0.38	95.92	0.51	1.11	93.48	0.50	0.78	95.00
	σ_{x_5}	0.50	0.50	-0.65	91.84	0.49	-1.54	97.83	0.50	0.60	100.00
	σ_{x_6}	0.50	0.50	0.36	100.00	0.50	0.42	95.65	0.50	-0.38	95.00
	σ_{y_1}	0.50	0.49	-1.43	87.76	0.50	-1.00	86.96	0.49	-2.15	87.50
	σ_{y_2}	0.50	0.50	-0.80	100.00	0.50	-0.15	95.65	0.50	-0.50	100.00
	σ_{y_3}	0.50	0.49	-1.97	93.88	0.49	-1.38	91.30	0.49	-1.84	95.00
	σ_{ξ_1}	1.00	1.01	1.16	93.88	1.01	1.00	82.61	1.02	1.92	95.00
	σ_{ξ_2}	1.00	1.00	-0.24	93.88	1.00	-0.12	97.83	0.99	-0.61	90.00
	ν_{x_1}	0.00	0.00	-0.23	93.88	-0.02	-1.64	95.65	0.01	0.77	97.50
	ν_{x_2}	0.00	-0.01	-0.70	91.84	-0.01	-1.03	89.13	0.01	1.32	95.00
	ν_{x_3}	0.00	0.00	0.17	97.96	-0.01	-0.71	91.30	0.00	0.34	95.00
	ν_{x_4}	0.00	0.00	0.09	95.92	0.00	-0.16	100.00	0.01	0.71	87.50
	ν_{x_5}	0.00	0.01	0.67	100.00	0.00	0.03	97.83	0.01	0.84	90.00
	ν_{x_6}	0.00	0.01	0.91	95.92	0.00	0.01	100.00	0.00	-0.01	87.50
	ν_{y_2}	0.00	0.01	0.60	97.96	0.00	0.14	95.65	0.00	0.47	100.00
	ν_{y_3}	0.00	-0.01	-0.98	93.88	0.00	-0.18	97.83	-0.01	-0.55	100.00
	λ_{x_2}	1.00	1.01	0.56	91.84	1.01	0.57	93.48	1.01	0.61	97.50
	λ_{x_3}	1.00	1.00	0.48	97.96	1.00	0.39	93.48	1.01	0.62	92.50
	λ_{x_4}	1.00	1.01	1.07	91.84	1.01	0.88	100.00	1.00	0.13	100.00
	λ_{x_5}	1.00	1.01	1.06	100.00	1.01	0.76	97.83	1.01	0.77	97.50
	λ_{y_2}	1.00	0.99	-1.43	85.71	0.98	-1.70	93.48	0.97	-2.60	95.00
	λ_{y_3}	1.00	0.99	-1.02	89.80	0.99	-1.15	91.30	1.00	0.49	97.50

[1] W^* is the population and analysis W condition.

[2] N is the simulated sample size.

[3] θ is the simulated population value of each parameter.

[4] $\rho_\eta = ...$ is the simulated spatial autocorrelation of the endogenous lag.

[5] $\bar{\hat{\theta}}$ is the obtained average posterior mean across simulated iterations.

[6] $Bias(\bar{\hat{\theta}})\%$ is the average percent bias across simulation iterations when $\theta \neq 0$, otherwise $Bias(\bar{\hat{\theta}})\%$ provides absolute bias.

[7] Cover% is the percent of simulated trials in which the population value falls within the central 95% density of the posterior estimate.

[8] $\phi_\zeta = 0.6$ at the population level.

A.3 Study 3

A.3.1 Study 3 Result Tables

Table A.9: Results table for Study 3 measurement lag population model and measurement lag analysis model

				W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^{*}		
ρ_{y2}	N		θ	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\theta)\%$	Cover%
0															
	49														
		ρ_{y2}	0.00	0.41	41.43	0.00	0.44	43.92	0.00	0.45	45.38	0.00	0.49	49.29	0.00
		α	0.00	-0.01	-0.52	95.91	0.00	-0.20	97.30	0.00	-0.42	96.35	0.00	0.26	96.24
		γ_1	0.30	0.32	6.76	96.52	0.32	6.34	97.40	0.33	8.92	98.02	0.32	7.71	96.97
		γ_2	0.30	0.32	6.83	97.14	0.32	6.90	96.57	0.32	7.04	97.39	0.32	6.49	94.99
		γ_3	0.15	0.16	8.24	95.40	0.16	8.29	96.37	0.16	8.37	96.14	0.16	9.38	97.39
		σ_{x_1}	0.50	0.51	1.12	92.84	0.51	2.96	95.33	0.51	1.49	93.95	0.51	1.57	93.11
		σ_{x_2}	0.50	0.51	1.69	94.48	0.51	1.05	95.22	0.51	1.81	92.60	0.51	1.54	93.95
		σ_{x_3}	0.50	0.51	1.70	95.30	0.50	0.55	93.77	0.50	0.28	94.16	0.50	0.53	95.30
		σ_{x_4}	0.50	0.51	2.58	93.46	0.51	2.22	93.77	0.51	2.07	95.52	0.52	3.77	94.36
		σ_{x_5}	0.50	0.50	-0.22	94.89	0.50	0.36	94.91	0.51	1.72	92.18	0.50	-0.38	94.78
		σ_{x_6}	0.50	0.51	2.12	95.09	0.50	0.65	94.91	0.50	0.62	94.58	0.51	1.26	94.57
		σ_{y_1}	0.50	0.49	-1.61	94.79	0.49	-2.34	93.15	0.49	-1.36	94.68	0.49	-2.03	95.30
		σ_{y_2}	0.50	0.50	0.10	94.99	0.50	0.44	94.70	0.50	0.43	94.58	0.50	0.90	93.74
		σ_{y_3}	0.50	0.38	-23.01	90.70	0.37	-26.44	90.34	0.36	-27.75	88.95	0.34	-32.07	88.94
		σ_{ξ_1}	1.00	1.01	1.41	93.76	1.00	-0.03	93.25	1.01	1.25	95.41	1.00	0.42	94.36
		σ_{ξ_2}	1.00	1.01	1.32	93.76	1.01	0.62	95.43	1.01	0.81	95.20	1.00	-0.11	93.42
		ν_{x_1}	0.00	0.00	-0.48	95.50	0.00	0.43	96.05	0.00	-0.34	95.93	0.00	0.12	95.09
		ν_{x_2}	0.00	-0.01	-0.60	96.01	0.00	-0.06	95.02	-0.01	-1.45	95.52	0.00	0.13	96.35
		ν_{x_3}	0.00	0.00	-0.29	96.22	0.01	0.67	95.43	-0.01	-0.78	95.41	0.00	0.17	93.95
		ν_{x_4}	0.00	0.01	0.65	94.58	0.01	0.64	95.12	0.00	0.16	94.16	0.01	0.81	95.41
		ν_{x_5}	0.00	0.01	0.75	95.40	-0.01	-0.52	95.43	0.00	0.19	95.20	0.00	0.23	95.62
		ν_{x_6}	0.00	0.01	0.61	94.07	0.00	-0.40	95.53	0.00	-0.26	94.37	0.00	0.47	96.03
		ν_{y_2}	0.00	0.00	0.24	94.17	0.00	0.49	95.22	0.00	-0.11	94.89	0.00	-0.10	94.89
		ν_{y_3}	0.00	0.00	0.49	96.22	0.00	0.15	97.30	-0.01	-0.89	97.71	0.00	0.09	97.91
		λ_{x_2}	1.00	1.01	1.41	94.07	1.03	2.69	95.22	1.02	2.07	94.89	1.02	2.39	96.24
		λ_{x_3}	1.00	1.01	1.48	94.58	1.03	2.89	96.16	1.02	2.15	95.72	1.02	2.11	95.72
		λ_{x_4}	1.00	1.02	2.02	94.48	1.02	2.03	95.33	1.02	2.16	96.25	1.03	3.25	95.51
		λ_{x_5}	1.00	1.02	1.60	97.03	1.02	1.86	95.22	1.02	2.45	96.25	1.03	2.74	94.47
		λ_{y_2}	1.00	0.92	-8.17	93.87	0.92	-7.89	92.73	0.92	-8.42	91.97	0.92	-7.51	92.69
		λ_{y_3}	1.00	0.91	-9.11	91.62	0.93	-7.11	92.63	0.90	-9.86	93.33	0.92	-8.36	93.74
0.3															
		ρ_{y2}	0.30	0.51	70.54	98.26	0.50	67.06	99.38	0.50	67.03	98.96	0.50	66.34	100.00
		α	0.00	0.00	0.10	94.68	0.00	0.16	96.26	0.00	0.16	95.41	-0.01	-0.91	95.61
		γ_1	0.30	0.32	7.69	96.42	0.32	7.79	96.26	0.32	8.30	96.45	0.32	6.95	97.91
		γ_2	0.30	0.32	6.11	96.83	0.32	7.56	96.47	0.32	7.08	97.29	0.32	7.45	96.34
		γ_3	0.15	0.16	9.65	96.52	0.16	9.88	96.37	0.16	5.97	97.08	0.17	11.94	96.24
		σ_{x_1}	0.50	0.51	1.21	94.89	0.51	2.64	93.67	0.51	1.20	94.99	0.51	2.29	93.83
		σ_{x_2}	0.50	0.51	1.02	93.87	0.50	-0.09	93.87	0.51	2.36	95.52	0.51	1.91	95.72
		σ_{x_3}	0.50	0.50	-0.09	93.87	0.51	1.75	92.83	0.51	1.16	96.04	0.51	1.24	96.66
		σ_{x_4}	0.50	0.51	1.38	94.89	0.50	0.95	94.70	0.51	1.60	94.89	0.51	1.85	94.25
		σ_{x_5}	0.50	0.51	1.11	94.89	0.51	2.40	94.50	0.51	1.31	94.58	0.50	-0.09	94.36
		σ_{x_6}	0.50	0.50	0.83	94.48	0.50	0.72	95.12	0.50	0.97	95.52	0.50	0.08	94.36
		σ_{y_1}	0.50	0.49	-1.84	96.22	0.49	-2.06	94.70	0.49	-1.66	95.10	0.49	-1.37	96.03
		σ_{y_2}	0.50	0.49	-1.01	95.40	0.50	-0.86	93.35	0.50	0.59	95.20	0.50	0.78	94.36
		σ_{y_3}	0.50	0.31	-37.29	79.24	0.33	-34.40	83.39	0.33	-34.69	84.15	0.33	-34.10	86.00
		σ_{ξ_1}	1.00	1.01	0.85	94.58	1.00	0.25	95.74	1.01	1.29	93.22	1.01	1.28	94.25

Table A.9: Results table for Study 3 measurement lag population model (D2) and measurement lag analysis model (A2) *(continued)*

			W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^*		
ρ_{y2}	N	θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
		σ_{ξ_2} 1.00	1.00	0.49	96.22	1.01	0.70	95.64	1.01	0.76	94.68	1.01	0.99	93.73
		ν_{x_1} 0.00	0.00	0.14	95.19	0.01	0.61	95.85	0.00	0.02	95.72	0.00	-0.25	96.24
		ν_{x_2} 0.00	0.00	-0.42	93.35	0.00	0.44	96.16	0.00	-0.21	95.83	0.00	-0.42	96.55
		ν_{x_3} 0.00	0.00	0.16	95.30	0.01	0.55	96.68	0.00	-0.13	96.66	0.00	-0.18	96.66
		ν_{x_4} 0.00	0.01	1.15	95.40	0.00	0.10	97.40	0.00	0.22	95.41	-0.01	-1.16	94.88
		ν_{x_5} 0.00	0.01	0.89	96.73	0.00	-0.08	96.99	0.00	0.41	95.62	-0.01	-1.22	96.45
		ν_{x_6} 0.00	0.01	1.37	96.11	0.00	-0.29	96.16	0.01	0.73	95.52	-0.01	-1.16	94.67
		ν_{y_2} 0.00	0.00	0.20	95.40	0.00	0.36	95.95	0.00	0.27	94.89	0.00	0.13	94.67
		ν_{y_3} 0.00	0.00	0.02	94.27	-0.01	-0.66	96.26	0.00	-0.16	96.14	0.00	0.20	95.09
		λ_{x_2} 1.00	1.02	2.08	95.81	1.03	2.84	95.12	1.02	2.23	96.66	1.03	2.66	94.67
		λ_{x_3} 1.00	1.03	2.51	96.22	1.02	2.31	96.16	1.03	2.70	94.26	1.02	2.42	96.24
		λ_{x_4} 1.00	1.03	2.64	95.71	1.02	2.33	94.50	1.02	2.32	95.83	1.03	2.73	94.25
		λ_{x_5} 1.00	1.02	2.47	95.19	1.02	2.35	94.60	1.02	2.08	95.52	1.02	2.48	93.73
		λ_{y_2} 1.00	0.94	-6.26	93.56	0.93	-7.24	91.07	0.91	-8.74	91.76	0.92	-7.88	93.83
		λ_{y_3} 1.00	0.92	-7.83	92.43	0.91	-9.49	92.83	0.92	-8.43	93.12	0.91	-8.81	93.21
0.6														
		ρ_{y2} 0.60	0.65	8.12	98.56	0.60	-0.44	99.79	0.57	-5.74	100.00	0.51	-15.64	100.00
		α 0.00	0.00	-0.08	95.28	0.00	-0.45	96.37	0.00	0.03	96.77	0.00	0.37	96.55
		γ_1 0.30	0.32	8.31	96.31	0.32	6.04	95.74	0.32	7.08	96.87	0.32	7.67	96.23
		γ_2 0.30	0.32	7.87	96.72	0.32	6.84	96.57	0.32	7.05	95.93	0.32	6.67	97.07
		γ_3 0.15	0.16	9.00	95.69	0.16	6.19	97.20	0.16	7.75	96.66	0.17	10.85	96.65
		σ_{x_1} 0.50	0.50	0.98	93.95	0.51	1.89	92.63	0.51	2.51	94.06	0.51	2.44	93.51
		σ_{x_2} 0.50	0.50	0.89	96.21	0.50	0.81	95.22	0.51	1.23	95.52	0.50	0.67	94.67
		σ_{x_3} 0.50	0.51	1.39	95.18	0.50	0.93	92.94	0.50	0.51	94.58	0.50	0.70	93.62
		σ_{x_4} 0.50	0.51	1.77	95.69	0.51	1.87	94.70	0.51	1.83	94.06	0.51	2.37	93.10
		σ_{x_5} 0.50	0.50	0.84	94.87	0.51	2.15	95.02	0.51	2.54	95.52	0.49	-1.34	94.35
		σ_{x_6} 0.50	0.50	0.98	94.77	0.50	-0.07	93.35	0.51	1.25	94.99	0.51	1.65	94.77
		σ_{y_1} 0.50	0.49	-1.41	94.46	0.49	-2.00	95.53	0.49	-2.09	94.68	0.49	-1.15	95.19
		σ_{y_2} 0.50	0.50	0.82	96.10	0.50	0.82	94.91	0.50	-0.55	95.41	0.50	-0.20	96.13
		σ_{y_3} 0.50	0.22	-56.29	37.95	0.27	-46.87	63.66	0.29	-42.56	74.14	0.32	-35.43	86.61
		σ_{ξ_1} 1.00	1.01	0.79	94.97	1.01	1.41	93.87	1.00	0.09	93.12	1.01	1.49	94.77
		σ_{ξ_2} 1.00	1.01	0.99	93.74	1.02	1.75	94.60	1.01	0.85	93.43	1.00	0.19	95.92
		ν_{x_1} 0.00	-0.01	-0.92	94.97	0.00	0.12	96.26	0.00	-0.31	95.52	0.00	0.23	95.92
		ν_{x_2} 0.00	-0.01	-0.87	93.85	0.00	-0.05	96.16	0.00	-0.25	95.72	0.00	0.01	96.34
		ν_{x_3} 0.00	0.00	-0.42	94.26	0.00	-0.23	95.95	-0.01	-0.71	95.52	0.00	0.43	96.55
		ν_{x_4} 0.00	0.00	0.27	94.46	-0.01	-0.55	95.33	-0.01	-0.69	93.22	0.01	0.92	95.19
		ν_{x_5} 0.00	0.00	-0.01	94.97	0.00	-0.39	96.16	-0.01	-0.63	94.37	0.01	0.51	96.44
		ν_{x_6} 0.00	0.00	0.35	96.10	-0.01	-1.00	96.16	-0.01	-0.67	94.47	0.00	0.33	95.29
		ν_{y_2} 0.00	0.00	-0.40	95.08	0.00	-0.27	94.29	0.00	-0.36	96.87	-0.01	-0.52	96.03
		ν_{y_3} 0.00	0.00	-0.18	96.82	0.00	0.18	93.15	0.00	-0.29	92.81	0.00	0.28	93.31
		λ_{x_2} 1.00	1.03	2.87	96.41	1.02	1.65	94.50	1.03	2.68	95.62	1.02	2.17	94.77
		λ_{x_3} 1.00	1.03	3.08	95.18	1.02	2.03	94.60	1.02	1.61	96.35	1.02	2.44	95.19
		λ_{x_4} 1.00	1.02	2.33	95.28	1.01	1.36	96.47	1.02	2.46	95.41	1.03	2.78	96.34
		λ_{x_5} 1.00	1.02	2.21	94.46	1.02	2.21	95.02	1.02	1.79	96.04	1.02	2.48	96.23
		λ_{y_2} 1.00	0.93	-7.19	92.62	0.93	-6.64	94.81	0.93	-7.12	92.18	0.92	-7.79	93.62
		λ_{y_3} 1.00	0.92	-8.44	93.85	0.91	-9.09	93.35	0.92	-8.43	94.89	0.90	-10.19	93.51
0														
	196													
		ρ_{y2} 0.00	0.37	37.37	0.00	0.39	39.01	0.00	0.40	39.91	0.00	0.48	48.06	0.00
		α 0.00	0.00	0.00	94.17	0.00	0.38	94.02	0.00	0.09	95.31	0.00	-0.26	95.61
		γ_1 0.30	0.30	1.58	95.58	0.30	1.07	94.99	0.30	0.84	97.05	0.30	1.13	96.38
		γ_2 0.30	0.31	1.71	93.96	0.30	1.29	95.97	0.30	0.83	95.20	0.30	1.42	96.93
		γ_3 0.15	0.15	2.90	96.55	0.15	1.78	93.47	0.15	2.40	95.31	0.15	1.90	94.08
		σ_{x_1} 0.50	0.50	0.17	95.47	0.50	0.69	94.45	0.50	0.33	92.79	0.51	1.15	94.08

Table A.9: Results table for Study 3 measurement lag population model (D2) and measurement lag analysis model (A2) *(continued)*

				W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^*		
ρ_{y2}	N		θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
		σ_{x_2}	0.50	0.50	0.45	94.07	0.50	0.24	94.34	0.50	0.23	94.65	0.50	0.42	93.86
		σ_{x_3}	0.50	0.50	0.19	94.71	0.50	0.66	94.45	0.50	0.48	95.31	0.50	-0.10	95.39
		σ_{x_4}	0.50	0.50	0.82	93.20	0.50	0.42	95.21	0.50	0.07	96.07	0.50	0.56	96.38
		σ_{x_5}	0.50	0.50	0.64	94.93	0.50	0.95	94.67	0.50	0.67	96.07	0.50	0.89	94.63
		σ_{x_6}	0.50	0.50	-0.84	95.04	0.50	-0.13	94.89	0.50	0.38	95.74	0.50	0.11	94.19
		σ_{y_1}	0.50	0.50	-0.62	94.61	0.49	-1.13	92.27	0.50	-0.53	95.09	0.49	-1.03	95.18
		σ_{y_2}	0.50	0.50	-0.46	95.69	0.50	-0.45	95.10	0.50	-0.62	95.63	0.50	-0.79	94.63
		σ_{y_3}	0.50	0.40	-19.30	82.63	0.40	-20.44	81.94	0.39	-22.57	80.24	0.33	-33.49	72.92
		σ_{ξ_1}	1.00	1.00	-0.08	94.39	1.00	0.44	95.32	1.00	0.02	94.10	1.00	0.11	94.41
		σ_{ξ_2}	1.00	1.00	0.18	95.04	1.00	0.29	93.69	1.01	0.52	94.65	1.00	-0.10	94.74
		ν_{x_1}	0.00	0.00	0.11	95.25	0.00	0.50	95.21	0.00	0.31	95.09	-0.01	-0.66	96.49
		ν_{x_2}	0.00	0.00	0.28	94.93	0.00	0.07	94.89	0.00	0.28	94.21	-0.01	-0.61	95.61
		ν_{x_3}	0.00	0.01	0.55	94.61	0.00	0.25	95.65	0.00	0.19	94.32	-0.01	-0.60	95.50
		ν_{x_4}	0.00	0.00	-0.19	94.93	0.00	-0.30	94.12	0.00	0.34	96.07	0.00	-0.36	95.83
		ν_{x_5}	0.00	0.00	-0.29	95.69	0.00	0.02	94.45	0.00	0.30	95.52	0.00	0.00	94.41
		ν_{x_6}	0.00	0.00	-0.12	95.47	0.00	0.15	94.12	0.00	0.07	96.29	0.00	-0.17	93.42
		ν_{y_2}	0.00	0.00	-0.16	93.74	0.00	-0.26	94.67	0.00	0.02	96.18	0.00	-0.26	95.50
		ν_{y_3}	0.00	0.00	-0.09	94.39	0.00	-0.18	95.87	0.00	-0.07	95.09	0.00	0.13	97.81
		λ_{x_2}	1.00	1.01	0.58	94.50	1.01	0.78	94.45	1.01	0.76	94.43	1.00	0.36	95.07
		λ_{x_3}	1.00	1.01	0.66	95.25	1.01	0.56	93.58	1.01	0.90	94.32	1.01	0.71	94.52
		λ_{x_4}	1.00	1.01	0.88	94.82	1.01	0.66	95.21	1.01	0.59	95.74	1.01	0.84	93.86
		λ_{x_5}	1.00	1.01	0.98	94.28	1.01	0.73	93.80	1.00	0.50	94.54	1.01	0.55	94.30
		λ_{y_2}	1.00	0.99	-1.12	94.28	0.99	-1.14	96.19	0.99	-0.92	94.98	0.99	-1.26	96.16
		λ_{y_3}	1.00	0.99	-1.07	92.99	0.99	-0.92	96.08	0.99	-0.62	94.54	0.97	-2.71	94.30
0.3															
		ρ_{y2}	0.30	0.51	71.45	89.83	0.50	67.65	95.75	0.50	66.77	96.62	0.50	65.35	100.00
		α	0.00	0.00	-0.25	94.59	0.00	-0.37	94.22	0.00	-0.03	94.87	0.00	0.21	94.62
		γ_1	0.30	0.30	1.37	94.81	0.30	0.97	95.86	0.30	1.22	96.51	0.31	1.71	95.82
		γ_2	0.30	0.31	1.98	94.05	0.30	0.43	95.64	0.30	0.82	94.65	0.31	1.81	94.51
		γ_3	0.15	0.15	2.26	95.02	0.15	0.77	95.53	0.15	2.23	94.98	0.15	2.93	96.59
		σ_{x_1}	0.50	0.50	0.44	94.05	0.50	0.60	94.44	0.50	0.76	94.76	0.50	0.18	94.40
		σ_{x_2}	0.50	0.50	-0.05	94.37	0.50	0.86	92.69	0.50	0.33	94.10	0.50	-0.22	95.05
		σ_{x_3}	0.50	0.50	0.44	95.02	0.50	-0.14	95.09	0.50	-0.10	94.32	0.50	0.83	94.40
		σ_{x_4}	0.50	0.50	0.59	95.78	0.50	-0.01	95.53	0.50	-0.10	93.89	0.50	0.84	94.84
		σ_{x_5}	0.50	0.50	0.23	92.97	0.50	-0.12	94.55	0.50	-0.19	95.41	0.50	0.65	94.84
		σ_{x_6}	0.50	0.50	0.56	94.91	0.50	0.60	95.42	0.50	0.78	95.31	0.50	0.66	94.73
		σ_{y_1}	0.50	0.49	-1.19	95.24	0.50	-0.97	95.42	0.50	-0.79	94.76	0.50	-0.94	96.04
		σ_{y_2}	0.50	0.50	-0.27	95.24	0.50	-0.70	94.22	0.50	-0.04	94.76	0.50	-0.36	95.71
		σ_{y_3}	0.50	0.29	-42.33	24.89	0.31	-37.90	46.78	0.32	-36.20	52.51	0.32	-35.26	69.89
		σ_{ξ_1}	1.00	1.00	0.24	95.89	1.00	0.19	93.24	1.00	0.24	94.98	1.00	0.30	94.62
		σ_{ξ_2}	1.00	1.01	0.53	94.81	1.01	0.52	93.57	1.00	0.10	96.07	1.00	-0.14	94.18
		ν_{x_1}	0.00	0.01	0.68	92.97	0.00	-0.32	94.77	0.00	-0.01	95.20	0.00	0.06	95.27
		ν_{x_2}	0.00	0.00	0.44	92.97	-0.01	-0.50	94.98	0.00	-0.31	96.40	0.00	0.14	95.49
		ν_{x_3}	0.00	0.00	0.34	92.32	0.00	-0.22	94.44	0.00	-0.35	96.07	0.00	0.33	96.81
		ν_{x_4}	0.00	-0.01	-0.53	94.81	0.00	-0.17	95.75	0.00	0.49	95.41	0.00	0.20	95.16
		ν_{x_5}	0.00	0.00	-0.46	93.29	0.00	-0.02	96.07	0.01	0.62	94.54	0.00	0.17	93.30
		ν_{x_6}	0.00	-0.01	-0.54	94.81	0.00	-0.14	95.86	0.00	0.40	94.98	0.00	-0.03	93.19
		ν_{y_2}	0.00	0.00	0.09	94.81	0.00	0.08	95.31	0.00	-0.01	96.18	0.00	-0.10	94.62
		ν_{y_3}	0.00	0.00	0.20	95.35	0.00	0.07	94.87	0.00	0.25	96.29	0.00	-0.23	94.07
		λ_{x_2}	1.00	1.00	0.36	95.13	1.01	0.69	95.97	1.01	0.88	94.98	1.00	0.43	94.40
		λ_{x_3}	1.00	1.00	0.45	94.91	1.01	0.61	93.89	1.01	0.78	94.00	1.01	0.71	94.73
		λ_{x_4}	1.00	1.00	0.31	95.67	1.00	0.47	96.07	1.00	0.47	93.89	1.00	0.42	95.60
		λ_{x_5}	1.00	1.00	0.28	93.83	1.00	0.08	95.09	1.00	0.22	95.96	1.01	0.54	95.05
		λ_{y_2}	1.00	0.98	-1.60	95.56	0.99	-1.08	93.78	0.99	-1.00	94.43	0.99	-1.02	94.62
		λ_{y_3}	1.00	0.99	-1.39	93.40	0.98	-1.60	95.20	0.99	-0.80	95.52	0.99	-1.13	94.18

Table A.9: Results table for Study 3 measurement lag population model (D2) and measurement lag analysis model (A2) *(continued)*

				W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^{*}		
ρ_{y2}	N		θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
0.6															
		ρ_{y2}	0.60	0.72	19.63	87.22	0.69	14.74	94.87	0.66	10.17	96.83	0.51	-14.27	100.00
		α	0.00	0.00	-0.49	96.64	0.00	-0.07	96.51	0.00	-0.18	97.05	0.00	0.03	95.36
		γ_1	0.30	0.30	1.61	96.53	0.30	1.02	95.09	0.30	0.85	95.74	0.30	0.69	93.71
		γ_2	0.30	0.30	1.01	96.53	0.30	0.74	95.64	0.30	0.67	94.76	0.30	1.56	97.02
		γ_3	0.15	0.15	1.73	95.56	0.15	0.19	95.75	0.15	1.88	93.45	0.15	2.01	96.14
		σ_{x_1}	0.50	0.50	0.40	94.15	0.50	0.28	93.24	0.50	0.24	95.63	0.50	0.91	94.70
		σ_{x_2}	0.50	0.50	-0.29	94.58	0.50	0.32	95.86	0.50	0.61	95.52	0.50	0.40	92.72
		σ_{x_3}	0.50	0.50	0.33	92.20	0.50	-0.27	95.31	0.50	0.24	96.07	0.50	0.12	95.70
		σ_{x_4}	0.50	0.50	0.29	93.93	0.50	0.31	95.75	0.50	0.29	95.31	0.51	1.07	94.48
		σ_{x_5}	0.50	0.50	0.27	95.45	0.50	0.47	94.11	0.50	0.37	93.78	0.50	0.17	95.03
		σ_{x_6}	0.50	0.50	0.86	93.93	0.50	0.26	93.24	0.50	0.43	95.31	0.50	0.50	94.81
		σ_{y_1}	0.50	0.50	-1.00	95.99	0.50	-0.91	93.78	0.50	-0.88	96.51	0.50	-0.60	94.70
		σ_{y_2}	0.50	0.50	-0.45	95.56	0.50	-0.46	92.80	0.50	-0.78	95.74	0.50	-0.74	93.93
		σ_{y_3}	0.50	0.16	-67.86	0.00	0.21	-58.60	4.91	0.23	-54.97	13.54	0.31	-37.45	65.34
		σ_{ξ_1}	1.00	1.00	0.43	96.10	1.00	0.37	95.09	1.01	0.51	92.47	1.00	0.23	94.59
		σ_{ξ_2}	1.00	1.00	0.35	94.69	1.01	0.75	94.66	1.00	0.41	94.21	1.00	-0.11	94.81
		ν_{x_1}	0.00	0.00	-0.34	95.02	0.00	-0.09	94.22	0.00	-0.46	94.76	0.00	-0.19	95.36
		ν_{x_2}	0.00	0.00	-0.26	92.74	0.00	0.16	93.68	-0.01	-0.52	94.65	0.00	-0.31	95.58
		ν_{x_3}	0.00	0.00	-0.44	94.58	0.00	-0.25	93.78	0.00	-0.09	94.87	0.00	-0.45	95.14
		ν_{x_4}	0.00	0.00	-0.07	94.80	0.00	-0.39	94.77	0.00	0.10	93.34	0.00	0.16	94.81
		ν_{x_5}	0.00	0.00	-0.22	94.47	0.00	-0.08	95.09	0.00	-0.34	94.21	0.00	0.32	95.81
		ν_{x_6}	0.00	0.00	0.13	93.93	0.00	-0.31	96.07	0.00	-0.26	92.79	0.00	0.18	96.14
		ν_{y_2}	0.00	0.00	0.45	95.56	0.00	-0.28	95.31	0.00	0.21	94.98	0.00	-0.13	95.03
		ν_{y_3}	0.00	0.01	0.55	95.99	0.00	-0.25	92.69	0.00	0.27	95.52	0.00	-0.33	89.85
		λ_{x_2}	1.00	1.01	0.71	96.53	1.00	0.37	94.77	1.00	0.48	93.45	1.01	0.66	93.82
		λ_{x_3}	1.00	1.01	0.53	95.99	1.01	0.62	95.86	1.00	0.48	95.41	1.01	0.59	93.93
		λ_{x_4}	1.00	1.00	0.46	94.58	1.00	0.33	96.40	1.00	0.13	95.31	1.01	0.82	93.49
		λ_{x_5}	1.00	1.01	0.77	94.15	1.00	0.29	95.42	1.00	0.44	95.09	1.01	0.93	94.59
		λ_{y_2}	1.00	0.99	-1.13	93.61	0.99	-0.61	95.09	0.99	-0.96	92.69	0.99	-0.70	94.37
		λ_{y_3}	1.00	0.99	-1.47	94.91	0.99	-0.83	94.11	0.99	-1.37	94.21	0.99	-0.70	94.15
0															
	400														
		ρ_{y2}	0.00	0.35	35.28	0.00	0.36	36.10	0.00	0.37	37.29	0.00	0.49	48.75	0.00
		α	0.00	0.00	-0.03	94.99	0.00	0.17	94.96	0.00	0.10	93.09	0.00	-0.04	96.01
		γ_1	0.30	0.30	-1.02	95.24	0.30	-0.14	95.45	0.30	0.43	95.31	0.30	0.17	93.39
		γ_2	0.30	0.30	0.84	95.36	0.30	0.78	97.05	0.30	0.39	95.44	0.30	0.39	96.63
		γ_3	0.15	0.15	-2.91	94.75	0.15	-1.38	93.85	0.15	0.26	95.19	0.15	-0.28	97.13
		σ_{x_1}	0.50	0.50	0.01	95.48	0.50	0.41	94.10	0.50	0.12	94.20	0.50	0.44	93.64
		σ_{x_2}	0.50	0.50	0.75	95.60	0.50	-0.07	94.10	0.50	0.06	94.08	0.50	0.43	95.64
		σ_{x_3}	0.50	0.50	0.03	94.63	0.50	0.08	94.22	0.50	0.11	94.82	0.50	-0.09	95.26
		σ_{x_4}	0.50	0.50	0.00	94.02	0.50	0.04	93.11	0.50	0.45	93.46	0.50	0.49	94.14
		σ_{x_5}	0.50	0.50	-0.06	94.99	0.50	-0.20	95.82	0.50	0.24	95.81	0.50	-0.16	94.26
		σ_{x_6}	0.50	0.50	0.04	94.14	0.50	-0.10	95.45	0.50	-0.03	96.30	0.50	0.06	95.01
		σ_{y_1}	0.50	0.50	-0.69	94.26	0.50	-0.97	93.48	0.50	-0.60	93.59	0.50	-0.67	95.14
		σ_{y_2}	0.50	0.50	-0.41	95.60	0.50	-0.57	95.45	0.50	-0.48	95.68	0.50	-0.71	93.52
		σ_{y_3}	0.50	0.42	-16.33	80.83	0.41	-18.89	81.06	0.40	-20.39	74.48	0.33	-33.60	59.48
		σ_{ξ_1}	1.00	1.00	0.14	94.87	1.00	-0.11	95.82	1.00	0.11	97.41	1.00	0.06	95.51
		σ_{ξ_2}	1.00	1.00	0.29	93.53	1.00	0.10	94.10	1.00	0.38	94.57	1.00	0.16	95.14
		ν_{x_1}	0.00	0.00	-0.01	96.46	0.00	0.09	94.59	0.00	0.20	93.34	0.00	-0.04	95.14
		ν_{x_2}	0.00	0.00	-0.03	96.46	0.00	-0.12	95.94	0.00	0.36	93.96	0.00	0.30	96.26
		ν_{x_3}	0.00	0.00	-0.05	95.36	0.00	0.15	94.96	0.00	0.40	93.83	0.00	0.04	94.89
		ν_{x_4}	0.00	0.00	-0.28	95.85	0.00	-0.16	94.83	0.00	-0.03	95.31	0.00	0.23	94.51
		ν_{x_5}	0.00	0.00	-0.41	93.89	0.00	-0.12	94.96	0.00	0.03	94.45	0.00	0.22	94.76

Table A.9: Results table for Study 3 measurement lag population model (D2) and measurement lag analysis model (A2) *(continued)*

				W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^{*}		
ρ_{y2}	N		θ	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\theta)\%$	Cover%
		ν_{x_6}	0.00	0.00	-0.37	94.51	0.00	-0.28	94.71	0.00	0.03	94.94	0.00	0.07	94.89
		ν_{y_2}	0.00	0.00	0.02	96.34	0.00	-0.04	96.31	0.00	-0.03	94.57	0.00	0.16	93.89
		ν_{y_3}	0.00	0.00	-0.07	96.46	0.00	-0.30	97.91	0.00	0.05	94.82	0.00	0.06	97.01
		λ_{x_2}	1.00	1.00	-0.14	94.99	1.00	0.37	93.48	1.00	0.28	91.99	1.00	0.26	95.26
		λ_{x_3}	1.00	1.00	0.05	94.87	1.00	0.32	96.31	1.00	0.18	94.33	1.00	0.24	94.51
		λ_{x_4}	1.00	1.00	-0.03	95.85	1.00	0.37	94.83	1.00	0.11	93.96	1.00	0.30	92.52
		λ_{x_5}	1.00	1.00	-0.39	94.38	1.00	0.07	94.34	1.00	0.25	94.08	1.01	0.53	93.14
		λ_{y_2}	1.00	1.00	-0.48	94.75	1.00	-0.45	97.79	0.99	-0.61	94.94	0.99	-0.67	95.26
		λ_{y_3}	1.00	0.99	-0.93	94.63	0.99	-0.73	96.68	0.99	-0.75	93.34	0.99	-0.65	96.63
0.3															
		ρ_{y2}	0.30	0.53	76.56	85.71	0.52	74.64	90.53	0.51	71.66	94.20	0.50	67.37	100.00
		α	0.00	0.00	0.02	94.26	0.00	0.08	93.97	0.00	-0.23	94.20	0.00	-0.24	94.58
		γ_1	0.30	0.30	0.34	94.26	0.30	0.35	94.34	0.30	-0.03	94.82	0.30	0.35	96.22
		γ_2	0.30	0.30	0.31	95.97	0.30	0.22	95.69	0.30	0.40	94.08	0.30	0.24	95.21
		γ_3	0.15	0.15	0.11	93.77	0.15	-0.32	95.20	0.15	0.45	95.56	0.15	-0.98	95.47
		σ_{x_1}	0.50	0.50	-0.10	95.48	0.50	0.21	95.20	0.50	0.41	94.33	0.50	-0.04	94.33
		σ_{x_2}	0.50	0.50	0.05	95.36	0.50	-0.06	94.71	0.50	0.29	95.44	0.50	0.62	94.84
		σ_{x_3}	0.50	0.50	0.51	94.75	0.50	0.27	95.45	0.50	0.34	95.68	0.50	-0.03	94.71
		σ_{x_4}	0.50	0.50	-0.17	95.36	0.50	0.48	96.68	0.50	0.39	97.04	0.50	-0.17	93.20
		σ_{x_5}	0.50	0.50	-0.26	95.36	0.50	0.43	94.83	0.50	-0.02	96.18	0.50	0.17	94.58
		σ_{x_6}	0.50	0.50	0.41	94.87	0.50	-0.19	94.96	0.50	0.04	94.70	0.50	0.40	94.71
		σ_{y_1}	0.50	0.50	-0.56	95.12	0.50	-0.60	94.96	0.50	-0.78	93.34	0.50	-0.60	94.33
		σ_{y_2}	0.50	0.50	-0.71	94.26	0.50	-0.33	95.82	0.50	-0.59	94.33	0.50	-0.60	95.84
		σ_{y_3}	0.50	0.28	-44.87	3.30	0.30	-40.36	16.48	0.30	-39.02	26.51	0.32	-36.14	54.91
		σ_{ξ_1}	1.00	1.00	0.05	94.75	1.00	0.24	95.69	1.00	-0.03	92.60	1.00	0.34	94.58
		σ_{ξ_2}	1.00	1.00	0.30	94.14	1.00	0.31	95.82	1.00	0.13	95.93	1.00	0.20	94.21
		ν_{x_1}	0.00	0.00	0.13	95.73	0.00	0.14	93.36	0.00	-0.23	93.59	0.00	-0.20	95.59
		ν_{x_2}	0.00	0.00	0.05	95.97	0.00	0.18	93.48	0.00	-0.22	94.70	0.00	-0.18	95.09
		ν_{x_3}	0.00	0.00	0.04	95.60	0.00	0.17	93.97	0.00	-0.02	93.71	0.00	-0.25	93.07
		ν_{x_4}	0.00	0.00	0.03	96.83	0.00	0.21	94.34	0.00	-0.05	93.83	0.00	-0.06	93.95
		ν_{x_5}	0.00	0.00	0.13	96.46	0.00	0.22	94.96	0.00	-0.02	94.45	0.00	-0.23	96.60
		ν_{x_6}	0.00	0.00	0.03	96.46	0.00	0.28	94.96	0.00	0.22	94.94	0.00	0.04	95.21
		ν_{y_2}	0.00	0.00	-0.04	94.87	0.00	-0.07	93.60	0.00	0.16	93.96	0.00	-0.20	93.20
		ν_{y_3}	0.00	0.00	-0.06	95.36	0.00	-0.05	93.85	0.00	0.21	93.22	0.00	0.03	94.58
		λ_{x_2}	1.00	1.00	0.14	94.99	1.00	0.29	95.08	1.00	0.17	95.68	1.00	0.05	93.32
		λ_{x_3}	1.00	1.00	0.20	94.75	1.00	0.22	95.08	1.00	0.13	93.34	1.00	0.27	94.21
		λ_{x_4}	1.00	1.00	0.04	95.24	1.00	0.06	95.33	1.00	0.37	95.93	1.00	0.06	96.47
		λ_{x_5}	1.00	1.00	0.05	94.87	1.00	0.05	95.45	1.01	0.54	93.71	1.00	0.01	94.71
		λ_{y_2}	1.00	0.99	-0.52	94.26	0.99	-0.60	95.20	1.00	0.18	95.68	0.99	-1.15	93.45
		λ_{y_3}	1.00	1.00	0.04	93.41	0.99	-0.63	94.10	1.00	-0.27	94.20	0.98	-1.65	92.70
0.6															
		ρ_{y2}	0.60	0.73	20.88	84.19	0.72	20.30	93.97	0.71	18.36	96.55	0.52	-13.78	100.00
		α	0.00	0.00	-0.03	95.22	0.00	0.29	94.70	0.00	-0.02	94.57	0.00	-0.03	95.69
		γ_1	0.30	0.30	0.02	95.71	0.30	0.98	95.69	0.30	-0.16	95.56	0.30	0.28	95.30
		γ_2	0.30	0.30	0.05	93.75	0.30	0.45	94.95	0.30	0.11	96.67	0.30	1.39	96.45
		γ_3	0.15	0.15	-1.94	96.20	0.15	0.40	94.46	0.15	-1.97	94.57	0.15	-0.84	94.67
		σ_{x_1}	0.50	0.50	0.53	94.24	0.50	-0.10	95.32	0.50	-0.18	94.70	0.50	0.87	95.69
		σ_{x_2}	0.50	0.50	0.18	95.83	0.50	0.02	93.84	0.50	0.52	95.19	0.50	0.19	95.30
		σ_{x_3}	0.50	0.50	0.61	94.73	0.50	-0.03	95.57	0.50	-0.20	94.82	0.50	0.54	94.04
		σ_{x_4}	0.50	0.50	0.13	95.10	0.50	0.28	93.60	0.50	0.34	94.45	0.50	-0.05	94.92
		σ_{x_5}	0.50	0.50	0.19	93.87	0.50	-0.10	95.81	0.50	0.07	96.18	0.50	0.26	95.69
		σ_{x_6}	0.50	0.50	0.15	93.50	0.50	0.05	96.92	0.50	0.51	94.20	0.50	0.46	95.18
		σ_{y_1}	0.50	0.50	-0.53	93.75	0.50	-0.65	95.32	0.50	-0.89	92.73	0.50	-0.64	94.80
		σ_{y_2}	0.50	0.50	-0.32	96.20	0.50	-0.30	95.81	0.50	-0.72	93.09	0.50	-0.48	95.05
		σ_{y_3}	0.50	0.15	-69.78	0.00	0.19	-61.42	0.12	0.20	-59.43	1.85	0.31	-37.72	50.38

Table A.9: Results table for Study 3 measurement lag population model (D2) and measurement lag analysis model (A2) *(continued)*

			W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^{*}		
ρ_{y2}	N	θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
	σ_{ξ_1}	1.00	1.00	-0.24	94.36	1.00	-0.23	94.46	1.00	0.03	95.68	1.00	0.05	95.43
	σ_{ξ_2}	1.00	1.00	0.07	95.22	1.00	0.30	95.94	1.00	0.38	95.81	1.00	0.06	96.32
	ν_{x_1}	0.00	0.01	0.52	95.71	0.00	0.49	95.69	0.00	0.29	94.57	0.00	0.14	94.80
	ν_{x_2}	0.00	0.00	0.38	95.47	0.00	0.31	95.44	0.01	0.62	95.68	0.00	0.07	95.30
	ν_{x_3}	0.00	0.00	0.30	95.47	0.00	0.38	94.95	0.01	0.54	94.33	0.00	0.02	95.69
	ν_{x_4}	0.00	0.00	0.06	94.85	0.01	0.55	93.60	0.00	0.02	95.07	0.00	-0.20	95.05
	ν_{x_5}	0.00	0.00	0.18	94.49	0.00	0.30	95.20	0.00	0.10	94.82	0.00	0.02	93.91
	ν_{x_6}	0.00	0.00	0.23	93.50	0.00	0.43	94.95	0.00	-0.02	94.57	0.00	-0.24	96.45
	ν_{y_2}	0.00	0.00	0.15	94.61	0.00	-0.15	93.60	0.00	0.17	94.57	0.00	0.01	95.05
	ν_{y_3}	0.00	0.00	0.12	96.20	0.00	0.02	93.35	0.00	0.26	95.31	0.00	0.01	90.86
	λ_{x_2}	1.00	1.00	0.04	94.24	1.01	0.63	95.20	1.00	-0.01	95.56	1.00	0.34	93.27
	λ_{x_3}	1.00	1.00	0.03	94.24	1.00	0.45	95.32	1.00	0.20	95.31	1.00	0.41	95.05
	λ_{x_4}	1.00	1.00	-0.16	95.47	1.00	0.28	96.67	1.00	0.11	94.82	1.00	0.38	96.19
	λ_{x_5}	1.00	1.00	-0.05	95.10	1.00	0.13	96.18	1.00	0.12	94.70	1.00	0.31	94.80
	λ_{y_2}	1.00	0.99	-0.73	94.61	0.99	-0.76	95.94	1.00	-0.47	93.34	0.99	-0.75	96.19
	λ_{y_3}	1.00	0.99	-0.93	92.89	1.00	-0.34	96.92	0.99	-0.52	92.48	0.99	-1.06	92.64

1 W^* is the population and analysis W condition.

2 N is the simulated sample size.

3 θ is the simulated population value of each parameter.

4 $\rho_{y2} = ...$ is the simulated spatial autocorrelation of the endogenous lag.

5 $\bar{\hat{\theta}}$ is the obtained average posterior mean across simulated iterations.

6 $Bias(\bar{\hat{\theta}})\%$ is the average percent bias across simulation iterations when $\theta \neq 0$, otherwise $Bias(\bar{\hat{\theta}})\%$ provides absolute bias.

7 Cover% is the percent of simulated trials in which the population value falls within the central 95% density of the posterior estimate.

Table A.10: Results table for Study 3 endogenous structural lag population model (D3) and endogenous structural lag analysis model (A3)

				W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^{*}		
ρ_η	N		θ	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%
0															
	49														
		ρ_η	0.00	0.20	19.99	0.00	0.31	31.06	0.00	0.36	35.87	0.00	0.48	48.20	0.00
		α	0.00	0.00	-0.14	97.73	-0.01	-0.62	99.60	0.00	0.04	98.66	0.00	-0.25	99.20
		γ_1	0.30	0.31	1.70	94.80	0.31	2.95	96.93	0.31	2.42	96.66	0.30	-1.65	94.65
		γ_2	0.30	0.30	0.21	95.73	0.28	-5.16	95.59	0.30	-1.52	96.12	0.29	-2.73	95.85
		γ_3	0.15	0.14	-6.12	95.47	0.15	1.25	94.92	0.15	0.93	96.52	0.14	-6.19	95.85
		σ_{x_1}	0.50	0.51	1.88	96.27	0.51	1.60	94.65	0.51	2.61	94.39	0.51	1.70	93.31
		σ_{x_2}	0.50	0.50	0.75	95.60	0.50	-0.80	94.52	0.51	1.29	95.05	0.51	2.12	93.57
		σ_{x_3}	0.50	0.50	0.33	92.80	0.50	0.16	95.45	0.50	-0.44	94.92	0.50	0.94	94.65
		σ_{x_4}	0.50	0.51	1.48	94.00	0.50	0.73	94.25	0.51	1.80	95.45	0.51	2.01	94.38
		σ_{x_5}	0.50	0.51	2.11	94.00	0.50	-0.10	93.58	0.50	0.78	94.92	0.50	-0.96	93.84
		σ_{x_6}	0.50	0.51	1.11	95.87	0.51	1.28	93.72	0.51	1.95	92.78	0.51	2.33	94.91
		σ_{y_1}	0.50	0.50	0.33	92.53	0.49	-1.78	95.59	0.49	-1.91	94.25	0.49	-1.62	94.65
		σ_{y_2}	0.50	0.52	3.33	96.40	0.52	3.06	93.98	0.52	3.39	93.85	0.52	3.04	95.45
		σ_{y_3}	0.50	0.51	2.20	94.93	0.52	3.12	95.45	0.51	2.38	94.39	0.51	1.18	93.31
		σ_{ξ_1}	1.00	1.01	0.53	96.13	1.01	0.98	95.72	1.01	1.40	92.65	1.01	0.65	94.65
		σ_{ξ_2}	1.00	1.00	0.05	96.40	1.01	0.80	96.12	1.01	0.91	93.18	1.01	1.33	94.24
		ν_{x_1}	0.00	-0.01	-0.75	95.20	-0.01	-0.55	95.86	0.00	0.24	94.79	0.00	0.06	94.91
		ν_{x_2}	0.00	-0.01	-0.65	95.07	-0.01	-0.77	95.19	0.00	0.36	94.25	0.00	-0.31	94.11
		ν_{x_3}	0.00	-0.01	-0.74	93.47	0.00	-0.41	96.66	0.00	0.24	94.92	0.00	-0.29	95.58
		ν_{x_4}	0.00	-0.01	-1.20	95.47	0.01	1.19	95.32	-0.02	-1.69	95.05	0.00	0.40	96.12
		ν_{x_5}	0.00	-0.01	-1.10	95.73	0.01	1.36	95.86	-0.02	-1.81	96.39	0.00	-0.37	94.78
		ν_{x_6}	0.00	-0.01	-0.97	96.00	0.01	1.26	95.86	-0.02	-1.56	94.39	0.00	0.33	94.91
		ν_{y_2}	0.00	0.00	-0.22	95.07	0.00	-0.29	95.86	0.00	0.36	97.06	0.01	0.75	96.79
		ν_{y_3}	0.00	0.00	-0.30	95.73	0.00	-0.04	95.19	-0.01	-0.56	96.79	0.00	0.08	97.46
		λ_{x_2}	1.00	1.03	2.56	96.40	1.02	1.86	96.66	1.02	1.79	96.52	1.02	1.97	95.18
		λ_{x_3}	1.00	1.02	2.34	96.80	1.02	1.72	95.99	1.03	2.73	93.72	1.02	2.25	96.92
		λ_{x_4}	1.00	1.03	2.53	95.47	1.02	2.23	96.66	1.03	2.91	94.65	1.03	2.78	95.98
		λ_{x_5}	1.00	1.03	2.80	95.47	1.02	1.85	96.93	1.02	1.76	95.05	1.01	1.48	95.85
		λ_{y_2}	1.00	0.98	-1.58	94.67	0.99	-1.20	94.39	0.98	-1.81	95.59	0.98	-1.73	95.31
		λ_{y_3}	1.00	0.99	-0.94	95.07	0.98	-1.68	94.79	0.98	-1.57	96.26	0.99	-1.40	96.25
0.3															
		ρ_η	0.30	0.35	15.45	98.27	0.42	38.63	99.33	0.44	45.14	99.06	0.49	64.03	100.00
		α	0.00	0.00	0.28	95.73	0.00	0.23	96.39	0.00	0.19	96.93	0.00	-0.04	98.26
		γ_1	0.30	0.30	0.82	96.53	0.29	-1.87	95.59	0.30	-0.89	95.72	0.30	0.01	96.65
		γ_2	0.30	0.31	3.66	96.00	0.30	-1.14	95.19	0.30	0.60	94.52	0.29	-2.65	94.78
		γ_3	0.15	0.16	9.00	95.07	0.15	1.41	94.92	0.15	1.92	96.66	0.16	5.54	96.12
		σ_{x_1}	0.50	0.51	1.58	94.53	0.51	1.56	94.65	0.51	2.06	94.25	0.51	2.53	94.78
		σ_{x_2}	0.50	0.50	0.66	94.40	0.50	0.89	94.92	0.51	1.31	94.52	0.50	0.28	93.84
		σ_{x_3}	0.50	0.50	0.85	96.67	0.50	0.24	94.79	0.50	-0.03	95.99	0.51	1.42	93.17
		σ_{x_4}	0.50	0.51	1.26	94.80	0.50	0.69	93.05	0.51	2.33	92.65	0.51	2.51	95.45
		σ_{x_5}	0.50	0.50	0.81	93.87	0.51	1.89	93.32	0.50	-0.64	94.65	0.51	1.95	94.51
		σ_{x_6}	0.50	0.50	-0.12	96.27	0.51	1.25	95.86	0.51	1.90	95.32	0.51	1.25	92.10
		σ_{y_1}	0.50	0.50	-0.46	95.87	0.50	0.43	92.25	0.50	0.38	92.38	0.50	-0.90	94.24
		σ_{y_2}	0.50	0.51	2.73	93.73	0.51	2.22	94.25	0.51	1.72	94.92	0.51	1.17	95.05
		σ_{y_3}	0.50	0.52	3.12	95.73	0.51	2.46	94.25	0.51	2.77	95.72	0.50	0.73	94.78
		σ_{ξ_1}	1.00	1.01	1.23	96.00	1.02	1.75	93.72	1.01	0.52	92.91	1.01	1.07	94.91
		σ_{ξ_2}	1.00	1.00	0.42	95.20	1.02	1.74	96.26	1.00	0.40	95.19	1.02	2.41	94.91
		ν_{x_1}	0.00	0.00	-0.26	95.07	0.00	0.40	95.19	0.00	0.03	96.79	0.00	-0.01	94.91
		ν_{x_2}	0.00	0.00	-0.10	96.13	0.00	0.07	95.72	0.00	0.33	96.39	0.00	-0.32	95.85
		ν_{x_3}	0.00	0.00	-0.22	95.47	0.01	0.75	94.79	0.00	0.01	95.59	0.00	-0.29	94.78
		ν_{x_4}	0.00	0.00	-0.15	96.67	-0.01	-0.92	96.66	-0.01	-0.56	95.86	0.00	0.18	94.38
		ν_{x_5}	0.00	0.00	0.12	96.00	-0.01	-1.32	96.39	-0.01	-1.18	95.59	-0.01	-0.57	96.25
		ν_{x_6}	0.00	0.00	-0.46	96.13	-0.01	-1.11	96.12	-0.01	-0.63	95.05	0.00	0.28	95.72

Table A.10: Results table for Study 3 endogenous structural lag population model (D3) and endogenous structural lag analysis model (A3) *(continued)*

				W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^{*}		
ρ_η	N		θ	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%
		ν_{y_2}	0.00	0.00	0.05	95.07	0.00	-0.05	95.86	0.01	0.51	96.79	0.00	-0.08	94.24
		ν_{y_3}	0.00	0.00	-0.06	96.93	-0.01	-1.09	95.99	0.00	0.11	96.12	0.00	-0.23	95.85
		λ_{x_2}	1.00	1.02	2.32	95.60	1.02	1.75	95.59	1.03	3.21	95.99	1.03	3.05	96.39
		λ_{x_3}	1.00	1.02	2.50	96.53	1.02	2.04	96.52	1.02	2.26	96.52	1.02	2.22	95.85
		λ_{x_4}	1.00	1.01	1.15	93.87	1.01	1.42	95.86	1.02	2.34	95.72	1.02	2.03	95.85
		λ_{x_5}	1.00	1.02	2.38	94.13	1.01	1.23	95.19	1.03	2.66	95.72	1.01	1.14	96.65
		λ_{y_2}	1.00	0.98	-1.76	93.87	0.99	-1.35	93.85	1.00	-0.27	96.79	0.99	-0.96	96.39
		λ_{y_3}	1.00	0.98	-1.75	95.33	1.00	-0.36	93.72	0.99	-0.98	95.99	0.99	-0.68	96.39
0.6															
		ρ_η	0.60	0.58	-3.21	97.33	0.55	-8.20	98.80	0.54	-9.70	99.73	0.50	-16.46	100.00
		α	0.00	0.00	-0.39	95.73	0.00	-0.44	95.05	0.02	1.57	95.72	0.01	0.82	94.78
		γ_1	0.30	0.31	2.93	95.20	0.30	1.24	94.39	0.28	-5.06	96.79	0.30	0.69	96.52
		γ_2	0.30	0.30	-0.90	95.73	0.30	1.63	95.05	0.31	2.61	96.79	0.29	-1.79	95.05
		γ_3	0.15	0.16	4.05	97.33	0.15	-0.22	95.19	0.14	-8.83	95.72	0.15	2.91	95.98
		σ_{x_1}	0.50	0.51	1.78	93.87	0.51	1.96	94.52	0.52	3.24	95.19	0.52	3.18	94.38
		σ_{x_2}	0.50	0.50	0.61	93.47	0.50	0.16	94.65	0.51	1.88	97.19	0.50	0.68	94.11
		σ_{x_3}	0.50	0.50	0.93	95.20	0.50	0.78	93.45	0.50	0.33	96.39	0.51	1.48	95.05
		σ_{x_4}	0.50	0.51	1.86	94.27	0.50	0.60	94.39	0.51	1.92	93.45	0.51	1.02	95.58
		σ_{x_5}	0.50	0.50	0.89	96.27	0.50	0.29	95.86	0.51	1.20	92.11	0.51	1.48	95.05
		σ_{x_6}	0.50	0.50	0.78	95.47	0.51	2.04	94.92	0.51	1.17	95.05	0.51	2.24	92.50
		σ_{y_1}	0.50	0.51	1.35	96.00	0.49	-1.02	94.12	0.51	1.39	95.99	0.50	0.03	93.98
		σ_{y_2}	0.50	0.50	0.96	92.93	0.51	2.44	95.72	0.51	2.28	94.52	0.51	1.94	95.31
		σ_{y_3}	0.50	0.51	2.57	93.73	0.52	3.47	95.86	0.50	0.83	95.19	0.51	2.04	95.05
		σ_{ξ_1}	1.00	1.01	0.55	95.87	1.01	1.24	96.52	1.02	2.35	94.39	1.01	0.69	94.24
		σ_{ξ_2}	1.00	1.00	-0.46	96.40	1.02	2.34	94.12	1.01	0.95	94.39	1.01	1.12	95.18
		ν_{x_1}	0.00	-0.01	-0.87	96.27	0.00	0.11	95.59	0.00	-0.46	96.93	-0.01	-0.89	95.58
		ν_{x_2}	0.00	0.00	-0.07	96.67	0.00	-0.17	96.93	0.00	0.13	96.12	-0.01	-0.77	95.98
		ν_{x_3}	0.00	0.00	-0.15	95.87	0.00	0.41	95.72	0.00	0.49	96.39	-0.01	-0.92	95.58
		ν_{x_4}	0.00	0.00	-0.20	96.13	0.00	-0.25	95.32	-0.01	-0.89	95.86	-0.01	-0.73	96.39
		ν_{x_5}	0.00	0.00	-0.08	94.67	-0.01	-0.66	93.98	-0.01	-1.33	96.26	0.00	-0.25	97.05
		ν_{x_6}	0.00	0.00	0.36	95.73	0.00	0.06	95.32	-0.01	-1.17	95.86	0.00	0.44	95.18
		ν_{y_2}	0.00	0.00	-0.40	93.07	-0.01	-0.68	94.39	0.00	0.13	95.45	0.00	0.43	95.18
		ν_{y_3}	0.00	0.00	-0.45	96.00	-0.01	-0.88	95.86	0.00	-0.04	96.12	0.01	1.00	96.52
		λ_{x_2}	1.00	1.02	2.17	95.07	1.02	1.88	96.26	1.02	2.08	96.39	1.03	2.86	96.12
		λ_{x_3}	1.00	1.02	2.09	96.00	1.02	1.78	94.65	1.02	1.59	96.26	1.03	2.67	95.05
		λ_{x_4}	1.00	1.02	2.41	95.07	1.02	1.76	94.79	1.02	2.36	95.72	1.02	2.17	95.45
		λ_{x_5}	1.00	1.02	2.19	95.87	1.02	2.00	95.45	1.02	1.93	95.72	1.02	2.17	95.45
		λ_{y_2}	1.00	0.99	-0.97	94.40	0.99	-1.38	95.72	0.99	-0.96	94.79	0.98	-1.56	95.58
		λ_{y_3}	1.00	0.99	-0.59	96.13	0.99	-1.28	94.39	0.99	-0.89	95.45	0.99	-1.17	94.51
0															
	196														
		ρ_η	0.00	0.10	10.04	0.00	0.15	14.92	0.00	0.18	18.21	0.00	0.47	46.62	0.00
		α	0.00	0.00	-0.16	97.04	0.00	0.13	97.31	0.00	0.43	98.16	0.00	0.07	99.86
		γ_1	0.30	0.29	-1.84	93.65	0.30	-0.03	94.34	0.30	-1.21	95.76	0.31	2.13	95.45
		γ_2	0.30	0.29	-1.75	96.33	0.30	0.72	93.21	0.30	0.11	95.90	0.30	-0.26	95.31
		γ_3	0.15	0.15	-1.13	94.22	0.15	1.28	94.63	0.15	-1.43	93.92	0.15	-0.56	96.45
		σ_{x_1}	0.50	0.50	0.60	94.08	0.50	0.64	92.93	0.50	0.20	94.34	0.51	1.18	94.60
		σ_{x_2}	0.50	0.50	0.34	95.06	0.50	-0.14	95.33	0.50	0.35	93.92	0.50	-0.25	96.16
		σ_{x_3}	0.50	0.50	0.47	96.05	0.50	0.05	95.05	0.50	0.33	95.05	0.50	-0.29	94.03
		σ_{x_4}	0.50	0.50	0.66	95.20	0.50	0.67	93.49	0.50	0.72	96.75	0.50	-0.01	93.75
		σ_{x_5}	0.50	0.50	0.28	93.23	0.50	0.58	94.06	0.50	0.54	95.19	0.50	0.42	94.32
		σ_{x_6}	0.50	0.50	0.53	92.81	0.50	0.20	94.48	0.50	0.48	94.34	0.50	0.73	94.60
		σ_{y_1}	0.50	0.50	0.42	94.64	0.50	-0.09	94.06	0.50	0.56	93.92	0.50	0.41	94.03
		σ_{y_2}	0.50	0.50	0.29	93.23	0.50	0.34	94.63	0.50	0.35	94.48	0.50	0.63	93.47

Table A.10: Results table for Study 3 endogenous structural lag population model (D3) and endogenous structural lag analysis model (A3) *(continued)*

				W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^{*}		
ρ_η	N		θ	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%
		σ_{y_3}	0.50	0.50	0.26	93.94	0.51	1.12	94.63	0.51	1.45	93.92	0.50	0.63	94.32
		σ_{ξ_1}	1.00	1.00	0.11	95.49	1.00	-0.02	94.91	1.00	0.31	94.63	1.00	0.30	94.89
		σ_{ξ_2}	1.00	1.01	0.70	95.91	1.00	-0.08	96.32	1.00	0.22	92.93	1.00	0.15	96.16
		ν_{x_1}	0.00	0.00	0.37	96.90	0.00	-0.30	93.07	-0.01	-0.57	95.62	0.00	-0.17	95.88
		ν_{x_2}	0.00	0.00	0.21	95.35	0.00	-0.15	96.04	0.00	-0.23	95.33	0.00	0.08	94.60
		ν_{x_3}	0.00	0.00	0.47	97.32	0.00	-0.01	95.19	-0.01	-0.64	94.77	0.00	-0.04	94.32
		ν_{x_4}	0.00	0.00	-0.42	95.49	0.00	-0.24	93.64	0.00	0.21	97.45	0.00	-0.05	93.18
		ν_{x_5}	0.00	0.00	-0.37	94.78	0.00	-0.18	95.05	0.00	0.22	96.46	0.00	0.11	94.74
		ν_{x_6}	0.00	0.00	-0.32	95.49	0.00	-0.25	93.49	0.00	0.15	95.47	0.00	-0.03	94.46
		ν_{y_2}	0.00	0.00	-0.38	96.33	0.00	-0.10	95.05	0.00	0.12	95.33	0.01	0.70	95.74
		ν_{y_3}	0.00	0.00	0.10	95.35	0.00	-0.09	95.19	0.00	0.19	94.77	0.01	0.63	96.02
		λ_{x_2}	1.00	1.01	0.74	96.33	1.01	0.85	96.75	1.01	0.81	94.77	1.01	0.62	95.17
		λ_{x_3}	1.00	1.00	0.25	94.08	1.00	0.32	95.19	1.00	0.40	92.08	1.01	0.69	95.31
		λ_{x_4}	1.00	1.00	0.45	95.77	1.00	0.36	95.47	1.01	0.66	94.48	1.01	0.61	95.60
		λ_{x_5}	1.00	1.01	0.61	94.22	1.00	0.20	93.49	1.01	1.03	93.21	1.00	0.22	93.89
		λ_{y_2}	1.00	1.00	-0.01	96.05	1.00	-0.29	93.49	1.00	-0.05	95.19	1.00	-0.02	95.31
		λ_{y_3}	1.00	1.00	0.35	96.33	1.00	-0.37	93.21	1.00	-0.16	95.90	1.00	-0.02	95.45
0.3															
		ρ_η	0.30	0.31	2.34	96.75	0.34	11.76	95.76	0.36	18.88	96.18	0.48	60.12	100.00
		α	0.00	0.00	0.00	95.90	0.00	-0.13	95.47	0.00	0.01	97.45	0.00	-0.34	97.44
		γ_1	0.30	0.30	0.56	95.90	0.30	0.24	94.48	0.30	-0.82	94.06	0.30	1.53	95.45
		γ_2	0.30	0.30	-1.12	93.36	0.31	2.87	94.77	0.30	0.94	95.05	0.30	0.05	94.60
		γ_3	0.15	0.16	4.43	95.06	0.15	-1.37	95.47	0.15	-1.45	96.18	0.16	6.82	93.18
		σ_{x_1}	0.50	0.50	0.91	94.92	0.50	-0.06	94.77	0.50	0.94	95.47	0.50	0.65	94.89
		σ_{x_2}	0.50	0.50	0.55	95.06	0.50	0.94	94.48	0.50	-0.09	95.05	0.50	-0.05	95.03
		σ_{x_3}	0.50	0.50	0.29	94.92	0.50	0.54	96.32	0.50	0.26	96.18	0.50	0.02	95.45
		σ_{x_4}	0.50	0.50	0.47	95.62	0.50	0.52	95.47	0.50	0.93	95.33	0.51	1.11	93.61
		σ_{x_5}	0.50	0.50	0.47	96.33	0.50	0.63	95.62	0.50	0.08	95.62	0.50	0.58	94.32
		σ_{x_6}	0.50	0.50	0.16	97.18	0.50	0.74	94.63	0.50	-0.11	94.20	0.50	0.46	95.74
		σ_{y_1}	0.50	0.50	0.43	94.21	0.50	0.05	91.94	0.50	0.47	94.77	0.50	0.37	94.60
		σ_{y_2}	0.50	0.50	0.90	95.48	0.50	-0.26	96.18	0.50	0.60	96.18	0.50	-0.24	96.31
		σ_{y_3}	0.50	0.50	0.72	95.48	0.50	0.53	95.19	0.50	0.79	95.90	0.51	1.12	94.60
		σ_{ξ_1}	1.00	1.00	0.13	95.62	1.01	0.71	96.04	1.00	0.09	95.90	1.01	0.65	93.18
		σ_{ξ_2}	1.00	1.00	0.11	93.36	1.00	0.40	94.63	1.00	0.45	94.06	1.00	0.40	95.45
		ν_{x_1}	0.00	0.00	0.08	95.90	0.00	-0.29	94.48	0.00	-0.24	95.05	0.00	-0.26	93.75
		ν_{x_2}	0.00	0.00	-0.16	96.19	0.00	-0.42	94.63	0.00	-0.49	94.48	-0.01	-0.59	96.16
		ν_{x_3}	0.00	0.00	0.10	96.89	0.00	-0.44	96.04	0.00	-0.10	94.20	0.00	-0.29	95.60
		ν_{x_4}	0.00	0.00	-0.20	94.21	0.00	0.09	93.21	0.00	-0.01	94.34	0.00	0.01	96.02
		ν_{x_5}	0.00	0.00	-0.12	95.06	0.00	-0.17	94.34	0.00	0.20	94.06	0.00	-0.02	94.74
		ν_{x_6}	0.00	0.00	-0.29	94.49	0.00	-0.19	93.07	0.00	-0.12	94.48	0.00	-0.48	95.17
		ν_{y_2}	0.00	0.00	-0.03	97.60	0.00	0.01	93.21	0.00	0.28	94.34	0.00	0.12	96.59
		ν_{y_3}	0.00	0.00	-0.08	95.06	0.00	0.12	95.90	0.00	0.26	95.19	0.00	0.05	95.31
		λ_{x_2}	1.00	1.01	1.06	94.63	1.00	0.05	93.21	1.01	0.63	95.19	1.01	0.72	95.03
		λ_{x_3}	1.00	1.01	0.62	93.93	1.00	0.25	93.92	1.00	0.38	93.49	1.01	0.74	94.32
		λ_{x_4}	1.00	1.00	0.49	95.20	1.00	0.47	95.76	1.01	0.61	92.79	1.00	0.03	94.60
		λ_{x_5}	1.00	1.00	0.38	96.19	1.01	1.10	93.49	1.01	0.98	95.19	1.00	0.28	95.45
		λ_{y_2}	1.00	1.00	-0.14	95.48	1.00	-0.50	94.48	1.00	-0.35	95.62	1.00	-0.07	95.17
		λ_{y_3}	1.00	1.00	-0.31	95.76	1.00	-0.48	94.48	1.00	-0.44	96.46	1.00	-0.24	95.88
0.6															
		ρ_η	0.60	0.60	-0.37	93.93	0.58	-2.75	95.62	0.58	-3.05	98.02	0.51	-14.68	100.00
		α	0.00	0.00	0.24	94.92	0.00	-0.09	95.33	0.00	0.25	93.35	-0.01	-0.95	96.73
		γ_1	0.30	0.30	1.59	94.49	0.30	0.22	95.33	0.31	2.46	94.06	0.29	-2.03	95.31
		γ_2	0.30	0.30	1.16	92.80	0.30	0.05	95.19	0.29	-2.69	96.18	0.30	-0.66	94.32
		γ_3	0.15	0.15	0.16	94.77	0.15	-0.11	96.75	0.16	4.78	94.77	0.15	-1.40	95.74
		σ_{x_1}	0.50	0.51	1.04	94.63	0.50	0.60	95.33	0.50	0.07	96.04	0.50	0.44	94.46

Table A.10: Results table for Study 3 endogenous structural lag population model (D3) and endogenous structural lag analysis model (A3) *(continued)*

				W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^{*}		
ρ_η	N		θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%
		σ_{x_2}	0.50	0.51	1.21	95.06	0.50	0.30	94.91	0.50	0.80	93.78	0.50	0.23	95.17
		σ_{x_3}	0.50	0.50	-0.08	94.49	0.50	-0.56	96.18	0.50	0.47	94.63	0.50	0.48	94.74
		σ_{x_4}	0.50	0.50	0.56	95.62	0.50	0.44	93.49	0.51	1.10	95.05	0.51	1.25	95.17
		σ_{x_5}	0.50	0.50	-0.02	95.48	0.50	0.65	95.05	0.50	0.09	93.92	0.50	0.62	94.60
		σ_{x_6}	0.50	0.51	1.05	94.92	0.51	1.09	95.62	0.50	-0.12	92.64	0.50	0.56	93.18
		σ_{y_1}	0.50	0.50	-0.02	96.47	0.50	-0.35	94.06	0.50	0.52	96.89	0.50	0.34	94.46
		σ_{y_2}	0.50	0.50	0.40	93.22	0.50	0.33	94.48	0.50	0.84	93.64	0.50	0.43	94.03
		σ_{y_3}	0.50	0.50	0.57	94.21	0.50	0.91	92.93	0.50	0.91	95.76	0.50	0.87	94.32
		σ_{ξ_1}	1.00	1.00	0.11	94.77	1.00	0.44	95.19	1.01	0.53	95.76	1.00	-0.18	95.17
		σ_{ξ_2}	1.00	1.00	0.02	94.92	1.00	0.47	95.05	1.00	-0.18	95.90	1.00	-0.24	95.74
		ν_{x_1}	0.00	0.00	0.44	95.62	0.00	0.39	95.05	0.00	0.13	95.47	0.00	-0.31	94.46
		ν_{x_2}	0.00	0.00	0.16	95.20	0.00	0.01	95.33	0.00	0.38	95.47	-0.01	-0.55	95.03
		ν_{x_3}	0.00	0.00	0.29	96.89	0.00	0.40	94.77	0.01	0.53	94.34	0.00	-0.18	95.74
		ν_{x_4}	0.00	0.00	0.04	95.20	0.00	-0.14	96.46	0.00	-0.47	95.47	0.00	-0.25	96.31
		ν_{x_5}	0.00	0.00	-0.28	95.48	0.00	-0.38	96.04	0.00	0.08	95.47	0.00	-0.50	94.89
		ν_{x_6}	0.00	0.00	0.08	93.22	-0.01	-0.61	94.77	0.00	0.10	94.91	0.00	-0.41	95.45
		ν_{y_2}	0.00	0.00	0.01	94.92	0.00	0.16	95.33	0.00	0.02	94.91	0.00	0.07	95.31
		ν_{y_3}	0.00	0.00	-0.08	94.07	0.00	-0.10	94.63	0.00	0.12	96.18	0.00	0.32	95.17
		λ_{x_2}	1.00	1.01	0.52	94.21	1.01	1.02	95.62	1.00	0.29	94.63	1.00	0.46	94.03
		λ_{x_3}	1.00	1.01	1.00	96.19	1.01	0.79	92.79	1.00	0.42	96.18	1.01	1.01	95.60
		λ_{x_4}	1.00	1.01	1.03	94.49	1.00	0.23	95.76	1.01	0.84	94.48	1.01	0.65	93.89
		λ_{x_5}	1.00	1.01	0.76	94.49	1.00	-0.08	96.18	1.01	0.71	93.64	1.01	0.79	93.18
		λ_{y_2}	1.00	1.00	-0.02	94.21	1.00	-0.15	94.48	1.00	-0.10	94.63	1.00	-0.12	94.46
		λ_{y_3}	1.00	1.00	-0.18	94.92	1.00	0.05	95.47	1.00	0.09	95.47	1.00	-0.01	94.32
0															
	400														
		ρ_η	0.00	0.07	6.82	0.00	0.10	10.24	0.00	0.13	12.52	0.00	0.45	44.90	0.00
		α	0.00	0.00	0.22	95.73	0.00	0.12	97.73	0.00	0.14	95.72	0.00	-0.03	98.39
		γ_1	0.30	0.30	0.59	95.46	0.30	-0.46	95.19	0.30	-1.15	94.25	0.30	-0.29	95.56
		γ_2	0.30	0.30	1.09	94.53	0.30	-0.48	94.39	0.30	-0.62	94.92	0.30	0.18	96.24
		γ_3	0.15	0.15	1.28	94.53	0.15	1.44	94.52	0.15	-3.28	94.65	0.16	5.49	93.01
		σ_{x_1}	0.50	0.50	0.42	94.13	0.50	0.74	92.78	0.50	-0.33	93.72	0.50	0.19	95.83
		σ_{x_2}	0.50	0.50	-0.04	95.19	0.50	0.08	94.12	0.50	0.10	94.25	0.50	0.17	96.51
		σ_{x_3}	0.50	0.50	0.17	94.53	0.50	0.18	94.39	0.50	0.67	94.39	0.50	-0.23	95.03
		σ_{x_4}	0.50	0.50	0.36	96.80	0.50	0.22	94.39	0.50	0.08	95.99	0.50	0.51	93.28
		σ_{x_5}	0.50	0.50	0.12	97.20	0.50	0.48	94.39	0.50	0.59	94.52	0.50	0.10	94.35
		σ_{x_6}	0.50	0.50	0.08	94.26	0.50	-0.33	95.45	0.50	-0.02	95.19	0.50	0.24	97.18
		σ_{y_1}	0.50	0.50	-0.29	95.99	0.50	0.22	94.12	0.50	0.03	95.05	0.50	0.31	94.62
		σ_{y_2}	0.50	0.50	0.63	96.26	0.50	-0.01	92.91	0.50	0.68	95.19	0.50	0.54	95.30
		σ_{y_3}	0.50	0.50	0.81	94.13	0.50	0.41	96.12	0.50	0.03	95.72	0.50	0.13	94.49
		σ_{ξ_1}	1.00	1.00	0.26	93.86	1.00	0.11	93.58	1.00	0.48	94.39	1.00	0.23	92.74
		σ_{ξ_2}	1.00	1.00	0.44	97.06	1.00	0.20	95.19	1.00	0.11	95.45	1.00	0.19	96.10
		ν_{x_1}	0.00	0.00	-0.19	94.53	0.00	0.48	94.12	0.00	0.26	96.39	0.00	0.04	93.95
		ν_{x_2}	0.00	0.00	-0.24	94.39	0.00	0.30	93.72	0.00	0.41	94.39	0.00	0.01	96.91
		ν_{x_3}	0.00	0.00	-0.08	94.13	0.00	0.32	92.65	0.00	0.36	95.45	0.00	-0.05	95.83
		ν_{x_4}	0.00	0.00	-0.15	94.53	0.00	0.17	95.72	0.00	0.28	95.86	0.00	0.14	94.22
		ν_{x_5}	0.00	0.00	-0.14	95.19	0.00	0.05	95.32	0.00	0.10	94.79	0.00	0.19	95.16
		ν_{x_6}	0.00	0.00	-0.21	95.59	0.00	0.20	96.12	0.00	-0.09	96.52	0.00	0.23	93.95
		ν_{y_2}	0.00	0.00	-0.12	95.19	0.00	0.09	95.05	0.00	0.01	95.19	0.00	-0.15	94.35
		ν_{y_3}	0.00	0.00	-0.01	95.06	0.00	0.10	95.32	0.00	-0.17	94.65	0.00	0.04	94.62
		λ_{x_2}	1.00	1.00	0.15	95.73	1.00	0.34	94.92	1.00	0.04	95.45	1.00	0.17	94.62
		λ_{x_3}	1.00	1.00	0.19	93.59	1.00	0.25	94.52	1.00	-0.30	94.65	1.00	0.33	94.09
		λ_{x_4}	1.00	1.00	0.22	95.99	1.00	0.22	95.05	1.00	0.10	94.79	1.00	0.27	97.18
		λ_{x_5}	1.00	1.00	0.26	95.33	1.01	0.60	91.98	1.00	0.25	94.65	1.00	0.17	95.43

Table A.10: Results table for Study 3 endogenous structural lag population model (D3) and endogenous structural lag analysis model (A3) *(continued)*

			W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^*		
ρ_η	N	θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%
	λ_{y_2}	1.00	1.00	-0.18	94.79	1.00	0.21	94.12	1.00	-0.46	95.99	1.00	0.00	94.62
	λ_{y_3}	1.00	1.00	-0.12	93.86	1.00	0.01	95.86	1.00	-0.12	95.05	1.00	0.05	93.68
0.3														
	ρ_η	0.30	0.30	0.33	94.39	0.31	2.93	96.12	0.33	8.48	95.86	0.47	58.26	99.73
	α	0.00	0.00	-0.34	97.06	0.00	-0.10	95.19	0.00	-0.20	97.06	0.00	0.10	96.37
	γ_1	0.30	0.30	0.73	94.65	0.30	-0.78	96.39	0.30	0.23	95.59	0.30	-0.39	94.76
	γ_2	0.30	0.30	-0.88	95.59	0.30	0.50	94.25	0.29	-2.15	93.05	0.30	0.87	94.76
	γ_3	0.15	0.15	-0.19	94.65	0.15	0.95	95.86	0.15	-1.28	94.79	0.15	-1.05	95.56
	σ_{x_1}	0.50	0.50	0.22	95.59	0.50	0.40	94.65	0.50	0.18	95.86	0.50	0.44	93.68
	σ_{x_2}	0.50	0.50	0.39	94.65	0.50	0.13	95.05	0.50	0.27	96.52	0.50	-0.05	94.89
	σ_{x_3}	0.50	0.50	0.21	95.32	0.50	0.02	93.18	0.50	0.73	95.99	0.50	-0.06	95.83
	σ_{x_4}	0.50	0.50	0.27	93.58	0.50	0.22	95.99	0.50	0.36	96.26	0.50	0.45	97.18
	σ_{x_5}	0.50	0.50	0.41	94.39	0.50	0.65	93.98	0.50	-0.33	92.51	0.50	0.51	95.97
	σ_{x_6}	0.50	0.50	-0.05	94.12	0.50	0.16	94.65	0.50	0.33	93.58	0.50	0.14	94.35
	σ_{y_1}	0.50	0.50	0.32	95.05	0.50	0.22	94.25	0.50	0.48	93.58	0.50	-0.01	95.56
	σ_{y_2}	0.50	0.50	0.19	93.85	0.50	0.21	95.72	0.50	-0.05	94.65	0.50	0.40	94.09
	σ_{y_3}	0.50	0.50	-0.05	95.05	0.50	0.06	94.52	0.50	0.08	94.25	0.50	0.00	96.24
	σ_{ξ_1}	1.00	1.00	0.05	93.32	1.00	0.28	95.32	1.00	0.04	94.25	1.00	0.01	93.41
	σ_{ξ_2}	1.00	1.00	0.32	94.92	1.00	0.12	94.39	1.00	0.43	95.05	1.00	0.13	95.30
	ν_{x_1}	0.00	0.00	-0.21	94.65	0.00	-0.06	93.98	0.00	-0.28	95.72	0.00	-0.01	95.83
	ν_{x_2}	0.00	0.00	-0.05	95.72	0.00	0.04	93.72	0.00	-0.48	94.92	0.00	0.02	95.56
	ν_{x_3}	0.00	0.00	-0.04	95.59	0.00	0.03	94.92	-0.01	-0.53	94.92	0.00	-0.03	96.24
	ν_{x_4}	0.00	0.00	0.28	94.92	0.00	0.35	93.85	0.00	-0.20	94.79	0.00	0.30	95.30
	ν_{x_5}	0.00	0.00	0.24	95.32	0.00	0.11	95.72	0.00	-0.21	95.45	0.00	0.15	95.03
	ν_{x_6}	0.00	0.00	0.24	95.45	0.00	0.14	95.32	0.00	-0.29	95.45	0.00	0.32	94.89
	ν_{y_2}	0.00	0.00	0.28	95.32	0.00	0.08	95.19	0.00	0.23	94.25	0.00	-0.06	95.30
	ν_{y_3}	0.00	0.01	0.54	95.86	0.00	0.00	95.05	0.00	0.24	95.99	0.00	-0.11	94.22
	λ_{x_2}	1.00	1.00	0.43	94.12	1.01	0.54	94.79	1.00	0.37	95.32	1.00	0.45	95.97
	λ_{x_3}	1.00	1.00	0.39	95.59	1.00	0.15	94.39	1.00	0.15	94.65	1.00	0.30	93.41
	λ_{x_4}	1.00	1.00	0.16	93.72	1.00	0.14	95.32	1.00	0.47	94.25	1.00	-0.18	94.89
	λ_{x_5}	1.00	1.00	0.29	94.12	1.00	0.14	94.39	1.00	0.35	95.86	1.00	-0.01	94.89
	λ_{y_2}	1.00	1.00	-0.13	94.12	1.00	0.06	96.79	1.00	-0.08	93.85	1.00	-0.39	94.35
	λ_{y_3}	1.00	1.00	0.05	94.65	1.00	0.04	96.39	1.00	0.00	94.79	1.00	-0.07	93.82
0.6														
	α	0.00	0.00	0.26	93.58	0.00	0.03	94.52	0.00	-0.35	94.92	0.00	0.00	95.56
	ρ_η	0.60	0.60	-0.20	95.99	0.60	0.00	96.26	0.60	0.38	96.93	0.51	-14.72	100.00
	γ_1	0.30	0.30	0.45	93.45	0.30	0.02	93.98	0.30	0.69	95.05	0.31	2.62	94.89
	γ_2	0.30	0.30	-0.37	95.45	0.30	0.03	94.52	0.30	0.63	93.58	0.30	-0.53	95.56
	γ_3	0.15	0.15	-2.87	94.52	0.15	-2.84	94.79	0.15	3.00	94.25	0.15	1.66	94.76
	σ_{x_1}	0.50	0.50	0.03	93.18	0.50	0.09	96.12	0.50	0.55	95.59	0.50	-0.23	94.22
	σ_{x_2}	0.50	0.50	0.48	93.98	0.50	0.42	94.79	0.50	0.03	95.32	0.50	0.23	94.76
	σ_{x_3}	0.50	0.50	0.03	95.86	0.50	0.08	94.65	0.50	0.14	95.72	0.50	0.21	93.41
	σ_{x_4}	0.50	0.50	0.54	95.19	0.50	0.46	95.05	0.50	0.20	94.52	0.50	0.52	94.09
	σ_{x_5}	0.50	0.50	0.12	95.19	0.50	0.09	94.25	0.50	0.14	95.99	0.50	0.52	95.43
	σ_{x_6}	0.50	0.50	-0.29	94.92	0.50	0.27	94.39	0.50	0.52	94.65	0.50	-0.50	93.68
	σ_{y_1}	0.50	0.50	0.07	93.45	0.50	-0.15	96.39	0.50	-0.52	95.72	0.50	-0.17	94.76
	σ_{y_2}	0.50	0.50	0.40	96.39	0.50	0.41	95.45	0.50	-0.28	95.86	0.50	0.66	94.62
	σ_{y_3}	0.50	0.50	0.37	97.06	0.50	0.00	93.45	0.50	0.67	94.25	0.50	0.29	95.03
	σ_{ξ_1}	1.00	1.00	0.14	95.59	1.00	0.03	94.12	1.00	0.06	95.32	1.00	0.13	95.30
	σ_{ξ_2}	1.00	1.00	0.09	94.12	1.00	0.16	95.45	1.00	0.20	95.72	1.00	-0.13	96.37
	ν_{x_1}	0.00	0.00	0.26	95.99	0.00	0.15	94.79	0.00	-0.09	93.58	0.00	0.06	94.22
	ν_{x_2}	0.00	0.00	0.02	95.32	0.00	0.18	94.39	0.00	-0.25	94.12	0.00	0.17	95.56
	ν_{x_3}	0.00	0.00	0.07	95.19	0.00	0.15	93.98	0.00	-0.23	93.18	0.00	0.05	96.51
	ν_{x_4}	0.00	0.00	0.02	94.52	0.00	0.45	94.65	0.00	0.14	93.72	0.00	-0.25	91.80
	ν_{x_5}	0.00	0.00	-0.05	92.65	0.00	0.38	96.79	0.00	0.08	94.39	0.00	-0.19	93.55

Table A.10: Results table for Study 3 endogenous structural lag population model (D3) and endogenous structural lag analysis model (A3) *(continued)*

			W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^{*}		
ρ_η	N	θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%
	ν_{x_6}	0.00	0.00	0.17	94.92	0.00	0.36	95.72	0.00	0.23	93.32	0.00	-0.13	95.03
	ν_{y_2}	0.00	0.00	-0.04	95.45	0.00	-0.15	95.59	0.00	-0.01	95.32	0.00	0.03	95.03
	ν_{y_3}	0.00	0.00	-0.25	94.25	0.00	-0.06	92.65	0.00	0.05	95.45	0.00	0.08	93.28
	λ_{x_2}	1.00	1.00	0.38	95.32	1.00	-0.01	94.79	1.00	0.28	95.05	1.00	0.20	94.09
	λ_{x_3}	1.00	1.00	0.12	96.12	1.00	0.30	95.32	1.00	0.34	95.19	1.00	0.17	95.83
	λ_{x_4}	1.00	1.01	0.55	94.12	1.00	0.21	94.39	1.00	0.35	95.32	1.00	0.35	95.03
	λ_{x_5}	1.00	1.00	0.50	95.32	1.00	0.20	94.25	1.00	0.36	94.92	1.00	0.20	96.10
	λ_{y_2}	1.00	1.00	0.19	93.32	1.00	-0.01	93.85	1.00	-0.09	94.79	0.99	-0.60	93.41
	λ_{y_3}	1.00	1.00	-0.07	93.32	1.00	0.07	95.86	1.00	-0.09	95.32	1.00	-0.45	95.16

[1] W^* is the population and analysis W condition.

[2] N is the simulated sample size.

[3] θ is the simulated population value of each parameter.

[4] $\rho_\eta = ...$ is the simulated spatial autocorrelation of the endogenous lag.

[5] $\bar{\hat{\theta}}$ is the obtained average posterior mean across simulated iterations.

[6] $Bias(\bar{\hat{\theta}})\%$ is the average percent bias across simulation iterations when $\theta \neq 0$, otherwise $Bias(\bar{\hat{\theta}})\%$ provides absolute bias.

[7] Cover% is the percent of simulated trials in which the population value falls within the central 95% density of the posterior estimate.

Table A.11: Results table for Study 3 simultaneous structural lag population model (D4) and simultaneous structural lag analysis model (A4) under $\phi_\zeta = 0.3$

				W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^{*}	
ρ_η	N		θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%
0															
	400														
		ρ_η	0.00	0.14	Inf	16.33	0.15	Inf	16.44	0.16	Inf	18.84	0.30	Inf	22.00
		ϕ_ζ	0.30	0.29	-11.90	98.98	0.29	-10.76	98.63	0.33	-0.67	100.00	0.49	46.57	100.00
		α	0.00	0.00	0.22	96.94	0.00	-0.18	93.15	0.00	-0.09	95.65	0.00	0.34	100.00
		γ_1	0.30	0.29	-2.46	94.90	0.31	1.94	93.15	0.30	1.07	89.86	0.30	-0.05	98.00
		γ_2	0.30	0.30	0.05	98.98	0.31	1.99	94.52	0.30	1.51	88.41	0.31	1.78	100.00
		γ_3	0.15	0.14	-5.17	95.92	0.15	2.15	93.15	0.15	-0.57	97.10	0.15	-0.60	94.00
		σ_{x_1}	0.50	0.51	1.31	98.98	0.50	-0.04	90.41	0.50	-0.33	97.10	0.49	-1.88	96.00
		σ_{x_2}	0.50	0.50	0.67	94.90	0.51	1.01	94.52	0.51	2.43	91.30	0.50	0.89	98.00
		σ_{x_3}	0.50	0.50	-0.16	97.96	0.49	-1.28	90.41	0.49	-1.71	97.10	0.50	0.46	94.00
		σ_{x_4}	0.50	0.49	-1.54	92.86	0.50	0.47	95.89	0.50	-0.09	97.10	0.49	-1.25	92.00
		σ_{x_5}	0.50	0.50	0.21	95.92	0.51	1.01	93.15	0.50	0.74	98.55	0.49	-2.43	96.00
		σ_{x_6}	0.50	0.51	1.42	95.92	0.50	-0.30	95.89	0.50	-0.23	98.55	0.51	1.58	90.00
		σ_{y_1}	0.50	0.50	-0.60	92.86	0.50	-0.95	93.15	0.49	-1.61	88.41	0.49	-2.74	82.00
		σ_{y_2}	0.50	0.49	-1.72	92.86	0.49	-1.15	91.78	0.49	-1.38	97.10	0.50	-0.36	92.00
		σ_{y_3}	0.50	0.50	-0.56	96.94	0.50	-0.68	98.63	0.50	-0.37	95.65	0.49	-2.10	92.00
		σ_{ξ_1}	1.00	1.01	0.65	97.96	1.01	0.89	94.52	1.00	0.38	98.55	1.01	0.88	98.00
		σ_{ξ_2}	1.00	1.01	0.88	97.96	1.00	-0.08	98.63	1.01	0.75	92.75	1.00	-0.06	94.00
		ν_{x_1}	0.00	-0.01	-0.56	90.82	0.01	0.85	93.15	0.00	-0.13	100.00	0.00	-0.05	96.00
		ν_{x_2}	0.00	0.00	-0.23	92.86	0.00	0.34	93.15	-0.01	-0.79	100.00	0.00	0.50	96.00
		ν_{x_3}	0.00	0.00	-0.07	91.84	0.00	0.42	91.78	0.00	0.23	100.00	0.00	0.35	90.00
		ν_{x_4}	0.00	0.00	-0.25	97.96	0.00	0.40	91.78	0.01	0.88	100.00	0.00	0.15	96.00
		ν_{x_5}	0.00	0.01	0.73	100.00	0.01	0.70	94.52	0.01	1.22	94.20	0.01	0.52	98.00
		ν_{x_6}	0.00	0.01	0.91	98.98	0.00	0.07	100.00	0.01	0.84	98.55	0.00	0.21	98.00
		ν_{y_2}	0.00	-0.01	-0.86	96.94	0.01	0.86	95.89	0.00	0.05	100.00	-0.01	-0.57	98.00
		ν_{y_3}	0.00	0.00	0.30	97.96	0.00	0.09	97.26	0.00	0.35	95.65	-0.01	-1.01	90.00
		λ_{x_2}	1.00	0.99	-0.87	94.90	1.00	-0.26	89.04	1.00	0.23	94.20	1.00	0.33	96.00
		λ_{x_3}	1.00	0.99	-0.65	95.92	1.01	1.25	97.26	1.01	0.52	98.55	0.99	-0.67	100.00
		λ_{x_4}	1.00	1.00	0.10	96.94	1.01	0.60	97.26	1.00	-0.16	86.96	1.00	0.34	96.00
		λ_{x_5}	1.00	0.99	-0.94	92.86	1.01	1.06	97.26	1.00	-0.25	89.86	1.01	0.74	100.00
		λ_{y_2}	1.00	1.00	0.33	95.92	0.99	-0.92	95.89	0.98	-2.29	92.75	1.00	0.24	94.00
		λ_{y_3}	1.00	1.01	0.76	95.92	1.00	-0.33	94.52	0.98	-1.56	89.86	0.99	-0.99	94.00
0.3															
		ρ_η	0.30	0.32	-11.17	93.62	0.32	-9.96	94.52	0.30	-17.37	92.75	0.32	-5.28	100.00
		ϕ_ζ	0.30	0.28	-14.28	98.94	0.28	-14.72	98.63	0.32	-2.82	100.00	0.49	44.37	100.00
		α	0.00	-0.06	-5.99	89.36	0.00	-0.02	94.52	0.00	0.10	95.65	0.00	0.04	100.00
		γ_1	0.30	0.31	1.89	94.68	0.31	2.08	97.26	0.30	-0.20	95.65	0.31	4.68	87.76
		γ_2	0.30	0.30	1.24	96.81	0.30	1.57	98.63	0.29	-2.66	95.65	0.31	2.32	95.92
		γ_3	0.15	0.15	2.40	96.81	0.15	0.61	95.89	0.14	-7.05	97.10	0.15	1.52	97.96
		σ_{x_1}	0.50	0.50	0.95	94.68	0.50	-0.13	93.15	0.50	0.35	95.65	0.50	0.15	95.92
		σ_{x_2}	0.50	0.50	0.64	94.68	0.50	0.09	95.89	0.50	0.17	95.65	0.50	-0.93	95.92
		σ_{x_3}	0.50	0.50	0.76	89.36	0.50	0.71	97.26	0.51	1.05	91.30	0.50	-0.41	97.96
		σ_{x_4}	0.50	0.50	0.24	95.74	0.50	0.11	98.63	0.50	0.15	97.10	0.50	-0.71	91.84
		σ_{x_5}	0.50	0.50	0.27	93.62	0.50	-0.39	95.89	0.51	1.60	98.55	0.50	0.57	85.71
		σ_{x_6}	0.50	0.50	0.72	89.36	0.50	0.42	91.78	0.50	0.13	94.20	0.49	-1.48	97.96
		σ_{y_1}	0.50	0.49	-1.70	91.49	0.50	-0.55	95.89	0.50	-0.61	98.55	0.49	-1.77	93.88
		σ_{y_2}	0.50	0.50	-0.65	95.74	0.50	-0.75	93.15	0.50	-0.41	95.65	0.50	0.29	100.00
		σ_{y_3}	0.50	0.50	-0.84	94.68	0.50	-0.60	94.52	0.50	-0.15	91.30	0.49	-1.51	89.80
		σ_{ξ_1}	1.00	1.00	0.05	94.68	1.00	0.21	95.89	1.00	0.16	92.75	1.01	1.08	97.96
		σ_{ξ_2}	1.00	1.01	0.51	98.94	1.00	0.23	89.04	1.00	-0.37	88.41	0.98	-1.51	87.76
		ν_{x_1}	0.00	0.00	-0.28	98.94	0.00	0.16	90.41	0.00	-0.29	95.65	-0.01	-1.04	93.88
		ν_{x_2}	0.00	-0.01	-0.74	96.81	0.00	-0.04	93.15	-0.01	-0.54	100.00	-0.01	-1.08	95.92
		ν_{x_3}	0.00	0.00	-0.29	95.74	0.00	-0.06	94.52	0.00	-0.42	94.20	0.00	0.37	93.88

Table A.11: Results table for Study 3 simultaneous structural lag population model (D4) and simultaneous structural lag analysis model (A4) under $\phi_\zeta = 0.3$ *(continued)*

				W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^{*}		
ρ_η	N		θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%
		ν_{x_4}	0.00	0.00	0.02	97.87	0.00	0.36	98.63	0.00	0.29	94.20	0.01	1.00	95.92
		ν_{x_5}	0.00	0.00	0.30	95.74	0.01	1.10	98.63	0.01	0.54	94.20	0.01	0.74	95.92
		ν_{x_6}	0.00	0.00	0.37	93.62	0.01	1.04	100.00	0.00	0.38	94.20	0.02	1.85	93.88
		ν_{y_2}	0.00	0.00	-0.24	95.74	0.00	-0.38	90.41	0.00	0.39	97.10	0.01	0.81	100.00
		ν_{y_3}	0.00	0.00	-0.14	96.81	0.00	-0.09	98.63	0.00	-0.37	100.00	0.00	-0.35	100.00
		λ_{x_2}	1.00	1.01	0.54	92.55	1.01	0.63	100.00	1.00	0.28	97.10	1.01	0.68	100.00
		λ_{x_3}	1.00	1.00	-0.17	96.81	1.01	1.24	95.89	1.00	-0.17	97.10	1.00	0.48	97.96
		λ_{x_4}	1.00	1.00	-0.27	94.68	1.00	-0.05	90.41	1.00	0.02	85.51	1.01	1.02	100.00
		λ_{x_5}	1.00	0.99	-0.50	98.94	0.99	-0.66	93.15	1.00	0.21	91.30	1.02	2.02	83.67
		λ_{y_2}	1.00	0.97	-2.59	87.23	0.98	-1.80	93.15	0.98	-1.59	94.20	0.96	-3.56	93.88
		λ_{y_3}	1.00	0.99	-1.10	94.68	0.98	-1.92	93.15	0.99	-1.43	95.65	0.98	-2.08	97.96
0.6															
		ρ_η	0.60	0.57	-4.83	93.48	0.56	-6.76	94.12	0.55	-8.12	98.48	0.37	-38.69	100.00
		ϕ_ζ	0.30	0.25	-26.18	98.91	0.24	-27.75	98.53	0.29	-15.60	100.00	0.50	49.18	100.00
		α	0.00	0.00	0.15	92.39	0.00	-0.21	95.59	-0.01	-0.61	95.45	-0.01	-0.65	100.00
		γ_1	0.30	0.30	0.76	98.91	0.31	2.04	97.06	0.31	2.09	96.97	0.31	3.35	93.88
		γ_2	0.30	0.30	0.30	95.65	0.31	2.12	98.53	0.30	1.53	98.48	0.31	2.67	93.88
		γ_3	0.15	0.15	1.07	93.48	0.15	2.89	98.53	0.15	2.64	98.48	0.15	-0.53	100.00
		σ_{x_1}	0.50	0.50	0.49	96.74	0.50	0.00	89.71	0.50	0.68	93.94	0.50	-0.13	95.92
		σ_{x_2}	0.50	0.50	-0.26	96.74	0.50	0.07	92.65	0.50	0.30	100.00	0.49	-1.80	89.80
		σ_{x_3}	0.50	0.50	0.16	92.39	0.50	0.71	91.18	0.50	-0.73	96.97	0.51	1.79	93.88
		σ_{x_4}	0.50	0.50	0.43	93.48	0.49	-1.02	97.06	0.50	0.32	95.45	0.51	1.27	91.84
		σ_{x_5}	0.50	0.50	-0.49	92.39	0.50	-0.80	94.12	0.50	0.25	93.94	0.50	-0.91	91.84
		σ_{x_6}	0.50	0.50	-0.40	91.30	0.50	0.60	100.00	0.50	0.62	92.42	0.51	1.23	95.92
		σ_{y_1}	0.50	0.50	-0.90	97.83	0.50	-0.41	95.59	0.50	-0.58	95.45	0.49	-1.92	91.84
		σ_{y_2}	0.50	0.50	-0.84	95.65	0.49	-1.32	89.71	0.49	-1.43	100.00	0.50	-0.49	97.96
		σ_{y_3}	0.50	0.50	-0.47	95.65	0.50	-0.93	94.12	0.49	-1.20	87.88	0.49	-1.19	93.88
		σ_{ξ_1}	1.00	1.01	0.51	98.91	1.01	0.64	95.59	1.00	0.28	92.42	1.00	0.09	95.92
		σ_{ξ_2}	1.00	1.01	0.57	97.83	1.00	-0.20	98.53	1.00	0.26	93.94	0.98	-1.80	89.80
		ν_{x_1}	0.00	-0.01	-0.54	92.39	-0.01	-1.04	85.29	-0.01	-0.92	98.48	-0.01	-0.96	95.92
		ν_{x_2}	0.00	-0.01	-0.61	88.04	-0.02	-1.56	91.18	-0.01	-1.09	96.97	-0.01	-0.85	89.80
		ν_{x_3}	0.00	-0.01	-0.80	92.39	-0.01	-1.18	92.65	0.00	-0.20	93.94	0.00	-0.30	93.88
		ν_{x_4}	0.00	0.01	0.63	97.83	0.00	0.29	94.12	0.00	0.24	98.48	0.00	0.33	97.96
		ν_{x_5}	0.00	0.01	0.99	94.57	0.01	1.00	88.24	-0.01	-0.64	95.45	0.00	-0.19	93.88
		ν_{x_6}	0.00	0.01	1.05	95.65	0.01	1.08	94.12	0.00	0.32	100.00	-0.01	-0.78	100.00
		ν_{y_2}	0.00	-0.01	-0.65	95.65	-0.01	-0.84	95.59	0.01	1.05	89.39	0.01	1.02	93.88
		ν_{y_3}	0.00	-0.01	-0.59	90.22	0.00	0.04	95.59	0.00	0.41	96.97	0.00	-0.20	97.96
		λ_{x_2}	1.00	1.00	0.46	98.91	1.00	-0.11	95.59	1.01	0.72	90.91	1.02	1.73	93.88
		λ_{x_3}	1.00	1.00	0.30	94.57	1.00	0.22	94.12	1.00	0.44	90.91	1.01	0.56	97.96
		λ_{x_4}	1.00	1.00	0.16	93.48	1.00	0.39	98.53	1.00	0.50	95.45	1.02	1.85	89.80
		λ_{x_5}	1.00	1.00	0.34	95.65	1.00	0.34	95.59	1.00	0.07	98.48	1.02	1.51	95.92
		λ_{y_2}	1.00	0.99	-1.01	95.65	0.98	-2.08	98.53	0.99	-1.46	96.97	0.97	-3.14	91.84
		λ_{y_3}	1.00	0.99	-0.76	89.13	0.98	-1.68	91.18	0.96	-3.74	96.97	0.98	-1.55	91.84
0															
	196														
		ρ_η	0.00	0.09	9.46	0.00	0.11	11.29	0.00	0.13	12.81	0.00	0.31	31.04	0.00
		ϕ_ζ	0.30	0.31	2.20	100.00	0.33	10.10	100.00	0.35	15.51	97.62	0.48	59.74	98.79
		α	0.00	0.00	-0.01	96.20	0.00	0.44	96.63	0.00	-0.28	95.83	0.00	-0.32	99.39
		γ_1	0.30	0.30	0.52	95.65	0.30	1.35	97.75	0.30	-0.26	98.81	0.30	-0.01	96.97
		γ_2	0.30	0.30	-0.11	96.20	0.31	2.81	94.94	0.30	0.67	97.02	0.30	1.43	96.36
		γ_3	0.15	0.15	-1.17	95.11	0.16	3.63	96.07	0.15	-0.27	95.24	0.15	-0.06	97.58
		σ_{x_1}	0.50	0.50	0.20	95.11	0.51	1.03	90.45	0.50	-0.60	98.21	0.51	1.23	94.55
		σ_{x_2}	0.50	0.51	1.35	95.65	0.50	0.56	93.82	0.51	1.46	94.05	0.50	0.89	95.76
		σ_{x_3}	0.50	0.50	0.97	96.20	0.50	-0.05	93.26	0.51	1.40	92.26	0.50	0.39	97.58

Table A.11: Results table for Study 3 simultaneous structural lag population model (D4) and simultaneous structural lag analysis model (A4) under $\phi_\zeta = 0.3$ *(continued)*

				W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^{*}		
ρ_η	N		θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%
		σ_{x_4}	0.50	0.50	-0.22	93.48	0.50	0.35	94.38	0.50	-0.11	92.86	0.50	0.32	96.36
		σ_{x_5}	0.50	0.51	1.04	97.28	0.50	0.54	97.75	0.51	1.13	92.86	0.50	-0.36	95.76
		σ_{x_6}	0.50	0.51	1.13	92.93	0.50	-0.11	93.26	0.50	0.11	94.64	0.51	1.38	90.91
		σ_{y_1}	0.50	0.50	-0.34	95.65	0.49	-1.12	94.94	0.49	-1.83	95.24	0.50	-0.67	93.33
		σ_{y_2}	0.50	0.49	-1.15	92.93	0.50	-0.89	96.07	0.49	-1.04	92.86	0.50	-0.64	94.55
		σ_{y_3}	0.50	0.50	-0.14	95.65	0.49	-1.23	94.94	0.50	-0.81	92.86	0.50	-0.61	96.36
		σ_{ξ_1}	1.00	1.00	0.15	94.57	1.01	0.56	95.51	1.00	0.34	93.45	1.00	0.39	95.76
		σ_{ξ_2}	1.00	1.01	1.44	92.93	1.01	0.55	92.13	1.00	-0.02	95.83	1.00	0.25	95.76
		ν_{x_1}	0.00	0.00	0.07	94.57	0.00	0.41	94.38	0.00	0.18	94.05	0.01	0.84	95.76
		ν_{x_2}	0.00	0.00	0.17	95.65	0.00	-0.03	94.94	0.00	0.22	95.83	0.00	0.03	93.94
		ν_{x_3}	0.00	0.00	-0.11	95.11	0.00	0.16	93.82	0.00	0.25	94.64	0.00	0.11	95.15
		ν_{x_4}	0.00	0.00	0.15	95.65	0.01	0.97	93.26	-0.01	-1.11	97.02	-0.01	-0.91	92.73
		ν_{x_5}	0.00	0.00	-0.10	94.02	0.00	0.39	96.63	0.00	0.13	95.83	0.00	-0.49	94.55
		ν_{x_6}	0.00	0.00	0.25	97.83	0.01	0.71	95.51	0.00	0.07	96.43	-0.01	-0.69	93.94
		ν_{y_2}	0.00	0.00	-0.01	91.85	0.00	-0.12	95.51	0.00	0.19	93.45	0.00	0.29	97.58
		ν_{y_3}	0.00	0.00	-0.38	95.11	0.00	-0.21	92.70	0.00	0.48	92.86	0.01	0.69	95.15
		λ_{x_2}	1.00	1.01	0.57	95.11	1.00	0.28	97.19	1.00	0.27	95.24	1.00	0.16	92.12
		λ_{x_3}	1.00	1.01	0.60	95.65	1.00	0.35	93.26	1.00	-0.04	94.05	1.00	0.26	95.15
		λ_{x_4}	1.00	1.00	-0.35	93.48	1.01	0.95	96.63	1.01	1.03	95.83	1.00	0.36	98.18
		λ_{x_5}	1.00	1.00	-0.47	96.20	1.01	0.69	96.07	1.01	0.98	95.83	1.01	0.95	93.33
		λ_{y_2}	1.00	0.97	-3.26	92.93	0.97	-2.95	93.26	0.99	-0.81	96.43	0.98	-2.41	94.55
		λ_{y_3}	1.00	0.97	-3.29	95.11	0.99	-1.07	96.07	0.98	-1.80	96.43	0.98	-2.21	93.94
0.3															
		ρ_η	0.30	0.26	-12.37	91.76	0.25	-16.83	96.59	0.25	-17.56	97.62	0.32	8.18	98.16
		ϕ_ζ	0.30	0.29	-2.70	99.45	0.32	5.96	98.86	0.34	12.34	98.21	0.48	60.28	98.16
		α	0.00	0.00	-0.20	95.05	-0.01	-0.55	97.73	0.00	-0.11	98.21	0.00	0.02	100.00
		γ_1	0.30	0.31	1.76	93.96	0.30	1.54	96.02	0.30	1.39	97.62	0.30	-0.41	96.32
		γ_2	0.30	0.30	1.55	95.05	0.30	0.91	94.32	0.30	0.74	95.83	0.30	1.32	96.93
		γ_3	0.15	0.15	1.21	97.25	0.15	-0.51	98.30	0.15	-1.67	97.02	0.14	-5.80	95.71
		σ_{x_1}	0.50	0.50	0.84	94.51	0.50	-0.75	94.89	0.50	-0.55	98.21	0.51	1.11	93.25
		σ_{x_2}	0.50	0.50	0.85	94.51	0.50	-0.28	93.18	0.50	0.25	97.62	0.50	0.77	93.87
		σ_{x_3}	0.50	0.50	0.32	93.41	0.50	0.88	93.75	0.50	-0.63	94.64	0.51	1.21	95.71
		σ_{x_4}	0.50	0.50	-0.41	94.51	0.50	0.17	96.02	0.50	0.85	95.83	0.50	-0.17	96.93
		σ_{x_5}	0.50	0.51	1.29	94.51	0.50	-0.01	94.89	0.50	-0.85	96.43	0.50	0.67	91.41
		σ_{x_6}	0.50	0.50	-0.71	96.15	0.50	0.40	93.18	0.50	0.67	95.24	0.50	0.95	94.48
		σ_{y_1}	0.50	0.50	-0.57	95.60	0.50	-0.93	98.30	0.50	-0.48	96.43	0.50	-0.50	94.48
		σ_{y_2}	0.50	0.50	-0.53	97.80	0.50	-0.50	97.16	0.50	-0.42	96.43	0.49	-1.07	95.71
		σ_{y_3}	0.50	0.50	-0.53	94.51	0.49	-1.54	95.45	0.50	-0.95	94.05	0.49	-1.33	94.48
		σ_{ξ_1}	1.00	0.99	-0.70	97.80	1.01	0.65	94.89	1.00	0.30	97.62	0.99	-1.06	96.32
		σ_{ξ_2}	1.00	1.00	-0.12	96.15	1.00	-0.16	93.18	1.01	0.76	94.05	1.01	0.50	98.77
		ν_{x_1}	0.00	0.01	0.72	91.76	-0.01	-0.97	96.59	0.01	1.03	97.62	0.00	0.30	95.09
		ν_{x_2}	0.00	0.00	0.07	92.86	-0.01	-1.14	95.45	0.01	1.15	96.43	0.00	-0.42	98.16
		ν_{x_3}	0.00	0.00	0.39	94.51	-0.01	-1.40	97.16	0.01	0.74	97.02	0.00	0.36	98.16
		ν_{x_4}	0.00	-0.01	-0.66	93.96	0.00	-0.24	94.89	0.00	0.46	97.02	0.00	0.40	96.93
		ν_{x_5}	0.00	0.00	-0.22	93.41	0.00	-0.10	97.73	0.01	0.81	94.64	0.01	0.84	96.32
		ν_{x_6}	0.00	0.00	-0.36	95.60	0.00	-0.42	97.16	0.00	0.25	95.24	0.00	0.36	95.71
		ν_{y_2}	0.00	0.00	-0.22	94.51	0.00	-0.02	96.59	0.00	-0.07	94.05	0.01	0.66	98.16
		ν_{y_3}	0.00	0.00	0.09	91.21	0.00	-0.02	96.02	0.00	-0.10	91.07	0.00	0.34	95.09
		λ_{x_2}	1.00	1.01	1.00	95.60	1.01	0.56	91.48	1.01	0.74	98.81	1.00	0.10	95.09
		λ_{x_3}	1.00	1.01	0.76	96.15	1.01	0.64	95.45	1.00	0.35	97.62	1.01	1.02	96.93
		λ_{x_4}	1.00	1.00	0.06	91.21	1.00	0.23	94.89	1.01	1.02	95.83	1.01	0.52	95.71
		λ_{x_5}	1.00	1.01	1.11	93.96	1.01	0.94	95.45	1.00	0.27	96.43	1.00	0.39	94.48
		λ_{y_2}	1.00	0.98	-2.18	95.60	0.98	-2.26	96.02	0.99	-0.75	97.62	0.95	-5.43	90.80
		λ_{y_3}	1.00	0.97	-3.00	90.11	0.98	-2.42	93.18	0.99	-1.39	95.24	0.97	-2.58	95.71
0.6															

Table A.11: Results table for Study 3 simultaneous structural lag population model (D4) and simultaneous structural lag analysis model (A4) under $\phi_\zeta = 0.3$ *(continued)*

			W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^{*}		
ρ_η	N	θ	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%
	ρ_η	0.60	0.53	-11.75	86.19	0.52	-12.64	88.51	0.49	-17.60	87.50	0.35	-41.23	97.53
	ϕ_ζ	0.30	0.27	-10.14	98.90	0.29	-3.77	98.85	0.32	7.49	98.81	0.48	60.27	98.15
	α	0.00	0.00	0.33	93.92	0.00	0.37	96.55	0.00	-0.13	97.02	0.01	0.50	99.38
	γ_1	0.30	0.31	1.73	97.24	0.31	3.16	90.80	0.31	2.43	93.45	0.30	1.22	94.44
	γ_2	0.30	0.31	1.70	94.48	0.31	2.63	94.83	0.31	2.47	96.43	0.29	-2.26	93.21
	γ_3	0.15	0.15	2.71	98.34	0.15	2.30	97.13	0.16	4.06	95.24	0.15	0.33	94.44
	σ_{x_1}	0.50	0.51	1.30	93.92	0.50	-0.15	97.13	0.50	0.27	93.45	0.51	1.37	96.30
	σ_{x_2}	0.50	0.50	0.68	93.92	0.50	-0.40	94.83	0.50	0.18	96.43	0.50	-0.07	96.30
	σ_{x_3}	0.50	0.50	-0.92	95.58	0.51	1.14	95.40	0.50	0.77	94.64	0.50	0.54	95.68
	σ_{x_4}	0.50	0.50	0.48	95.58	0.50	0.06	94.83	0.50	0.57	98.21	0.50	0.41	95.68
	σ_{x_5}	0.50	0.50	-0.19	97.24	0.50	-0.05	97.13	0.50	-0.83	94.64	0.50	0.68	93.21
	σ_{x_6}	0.50	0.50	0.62	95.03	0.50	0.09	91.95	0.51	1.58	94.05	0.50	-0.05	94.44
	σ_{y_1}	0.50	0.50	-0.95	97.79	0.50	-0.26	93.10	0.50	-0.80	93.45	0.49	-1.35	93.83
	σ_{y_2}	0.50	0.50	-0.24	96.13	0.50	-0.69	97.13	0.50	-0.97	94.64	0.50	-0.89	95.06
	σ_{y_3}	0.50	0.50	0.03	94.48	0.50	-0.27	95.98	0.50	-0.70	94.64	0.49	-1.22	95.06
	σ_{ξ_1}	1.00	1.00	0.46	96.13	1.00	0.01	95.98	1.01	0.65	97.02	1.00	0.18	95.68
	σ_{ξ_2}	1.00	1.01	0.90	95.58	1.01	1.00	95.40	1.01	1.32	96.43	1.00	0.45	97.53
	ν_{x_1}	0.00	0.00	-0.14	96.13	0.00	-0.40	90.80	0.00	-0.31	96.43	-0.01	-0.58	92.59
	ν_{x_2}	0.00	-0.01	-0.67	96.13	-0.01	-0.63	93.10	0.00	0.08	97.02	0.00	-0.25	92.59
	ν_{x_3}	0.00	0.00	-0.25	95.03	-0.01	-0.71	93.68	-0.01	-0.50	97.62	0.00	-0.28	95.06
	ν_{x_4}	0.00	0.02	1.50	93.37	0.01	0.78	94.83	0.00	-0.13	98.21	0.01	1.16	94.44
	ν_{x_5}	0.00	0.01	0.98	92.27	0.01	0.63	94.25	0.00	0.00	95.24	0.01	1.20	97.53
	ν_{x_6}	0.00	0.01	0.98	93.92	0.01	0.53	95.40	0.00	-0.08	98.21	0.01	0.88	93.21
	ν_{y_2}	0.00	-0.01	-0.67	96.13	0.00	-0.27	93.68	0.00	0.03	95.24	0.00	0.26	96.91
	ν_{y_3}	0.00	-0.01	-0.56	95.03	0.00	-0.08	93.10	0.01	0.74	94.64	0.01	0.58	92.59
	λ_{x_2}	1.00	1.00	-0.11	87.85	1.00	0.50	96.55	1.00	0.18	95.24	1.00	0.40	96.91
	λ_{x_3}	1.00	1.00	-0.29	95.03	1.00	0.47	94.83	1.00	-0.43	92.86	1.01	0.95	95.06
	λ_{x_4}	1.00	1.00	-0.17	97.79	1.00	0.25	96.55	1.00	0.37	98.81	1.00	-0.13	95.68
	λ_{x_5}	1.00	1.00	-0.31	93.92	1.01	0.63	94.25	0.99	-0.69	95.24	1.00	-0.10	95.68
	λ_{y_2}	1.00	0.97	-2.72	95.58	0.98	-2.26	90.23	0.98	-2.00	92.86	0.99	-1.29	95.06
	λ_{y_3}	1.00	0.97	-2.98	95.58	0.97	-2.55	95.40	0.98	-2.38	96.43	0.99	-0.89	93.83
0														
	49													
	ρ_η	0.00	0.18	17.61	0.00	0.20	19.73	0.00	0.22	22.48	0.00	0.32	32.05	0.00
	ϕ_ζ	0.30	0.36	18.64	100.00	0.39	29.17	100.00	0.40	34.61	98.82	0.48	58.46	99.40
	α	0.00	0.00	-0.05	100.00	0.01	1.15	98.88	0.00	-0.22	100.00	0.00	-0.21	100.00
	γ_1	0.30	0.32	5.22	96.77	0.32	7.30	94.41	0.33	10.46	97.65	0.32	8.23	95.81
	γ_2	0.30	0.31	4.73	97.31	0.32	7.33	97.21	0.32	7.04	97.65	0.34	12.12	95.21
	γ_3	0.15	0.16	4.95	96.77	0.17	13.45	94.97	0.17	12.39	94.12	0.17	10.42	95.81
	σ_{x_1}	0.50	0.50	0.65	92.47	0.51	1.93	87.71	0.52	3.09	95.29	0.51	1.84	97.01
	σ_{x_2}	0.50	0.50	-0.24	95.16	0.50	0.46	94.41	0.49	-1.61	94.12	0.51	2.26	90.42
	σ_{x_3}	0.50	0.51	1.25	95.70	0.50	0.61	95.53	0.52	3.04	95.29	0.51	2.04	90.42
	σ_{x_4}	0.50	0.52	3.36	93.01	0.52	3.24	94.41	0.50	0.51	95.88	0.51	2.23	95.21
	σ_{x_5}	0.50	0.50	0.75	95.16	0.51	2.79	95.53	0.51	1.19	97.06	0.51	1.41	94.01
	σ_{x_6}	0.50	0.50	0.28	94.09	0.50	0.93	93.30	0.51	2.47	94.12	0.51	1.11	91.62
	σ_{y_1}	0.50	0.50	0.04	97.85	0.50	0.04	91.06	0.50	0.67	97.65	0.48	-4.07	94.61
	σ_{y_2}	0.50	0.51	1.18	95.16	0.51	2.80	89.94	0.50	-0.53	94.71	0.50	0.75	97.01
	σ_{y_3}	0.50	0.50	0.78	94.62	0.51	1.06	97.77	0.50	0.37	93.53	0.50	-0.43	95.21
	σ_{ξ_1}	1.00	1.00	0.12	94.62	1.02	1.76	97.77	1.01	1.11	95.88	1.03	3.11	95.81
	σ_{ξ_2}	1.00	1.02	2.24	94.09	1.02	1.90	96.65	0.99	-0.84	93.53	1.03	2.87	93.41
	ν_{x_1}	0.00	0.00	-0.39	97.85	0.00	0.19	91.62	0.00	-0.45	94.71	0.00	0.44	94.01
	ν_{x_2}	0.00	0.00	-0.49	95.16	0.00	0.45	92.18	0.00	-0.04	96.47	0.01	0.67	95.81
	ν_{x_3}	0.00	-0.01	-0.99	96.77	0.00	-0.41	94.41	-0.02	-1.67	96.47	0.01	0.80	95.21
	ν_{x_4}	0.00	-0.01	-0.68	95.70	0.00	0.32	95.53	0.00	-0.34	95.29	0.00	0.19	95.21

Table A.11: Results table for Study 3 simultaneous structural lag population model (D4) and simultaneous structural lag analysis model (A4) under $\phi_\zeta = 0.3$ *(continued)*

				W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^{*}		
ρ_η	N		θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%
		ν_{x_5}	0.00	-0.01	-0.71	97.31	0.00	-0.03	97.21	0.00	0.27	95.88	0.00	-0.48	94.61
		ν_{x_6}	0.00	0.00	-0.36	95.70	-0.01	-0.81	96.65	0.00	0.05	94.71	0.00	-0.23	95.81
		ν_{y_2}	0.00	0.00	-0.44	96.24	0.01	0.59	95.53	0.00	-0.41	96.47	0.01	0.93	92.22
		ν_{y_3}	0.00	0.00	0.08	98.39	0.00	-0.22	93.30	0.00	-0.35	94.71	0.00	0.31	95.21
		λ_{x_2}	1.00	1.02	2.30	95.70	1.02	1.94	94.97	1.04	3.85	95.88	1.02	2.00	95.81
		λ_{x_3}	1.00	1.03	2.53	95.16	1.01	1.39	94.97	1.02	2.20	95.29	1.01	0.92	95.81
		λ_{x_4}	1.00	1.02	2.07	95.16	1.02	1.67	94.97	1.03	2.57	98.24	1.02	1.95	94.61
		λ_{x_5}	1.00	1.02	2.02	96.24	1.01	1.39	94.41	1.02	2.05	95.29	1.00	0.27	94.01
		λ_{y_2}	1.00	0.91	-9.11	92.47	0.90	-10.05	91.06	0.89	-11.36	92.94	0.91	-9.42	89.22
		λ_{y_3}	1.00	0.88	-11.60	92.47	0.92	-8.03	93.85	0.93	-7.05	94.71	0.90	-10.26	92.22
0.3															
		ρ_η	0.30	0.24	-19.56	98.92	0.26	-14.20	98.88	0.27	-9.40	100.00	0.33	9.41	100.00
		ϕ_ζ	0.30	0.37	23.20	98.39	0.38	27.26	99.44	0.39	31.04	97.65	0.48	60.18	99.40
		α	0.00	0.00	-0.07	98.92	0.00	-0.08	97.21	0.00	-0.39	98.82	0.00	0.05	99.40
		γ_1	0.30	0.32	7.95	96.77	0.33	9.88	97.21	0.32	5.90	97.06	0.32	6.40	97.60
		γ_2	0.30	0.32	5.42	99.46	0.32	7.73	97.21	0.32	7.79	97.65	0.33	8.53	97.01
		γ_3	0.15	0.17	10.36	95.16	0.17	13.97	97.21	0.18	18.02	95.88	0.16	9.37	95.81
		σ_{x_1}	0.50	0.52	3.51	96.77	0.52	3.67	94.97	0.51	2.63	92.94	0.51	1.83	97.01
		σ_{x_2}	0.50	0.49	-1.31	91.94	0.50	0.18	95.53	0.52	3.39	96.47	0.51	2.26	98.80
		σ_{x_3}	0.50	0.51	2.54	92.47	0.50	0.58	96.09	0.49	-1.95	97.06	0.50	-0.72	96.41
		σ_{x_4}	0.50	0.51	2.69	93.55	0.51	1.86	97.21	0.51	1.80	94.12	0.50	0.70	95.81
		σ_{x_5}	0.50	0.49	-1.21	97.31	0.51	3.00	92.18	0.51	2.21	98.82	0.51	1.14	92.81
		σ_{x_6}	0.50	0.50	0.61	97.85	0.49	-2.09	91.06	0.50	0.88	95.88	0.50	-0.68	96.41
		σ_{y_1}	0.50	0.50	0.00	93.55	0.49	-1.17	95.53	0.50	-0.62	95.29	0.49	-2.22	95.81
		σ_{y_2}	0.50	0.50	0.30	93.55	0.50	0.97	94.41	0.51	1.04	94.71	0.50	0.76	95.21
		σ_{y_3}	0.50	0.49	-1.06	94.09	0.50	0.84	94.97	0.50	0.21	97.65	0.50	0.29	94.01
		σ_{ξ_1}	1.00	0.99	-1.48	93.01	1.03	2.77	92.74	1.01	1.06	93.53	0.99	-0.92	93.41
		σ_{ξ_2}	1.00	1.00	-0.04	95.16	1.02	1.60	93.85	1.00	-0.07	96.47	1.00	0.02	95.21
		ν_{x_1}	0.00	0.00	-0.39	96.24	0.02	1.72	97.77	0.01	0.61	95.88	0.02	1.90	98.20
		ν_{x_2}	0.00	0.00	-0.34	92.47	0.01	0.91	96.65	0.01	0.56	96.47	0.02	1.99	96.41
		ν_{x_3}	0.00	-0.01	-0.73	97.31	0.02	1.66	97.77	0.00	0.02	94.12	0.01	1.32	96.41
		ν_{x_4}	0.00	0.00	0.44	97.31	0.01	1.01	93.85	0.00	0.44	93.53	-0.02	-1.51	98.20
		ν_{x_5}	0.00	0.00	0.15	98.92	0.00	0.44	96.65	0.00	0.11	91.76	-0.02	-1.59	98.80
		ν_{x_6}	0.00	0.00	0.03	98.39	0.02	2.05	93.85	0.00	0.14	95.29	-0.02	-1.75	97.60
		ν_{y_2}	0.00	-0.01	-1.14	96.77	0.01	1.33	93.85	0.01	0.66	97.65	0.00	-0.40	94.61
		ν_{y_3}	0.00	0.00	-0.23	93.01	0.01	0.75	92.74	0.00	-0.08	97.06	0.00	-0.49	98.20
		λ_{x_2}	1.00	1.03	3.30	97.31	1.02	2.30	97.21	1.03	2.51	95.88	1.02	1.66	98.20
		λ_{x_3}	1.00	1.03	2.91	96.77	1.02	1.77	97.77	1.02	2.48	95.88	1.03	2.79	98.80
		λ_{x_4}	1.00	1.03	3.28	94.09	1.02	2.16	96.09	1.02	2.06	91.76	1.02	1.89	95.81
		λ_{x_5}	1.00	1.02	2.25	96.24	1.02	2.19	96.09	1.03	3.09	96.47	1.02	2.15	95.81
		λ_{y_2}	1.00	0.90	-9.87	94.62	0.88	-11.82	89.94	0.91	-8.87	91.18	0.94	-6.45	91.62
		λ_{y_3}	1.00	0.88	-11.76	95.70	0.89	-11.37	90.50	0.93	-6.66	90.59	0.95	-5.26	89.22
0.6															
		ρ_η	0.60	0.42	-29.35	89.78	0.40	-33.59	88.83	0.37	-37.75	92.94	0.35	-42.14	98.80
		ϕ_ζ	0.30	0.37	22.45	98.92	0.40	32.95	99.44	0.42	40.48	99.41	0.48	60.09	100.00
		α	0.00	0.01	0.78	96.77	0.00	-0.39	96.09	-0.01	-0.75	97.06	0.01	0.65	100.00
		γ_1	0.30	0.32	7.13	97.31	0.32	5.46	98.32	0.33	8.83	99.41	0.32	7.91	96.41
		γ_2	0.30	0.33	9.62	96.77	0.32	6.95	98.88	0.32	5.69	98.82	0.34	12.17	93.41
		γ_3	0.15	0.17	11.36	97.31	0.16	7.12	96.09	0.16	5.67	95.88	0.17	14.61	96.41
		σ_{x_1}	0.50	0.50	0.68	95.70	0.49	-1.60	96.09	0.52	3.86	95.29	0.53	5.11	93.41
		σ_{x_2}	0.50	0.51	1.14	96.24	0.52	4.02	93.85	0.50	0.28	97.06	0.50	-0.47	91.62
		σ_{x_3}	0.50	0.51	1.10	94.62	0.50	-0.10	94.97	0.51	2.68	96.47	0.50	-0.61	97.01
		σ_{x_4}	0.50	0.51	2.20	96.24	0.51	2.85	94.97	0.50	0.03	91.18	0.50	-0.48	94.61
		σ_{x_5}	0.50	0.50	-0.01	96.24	0.50	-0.22	94.41	0.49	-1.39	96.47	0.50	0.77	91.62
		σ_{x_6}	0.50	0.50	0.12	93.01	0.51	1.14	96.09	0.51	1.38	91.76	0.51	2.91	94.61

Table A.11: Results table for Study 3 simultaneous structural lag population model (D4) and simultaneous structural lag analysis model (A4) under $\phi_\zeta = 0.3$ *(continued)*

				W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^*		
ρ_η	N		θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%
		σ_{y_1}	0.50	0.51	2.56	93.55	0.50	0.06	93.30	0.50	-0.31	92.35	0.50	-0.74	96.41
		σ_{y_2}	0.50	0.50	-0.28	94.62	0.51	1.24	93.30	0.51	1.23	93.53	0.50	-0.05	92.81
		σ_{y_3}	0.50	0.50	-0.20	94.09	0.49	-1.17	95.53	0.52	3.86	95.88	0.51	1.47	94.01
		σ_{ξ_1}	1.00	1.02	2.44	96.77	1.02	2.16	94.41	0.99	-1.04	94.12	1.00	-0.36	95.81
		σ_{ξ_2}	1.00	1.01	0.87	90.32	1.01	1.33	95.53	1.02	2.32	96.47	1.01	1.39	94.01
		ν_{x_1}	0.00	0.04	4.11	95.70	0.02	2.18	92.18	-0.01	-1.08	94.71	0.01	1.22	95.21
		ν_{x_2}	0.00	0.02	2.22	96.77	0.02	2.42	91.06	-0.01	-0.55	97.06	0.01	1.29	97.01
		ν_{x_3}	0.00	0.04	3.83	95.70	0.00	0.21	89.94	-0.01	-0.73	95.88	0.01	0.70	97.01
		ν_{x_4}	0.00	-0.01	-0.77	95.16	-0.01	-1.27	96.09	-0.01	-1.12	97.65	-0.02	-1.84	95.21
		ν_{x_5}	0.00	-0.01	-1.15	95.70	-0.02	-1.78	94.97	-0.01	-0.74	97.65	-0.02	-1.85	94.01
		ν_{x_6}	0.00	-0.01	-1.13	93.01	-0.01	-1.34	94.97	-0.01	-0.57	97.06	-0.03	-2.51	94.61
		ν_{y_2}	0.00	0.01	1.05	98.39	-0.01	-1.00	94.41	-0.01	-0.64	92.94	0.00	-0.44	95.21
		ν_{y_3}	0.00	0.01	1.19	97.31	0.00	0.43	97.21	0.00	0.14	96.47	0.00	-0.22	94.61
		λ_{x_2}	1.00	1.02	1.65	95.16	1.00	0.37	96.09	1.04	3.75	95.29	1.04	3.98	97.01
		λ_{x_3}	1.00	1.02	1.65	95.16	1.02	1.51	94.97	1.03	2.83	95.29	1.05	4.89	95.81
		λ_{x_4}	1.00	1.01	1.37	96.77	1.02	1.66	93.30	1.01	1.12	95.29	1.01	0.99	97.01
		λ_{x_5}	1.00	1.02	2.03	95.16	1.00	0.47	95.53	1.01	1.02	95.29	1.01	0.81	96.41
		λ_{y_2}	1.00	0.93	-7.33	94.62	0.91	-9.33	92.18	0.92	-8.49	92.94	0.88	-11.50	86.23
		λ_{y_3}	1.00	0.92	-8.13	95.16	0.92	-7.98	94.41	0.91	-8.92	93.53	0.86	-13.83	86.23

[1] W^* is the population and analysis W condition.

[2] N is the simulated sample size.

[3] θ is the simulated population value of each parameter.

[4] $\rho_\eta = ...$ is the simulated spatial autocorrelation of the endogenous lag.

[5] $\bar{\hat{\theta}}$ is the obtained average posterior mean across simulated iterations.

[6] $Bias(\bar{\hat{\theta}})\%$ is the average percent bias across simulation iterations when $\theta \neq 0$, otherwise $Bias(\bar{\hat{\theta}})\%$ provides absolute bias.

[7] Cover% is the percent of simulated trials in which the population value falls within the central 95% density of the posterior estimate.

[8] $\phi_\zeta = 0.3$ at the population level.

Table A.12: Results table for Study 3 simultaneous structural lag population model (D4) and simultaneous structural lag analysis model (A4) under $\phi_\zeta = 0.6$

			W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^{*}		
ρ_η	N	θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%
0														
	49													
	ρ_η	0.00	0.16	15.80	0.00	0.19	19.06	0.00	0.21	21.20	0.00	0.32	32.41	0.00
	ϕ_ζ	0.60	0.36	-39.79	98.39	0.38	-36.49	97.77	0.42	-30.33	100.00	0.48	-20.58	98.20
	α	0.00	0.00	-0.36	99.46	0.00	0.34	99.44	0.00	-0.18	99.41	-0.01	-1.30	99.40
	γ_1	0.30	0.32	6.82	97.31	0.33	9.84	98.32	0.33	9.21	98.24	0.33	8.56	98.20
	γ_2	0.30	0.31	2.68	98.39	0.32	6.38	97.21	0.32	6.73	98.24	0.33	10.12	95.81
	γ_3	0.15	0.16	3.57	96.24	0.18	18.96	93.85	0.18	16.68	94.71	0.17	10.92	96.41
	σ_{x_1}	0.50	0.52	3.84	95.70	0.52	4.21	97.77	0.51	2.62	93.53	0.50	-0.20	96.41
	σ_{x_2}	0.50	0.51	2.01	93.55	0.51	1.98	93.85	0.50	-0.54	97.06	0.51	1.69	93.41
	σ_{x_3}	0.50	0.50	0.06	95.16	0.51	1.04	95.53	0.51	1.44	95.29	0.50	0.58	96.41
	σ_{x_4}	0.50	0.49	-1.65	96.77	0.50	0.73	95.53	0.52	3.59	91.76	0.51	1.46	94.01
	σ_{x_5}	0.50	0.50	0.91	94.62	0.51	1.93	97.77	0.51	2.93	91.76	0.51	2.21	92.22
	σ_{x_6}	0.50	0.50	0.97	93.01	0.50	0.98	96.09	0.51	1.07	95.29	0.50	-0.05	97.01
	σ_{y_1}	0.50	0.48	-3.36	92.47	0.51	1.52	96.65	0.50	-0.91	94.71	0.49	-1.46	94.61
	σ_{y_2}	0.50	0.51	1.46	96.24	0.50	-0.32	93.85	0.50	0.21	92.35	0.51	1.40	96.41
	σ_{y_3}	0.50	0.50	0.40	97.31	0.50	0.84	94.41	0.50	0.14	95.29	0.50	0.73	92.81
	σ_{ξ_1}	1.00	1.00	0.14	94.09	1.01	1.35	94.41	1.01	1.31	95.29	1.00	0.29	95.21
	σ_{ξ_2}	1.00	1.03	2.52	93.55	1.02	1.97	97.21	1.00	0.18	95.29	1.00	-0.25	96.41
	v_{x_1}	0.00	-0.03	-2.52	95.70	0.01	0.79	92.74	-0.02	-1.56	97.65	-0.01	-0.89	96.41
	v_{x_2}	0.00	-0.02	-1.68	93.55	0.00	0.14	94.41	-0.02	-1.59	97.06	0.00	0.06	95.81
	v_{x_3}	0.00	-0.02	-2.01	96.77	0.00	0.19	97.21	-0.03	-2.53	97.06	0.00	-0.35	95.81
	v_{x_4}	0.00	0.00	-0.12	97.31	0.02	1.64	97.21	-0.01	-1.49	92.35	-0.02	-2.15	95.81
	v_{x_5}	0.00	0.01	0.51	95.70	0.02	2.11	99.44	-0.01	-1.40	93.53	-0.02	-1.79	97.01
	v_{x_6}	0.00	0.00	-0.03	93.55	0.02	1.76	92.74	-0.01	-1.28	91.76	-0.01	-1.42	96.41
	v_{y_2}	0.00	0.00	-0.21	96.77	0.00	0.19	96.09	0.01	0.72	94.12	0.01	1.07	94.61
	v_{y_3}	0.00	0.00	-0.11	94.09	-0.01	-1.25	93.30	-0.01	-0.98	93.53	0.00	-0.44	94.01
	λ_{x_2}	1.00	1.02	2.05	97.31	1.01	1.49	96.65	1.02	2.02	95.88	1.03	2.63	96.41
	λ_{x_3}	1.00	1.03	2.86	95.70	1.03	3.29	96.09	1.03	2.82	95.88	1.02	2.20	95.81
	λ_{x_4}	1.00	1.01	0.57	96.77	1.03	2.52	94.41	1.03	2.99	95.88	1.02	2.26	95.81
	λ_{x_5}	1.00	1.01	0.59	94.62	1.02	2.03	94.41	1.05	4.67	94.71	1.05	4.53	95.21
	λ_{y_2}	1.00	0.91	-8.50	91.94	0.89	-11.26	92.18	0.92	-8.40	93.53	0.90	-10.41	88.62
	λ_{y_3}	1.00	0.93	-7.47	90.86	0.91	-9.17	95.53	0.89	-10.94	91.76	0.93	-7.21	91.62
0.3														
	ρ_η	0.30	0.25	-17.87	97.85	0.25	-16.28	100.00	0.28	-7.48	98.24	0.32	8.07	99.40
	ϕ_ζ	0.60	0.36	-40.04	97.85	0.38	-37.38	98.32	0.42	-30.18	97.06	0.48	-20.25	97.60
	α	0.00	-0.02	-1.73	99.46	0.01	0.67	98.88	0.01	0.60	99.41	0.02	1.53	99.40
	γ_1	0.30	0.31	1.88	96.24	0.32	5.78	94.41	0.32	7.15	95.88	0.33	8.76	94.61
	γ_2	0.30	0.31	3.32	100.00	0.32	7.00	98.32	0.32	6.13	98.82	0.33	10.93	97.60
	γ_3	0.15	0.17	14.60	97.85	0.16	4.72	97.77	0.16	6.46	97.06	0.16	9.86	95.21
	σ_{x_1}	0.50	0.51	1.93	94.09	0.51	2.96	93.85	0.50	-0.48	93.53	0.50	0.15	93.41
	σ_{x_2}	0.50	0.50	0.92	94.09	0.50	-0.41	95.53	0.52	4.08	93.53	0.51	1.73	94.61
	σ_{x_3}	0.50	0.49	-1.63	94.09	0.50	0.20	94.41	0.50	0.29	93.53	0.52	3.48	95.21
	σ_{x_4}	0.50	0.51	2.28	93.55	0.51	2.75	93.30	0.51	1.42	97.65	0.52	4.10	97.01
	σ_{x_5}	0.50	0.51	1.73	96.24	0.49	-1.29	96.09	0.49	-1.45	94.12	0.50	-0.68	96.41
	σ_{x_6}	0.50	0.50	0.39	96.77	0.51	1.12	94.97	0.52	3.02	94.12	0.51	2.86	92.22
	σ_{y_1}	0.50	0.50	-0.08	96.24	0.49	-1.40	94.97	0.49	-1.27	94.71	0.50	0.00	94.61
	σ_{y_2}	0.50	0.51	1.24	94.62	0.50	0.09	97.21	0.50	-0.59	94.12	0.50	-0.29	96.41
	σ_{y_3}	0.50	0.49	-1.03	95.16	0.51	1.48	95.53	0.52	3.39	95.88	0.50	0.94	96.41
	σ_{ξ_1}	1.00	1.00	0.33	95.70	1.02	2.30	96.09	1.02	2.11	91.76	1.01	1.21	97.01
	σ_{ξ_2}	1.00	1.02	1.60	95.16	1.00	0.29	94.41	1.01	1.46	96.47	0.98	-1.96	96.41
	v_{x_1}	0.00	-0.01	-0.88	97.85	0.02	1.95	95.53	0.01	0.64	93.53	0.02	1.77	95.21
	v_{x_2}	0.00	-0.02	-1.66	98.39	0.01	1.29	96.65	0.01	1.35	94.12	0.02	2.15	96.41
	v_{x_3}	0.00	-0.01	-1.16	96.24	0.01	1.17	95.53	0.01	0.71	94.71	0.02	1.75	98.80

Table A.12: Results table for Study 3 simultaneous structural lag population model (D4) and simultaneous structural lag analysis model (A4) under $\phi_\zeta = 0.6$ *(continued)*

				W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^{*}		
ρ_η	N		θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%
		ν_{x_4}	0.00	-0.02	-1.91	95.70	0.00	-0.29	96.09	0.01	0.60	93.53	0.00	0.41	94.61
		ν_{x_5}	0.00	-0.02	-1.73	96.77	0.00	-0.02	94.97	0.01	0.71	97.06	0.01	1.15	96.41
		ν_{x_6}	0.00	-0.01	-1.08	98.39	0.01	1.07	93.85	0.01	0.88	95.88	0.01	1.04	97.60
		ν_{y_2}	0.00	0.01	1.40	95.70	-0.01	-0.58	96.09	-0.01	-0.80	95.29	0.00	0.02	95.81
		ν_{y_3}	0.00	0.01	0.91	91.94	-0.01	-0.69	96.65	-0.03	-2.57	92.35	0.01	1.14	96.41
		λ_{x_2}	1.00	1.03	2.92	93.55	1.01	1.29	94.97	1.02	2.45	92.94	1.03	2.57	95.81
		λ_{x_3}	1.00	1.04	4.05	93.01	1.02	1.59	97.21	1.02	1.61	94.12	1.02	2.45	95.81
		λ_{x_4}	1.00	1.02	2.44	97.31	1.02	1.95	93.30	1.01	1.40	97.65	1.04	4.22	95.81
		λ_{x_5}	1.00	1.03	3.14	97.31	1.02	1.56	94.97	1.02	1.77	97.06	1.02	2.38	96.41
		λ_{y_2}	1.00	0.93	-7.16	94.09	0.93	-7.20	96.09	0.93	-7.29	92.94	0.92	-7.60	93.41
		λ_{y_3}	1.00	0.90	-10.07	90.86	0.88	-11.55	91.06	0.93	-7.45	92.94	0.90	-10.34	91.62
0.6															
		ρ_η	0.60	0.42	-29.44	81.72	0.40	-33.36	88.27	0.39	-34.83	92.94	0.35	-42.33	98.80
		ϕ_ζ	0.60	0.36	-39.43	97.31	0.39	-34.27	98.88	0.43	-28.48	97.06	0.48	-20.11	97.60
		α	0.00	0.01	0.92	96.77	0.00	0.13	97.21	-0.01	-1.46	98.24	0.01	1.36	100.00
		γ_1	0.30	0.33	9.33	95.70	0.32	7.79	96.65	0.32	5.19	95.29	0.32	6.66	97.60
		γ_2	0.30	0.32	7.20	93.55	0.31	4.35	100.00	0.31	4.75	95.29	0.33	8.64	95.21
		γ_3	0.15	0.17	11.01	97.85	0.16	8.24	97.77	0.16	6.92	95.29	0.16	6.07	97.01
		σ_{x_1}	0.50	0.50	0.87	94.09	0.50	-0.58	97.21	0.52	4.00	94.71	0.51	1.65	95.81
		σ_{x_2}	0.50	0.51	2.71	96.24	0.50	-0.12	93.85	0.51	1.86	97.06	0.50	-0.56	91.02
		σ_{x_3}	0.50	0.49	-1.19	93.01	0.49	-1.39	94.41	0.50	-0.70	93.53	0.53	5.15	95.21
		σ_{x_4}	0.50	0.51	2.87	96.24	0.50	0.70	94.97	0.52	3.47	97.06	0.51	1.05	92.22
		σ_{x_5}	0.50	0.50	0.14	93.55	0.51	1.06	93.85	0.50	0.78	92.94	0.50	0.93	98.20
		σ_{x_6}	0.50	0.51	2.47	96.24	0.51	2.77	92.74	0.51	1.63	96.47	0.50	0.60	94.61
		σ_{y_1}	0.50	0.50	0.42	97.31	0.50	-0.76	97.21	0.49	-1.54	94.12	0.50	-0.21	98.80
		σ_{y_2}	0.50	0.50	0.38	90.32	0.50	-0.58	93.85	0.51	1.56	93.53	0.51	2.48	94.61
		σ_{y_3}	0.50	0.50	-0.44	93.55	0.50	0.33	97.21	0.50	0.58	98.24	0.50	-0.92	95.21
		σ_{ξ_1}	1.00	1.01	1.29	94.09	1.02	1.69	93.30	1.01	1.04	95.88	1.00	0.39	95.21
		σ_{ξ_2}	1.00	1.00	-0.01	95.16	1.02	1.78	91.62	1.02	2.27	93.53	1.01	0.71	95.81
		ν_{x_1}	0.00	0.02	1.88	97.31	-0.03	-2.66	94.41	-0.03	-3.43	95.29	0.02	1.52	97.60
		ν_{x_2}	0.00	0.02	1.80	97.85	-0.03	-3.04	95.53	-0.03	-2.63	94.12	0.01	0.55	92.81
		ν_{x_3}	0.00	0.02	1.85	97.31	-0.03	-2.50	95.53	-0.03	-2.84	93.53	0.01	1.06	98.20
		ν_{x_4}	0.00	-0.01	-1.40	96.24	0.01	0.55	94.41	-0.01	-0.64	97.06	0.01	1.49	97.01
		ν_{x_5}	0.00	-0.01	-0.86	96.24	0.00	0.35	93.85	0.00	-0.49	97.06	0.02	2.32	94.01
		ν_{x_6}	0.00	-0.01	-0.87	94.09	0.01	0.63	94.41	-0.01	-1.00	95.88	0.01	1.01	95.21
		ν_{y_2}	0.00	-0.02	-1.52	94.09	0.00	0.42	93.85	0.00	-0.26	95.29	-0.01	-1.08	95.81
		ν_{y_3}	0.00	0.00	-0.43	95.70	-0.01	-0.65	95.53	0.00	0.42	98.82	0.00	-0.36	94.61
		λ_{x_2}	1.00	1.02	2.02	95.16	1.03	2.92	96.65	1.02	2.17	95.88	1.04	3.63	94.01
		λ_{x_3}	1.00	1.02	2.41	96.24	1.02	2.38	94.97	1.03	2.62	95.88	1.03	3.30	95.21
		λ_{x_4}	1.00	1.03	3.36	97.31	1.02	1.91	93.30	1.02	2.03	94.12	1.02	2.03	96.41
		λ_{x_5}	1.00	1.04	3.76	95.16	1.02	2.02	94.97	1.02	2.32	95.29	1.01	0.99	94.61
		λ_{y_2}	1.00	0.96	-3.53	98.39	0.92	-7.57	94.97	0.93	-6.69	96.47	0.92	-8.06	94.01
		λ_{y_3}	1.00	0.94	-5.60	94.09	0.93	-6.80	91.62	0.92	-7.53	94.12	0.92	-7.82	92.22
0															
	196														
		ρ_η	0.00	0.10	9.82	0.00	0.12	11.86	0.00	0.13	13.29	0.00	0.31	30.71	0.00
		ϕ_ζ	0.60	0.31	-48.42	96.69	0.32	-46.11	95.38	0.34	-44.16	95.81	0.48	-19.64	97.52
		α	0.00	0.00	-0.44	93.37	0.01	0.56	95.95	0.00	0.25	98.80	0.00	0.28	100.00
		γ_1	0.30	0.30	0.49	96.69	0.31	2.31	94.22	0.30	1.17	95.81	0.31	3.80	95.03
		γ_2	0.30	0.30	0.91	93.92	0.30	0.37	93.06	0.30	0.57	98.20	0.31	3.96	93.17
		γ_3	0.15	0.15	0.28	94.48	0.15	0.16	93.06	0.15	2.77	97.01	0.15	1.05	93.17
		σ_{x_1}	0.50	0.50	0.96	94.48	0.50	0.16	95.38	0.50	0.95	97.01	0.50	0.19	95.03
		σ_{x_2}	0.50	0.50	-0.65	95.58	0.50	0.61	88.44	0.50	0.53	97.60	0.50	0.62	95.65
		σ_{x_3}	0.50	0.50	0.00	92.27	0.51	1.20	95.38	0.50	0.52	88.62	0.50	-0.24	93.79

Table A.12: Results table for Study 3 simultaneous structural lag population model (D4) and simultaneous structural lag analysis model (A4) under $\phi_\zeta = 0.6$ *(continued)*

				W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^{*}		
ρ_η	N		θ	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%
		σ_{x_4}	0.50	0.50	0.53	95.03	0.50	0.24	94.80	0.50	0.14	95.81	0.50	0.09	95.65
		σ_{x_5}	0.50	0.50	0.17	96.69	0.50	0.98	97.11	0.50	0.74	95.21	0.50	1.00	94.41
		σ_{x_6}	0.50	0.50	0.16	93.92	0.50	0.47	95.38	0.50	0.55	96.41	0.50	1.00	93.79
		σ_{y_1}	0.50	0.50	-0.97	95.58	0.50	-0.76	95.38	0.50	-0.30	95.21	0.49	-1.25	94.41
		σ_{y_2}	0.50	0.50	-0.21	94.48	0.50	-0.84	95.38	0.50	-0.88	96.41	0.49	-1.10	98.14
		σ_{y_3}	0.50	0.50	-0.97	95.58	0.50	0.42	97.11	0.50	-0.69	97.60	0.50	-0.68	95.03
		σ_{ξ_1}	1.00	1.01	0.89	95.58	1.00	0.17	96.53	1.00	0.25	98.20	1.00	0.34	96.27
		σ_{ξ_2}	1.00	1.01	0.92	92.82	1.01	0.92	93.64	1.00	-0.22	96.41	1.00	0.30	95.65
		ν_{x_1}	0.00	0.00	-0.13	92.27	-0.01	-0.52	92.49	0.00	0.05	97.60	0.00	0.26	96.27
		ν_{x_2}	0.00	0.01	0.50	91.71	0.00	0.45	93.06	0.00	0.42	97.01	0.00	0.48	95.65
		ν_{x_3}	0.00	0.00	0.48	92.27	0.00	-0.13	93.06	0.01	0.79	98.80	0.00	-0.19	94.41
		ν_{x_4}	0.00	0.00	-0.31	93.37	0.00	0.42	91.33	0.00	-0.43	94.01	0.00	0.28	98.14
		ν_{x_5}	0.00	0.00	-0.15	97.24	0.01	1.08	91.33	0.00	-0.42	94.01	0.00	0.12	97.52
		ν_{x_6}	0.00	0.00	0.01	96.13	0.01	0.94	94.22	-0.01	-0.76	97.60	0.00	0.44	98.14
		ν_{y_2}	0.00	0.00	0.36	96.13	0.00	-0.10	95.38	0.00	-0.46	96.41	-0.01	-0.53	95.65
		ν_{y_3}	0.00	0.01	0.51	93.37	-0.01	-0.89	89.60	0.00	-0.10	95.21	-0.01	-0.78	95.03
		λ_{x_2}	1.00	1.01	0.89	92.82	1.00	-0.23	94.22	1.00	0.13	96.41	1.00	-0.41	96.27
		λ_{x_3}	1.00	1.00	0.32	96.13	1.00	0.08	93.64	1.01	0.53	95.81	0.99	-0.63	94.41
		λ_{x_4}	1.00	1.01	0.79	97.24	1.00	0.28	94.22	1.01	0.52	95.21	1.01	0.64	95.65
		λ_{x_5}	1.00	1.01	0.81	95.03	1.00	0.14	95.38	1.01	1.00	95.21	1.01	0.92	91.93
		λ_{y_2}	1.00	0.98	-2.36	92.82	0.99	-1.27	91.91	0.97	-2.61	93.41	0.97	-2.59	95.03
		λ_{y_3}	1.00	0.97	-2.95	92.82	0.99	-1.23	95.95	0.97	-3.07	94.01	0.96	-3.55	92.55
0.3															
		ρ_η	0.30	0.27	-11.40	96.13	0.24	-18.51	95.91	0.24	-20.49	97.01	0.33	10.39	98.75
		ϕ_ζ	0.60	0.30	-49.79	92.27	0.32	-47.39	98.25	0.34	-43.11	97.01	0.48	-19.51	97.50
		α	0.00	0.00	0.50	96.13	0.00	-0.05	95.32	0.00	0.35	98.80	0.00	-0.14	100.00
		γ_1	0.30	0.30	0.50	97.24	0.31	1.99	97.08	0.30	-1.01	99.40	0.31	3.09	95.00
		γ_2	0.30	0.30	0.65	95.03	0.30	0.16	95.32	0.30	1.35	94.01	0.30	-0.32	96.25
		γ_3	0.15	0.15	-2.42	96.13	0.15	-2.69	95.91	0.15	-1.69	96.41	0.15	1.87	98.75
		σ_{x_1}	0.50	0.51	1.36	92.82	0.50	0.23	93.57	0.50	-0.59	97.60	0.50	0.87	96.88
		σ_{x_2}	0.50	0.50	0.04	94.48	0.50	0.55	92.98	0.50	-0.11	97.60	0.50	0.53	96.88
		σ_{x_3}	0.50	0.50	-0.44	91.16	0.49	-1.65	97.08	0.51	1.23	94.01	0.50	-0.51	96.25
		σ_{x_4}	0.50	0.50	0.52	93.37	0.50	0.83	94.15	0.50	-0.72	97.01	0.50	0.85	93.12
		σ_{x_5}	0.50	0.50	0.31	95.58	0.50	0.28	98.25	0.51	1.35	94.61	0.51	1.06	91.88
		σ_{x_6}	0.50	0.50	0.14	92.82	0.50	0.54	93.57	0.50	-0.17	95.81	0.50	-0.08	96.88
		σ_{y_1}	0.50	0.49	-1.14	91.71	0.49	-1.72	95.91	0.50	-0.43	97.01	0.49	-1.23	93.75
		σ_{y_2}	0.50	0.49	-1.04	93.37	0.49	-1.34	96.49	0.50	0.44	97.01	0.49	-1.48	90.62
		σ_{y_3}	0.50	0.50	-0.53	90.06	0.50	0.12	95.91	0.50	-0.29	97.60	0.50	-0.70	93.75
		σ_{ξ_1}	1.00	1.00	-0.44	91.71	1.00	0.18	94.15	1.01	0.57	95.21	1.00	-0.47	95.62
		σ_{ξ_2}	1.00	1.00	0.15	96.13	1.00	0.02	97.08	1.01	0.55	95.21	1.00	0.21	95.00
		ν_{x_1}	0.00	0.01	0.90	96.13	0.00	-0.12	95.32	0.01	0.94	95.81	-0.01	-0.90	95.62
		ν_{x_2}	0.00	0.01	1.18	93.92	0.00	-0.48	96.49	0.01	0.69	95.21	0.00	-0.32	92.50
		ν_{x_3}	0.00	0.02	1.52	96.13	0.00	-0.39	97.66	0.01	0.94	97.01	-0.01	-1.29	90.00
		ν_{x_4}	0.00	0.00	-0.32	91.16	0.00	0.06	96.49	-0.01	-0.60	97.60	0.00	-0.26	97.50
		ν_{x_5}	0.00	0.01	0.67	93.37	0.00	0.16	96.49	-0.01	-0.51	93.41	0.00	0.17	96.25
		ν_{x_6}	0.00	0.00	-0.08	90.61	0.00	-0.12	97.08	0.00	-0.39	95.21	0.00	-0.01	96.88
		ν_{y_2}	0.00	0.00	0.18	93.37	-0.01	-0.57	94.15	0.00	0.21	94.61	0.00	0.45	94.38
		ν_{y_3}	0.00	0.00	-0.22	92.82	0.00	-0.40	93.57	-0.01	-0.58	95.21	0.00	-0.04	94.38
		λ_{x_2}	1.00	1.00	0.34	94.48	1.01	0.81	92.40	1.01	0.87	96.41	1.01	0.83	93.75
		λ_{x_3}	1.00	1.01	0.83	96.69	1.01	0.61	95.32	1.00	-0.03	98.20	1.01	0.88	93.75
		λ_{x_4}	1.00	1.00	0.01	93.92	1.00	0.11	93.57	1.00	0.24	95.21	1.00	0.30	97.50
		λ_{x_5}	1.00	1.01	0.85	95.58	1.00	0.23	93.57	1.01	1.05	95.21	1.01	1.02	95.62
		λ_{y_2}	1.00	0.97	-2.93	96.13	0.97	-2.85	92.40	0.97	-2.83	95.81	0.98	-1.83	93.75
		λ_{y_3}	1.00	0.97	-2.74	94.48	0.98	-1.59	94.74	0.98	-2.34	95.21	0.99	-0.72	95.00
0.6															

Table A.12: Results table for Study 3 simultaneous structural lag population model (D4) and simultaneous structural lag analysis model (A4) under $\phi_\zeta = 0.6$ *(continued)*

				W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^*		
ρ_η	N		θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%
		ρ_η	0.60	0.53	-12.45	82.12	0.52	-13.47	86.47	0.50	-16.92	90.42	0.35	-41.94	97.47
		ϕ_ζ	0.60	0.27	-54.88	91.06	0.29	-51.25	96.47	0.33	-45.41	97.01	0.48	-20.41	96.84
		α	0.00	0.00	-0.24	96.09	0.00	0.36	92.35	0.00	0.46	95.21	0.02	1.78	98.73
		γ_1	0.30	0.31	2.45	96.09	0.31	2.93	92.94	0.30	1.27	95.81	0.31	3.51	96.20
		γ_2	0.30	0.31	3.16	97.21	0.30	0.78	97.06	0.31	3.84	91.62	0.31	2.52	95.57
		γ_3	0.15	0.15	3.18	98.32	0.15	3.19	98.24	0.16	4.14	95.81	0.16	4.62	94.94
		σ_{x_1}	0.50	0.50	-0.09	96.09	0.50	0.29	94.71	0.50	0.57	92.81	0.50	0.20	94.30
		σ_{x_2}	0.50	0.50	0.32	91.62	0.50	0.48	95.29	0.50	0.02	95.81	0.50	0.66	96.20
		σ_{x_3}	0.50	0.50	-0.03	94.41	0.50	0.47	94.12	0.51	1.23	95.81	0.50	0.08	95.57
		σ_{x_4}	0.50	0.50	0.30	93.85	0.50	0.43	96.47	0.50	0.29	94.61	0.51	1.28	94.30
		σ_{x_5}	0.50	0.50	-0.35	97.77	0.50	0.90	95.88	0.50	-0.69	94.01	0.50	-0.31	96.84
		σ_{x_6}	0.50	0.50	0.07	94.97	0.50	0.86	97.65	0.50	-0.22	95.21	0.50	0.71	97.47
		σ_{y_1}	0.50	0.50	-0.48	93.85	0.50	-0.43	96.47	0.50	-0.77	92.22	0.49	-1.07	93.04
		σ_{y_2}	0.50	0.49	-1.40	93.85	0.50	-0.05	96.47	0.50	-0.65	95.21	0.50	-0.21	95.57
		σ_{y_3}	0.50	0.50	-0.28	98.32	0.50	0.20	97.65	0.50	-0.38	93.41	0.50	0.11	94.94
		σ_{ξ_1}	1.00	1.01	0.98	92.18	1.00	0.33	95.29	1.00	0.24	89.82	1.01	0.76	94.94
		σ_{ξ_2}	1.00	1.00	0.18	94.97	1.01	0.77	96.47	1.00	0.31	92.22	1.01	0.84	93.04
		ν_{x_1}	0.00	-0.01	-1.07	96.09	-0.01	-1.09	90.59	0.01	0.67	92.81	0.00	-0.14	96.84
		ν_{x_2}	0.00	0.00	-0.34	93.85	-0.01	-0.99	91.76	0.01	1.36	94.61	0.00	0.10	91.77
		ν_{x_3}	0.00	0.00	-0.46	93.30	-0.01	-0.55	94.12	0.01	1.23	93.41	0.00	0.34	94.30
		ν_{x_4}	0.00	0.00	0.43	97.21	0.00	-0.29	95.88	0.00	0.47	95.81	0.00	0.41	91.77
		ν_{x_5}	0.00	0.00	0.42	96.09	0.00	-0.49	93.53	0.01	0.59	95.81	0.00	0.22	94.30
		ν_{x_6}	0.00	0.01	0.71	97.77	0.00	0.33	94.12	0.01	0.74	95.21	0.00	0.48	94.30
		ν_{y_2}	0.00	0.00	0.00	97.77	0.00	-0.47	93.53	0.00	0.31	97.01	-0.01	-0.68	92.41
		ν_{y_3}	0.00	0.00	-0.26	94.41	0.00	-0.28	98.24	0.00	-0.38	95.21	-0.01	-1.39	95.57
		λ_{x_2}	1.00	1.01	0.85	92.74	1.00	0.32	96.47	1.01	1.31	91.62	1.01	0.79	94.94
		λ_{x_3}	1.00	1.00	-0.02	97.21	1.01	0.72	97.06	1.01	1.08	93.41	1.01	1.36	93.67
		λ_{x_4}	1.00	1.01	1.02	96.65	1.01	0.60	97.65	1.01	1.08	96.41	1.01	0.97	97.47
		λ_{x_5}	1.00	1.01	0.78	92.18	1.00	0.29	95.29	1.01	0.67	95.81	1.00	0.28	96.84
		λ_{y_2}	1.00	0.99	-1.09	95.53	0.98	-1.72	94.71	0.96	-3.85	92.81	0.96	-3.57	90.51
		λ_{y_3}	1.00	0.98	-1.55	92.74	0.98	-1.98	95.29	0.97	-2.50	93.41	0.98	-2.23	93.04
0															
	400														
		ρ_η	0.00	0.16	16.13	18.18	0.16	16.11	17.65	0.17	17.32	19.70	0.29	29.31	22.45
		ϕ_ζ	0.60	0.29	-51.55	95.45	0.30	-50.17	95.59	0.33	-45.48	98.48	0.49	-17.63	100.00
		α	0.00	0.00	-0.26	94.32	0.00	-0.18	98.53	0.00	-0.35	98.48	0.00	-0.04	100.00
		γ_1	0.30	0.30	1.22	93.18	0.30	1.09	89.71	0.28	-5.58	98.48	0.31	3.02	89.80
		γ_2	0.30	0.31	2.02	96.59	0.31	2.10	98.53	0.28	-5.93	95.45	0.31	2.11	93.88
		γ_3	0.15	0.16	3.65	96.59	0.14	-3.92	100.00	0.12	-20.88	96.97	0.15	-1.47	100.00
		σ_{x_1}	0.50	0.50	0.73	92.05	0.51	1.47	94.12	0.51	1.76	93.94	0.50	0.68	93.88
		σ_{x_2}	0.50	0.51	1.05	88.64	0.50	0.81	98.53	0.50	0.86	89.39	0.50	-0.03	97.96
		σ_{x_3}	0.50	0.50	-0.93	96.59	0.50	0.55	94.12	0.51	1.23	95.45	0.50	0.10	89.80
		σ_{x_4}	0.50	0.50	0.11	88.64	0.51	1.11	95.59	0.51	1.14	95.45	0.50	-0.38	95.92
		σ_{x_5}	0.50	0.50	-0.10	95.45	0.50	-0.49	98.53	0.51	1.06	95.45	0.50	-0.65	91.84
		σ_{x_6}	0.50	0.50	-0.07	95.45	0.50	0.90	98.53	0.50	0.48	92.42	0.50	0.36	100.00
		σ_{y_1}	0.50	0.49	-1.33	92.05	0.49	-1.38	94.12	0.49	-1.85	86.36	0.49	-1.43	87.76
		σ_{y_2}	0.50	0.49	-1.33	95.45	0.50	0.03	97.06	0.49	-1.15	96.97	0.50	-0.80	100.00
		σ_{y_3}	0.50	0.49	-1.29	95.45	0.49	-1.03	100.00	0.50	-0.03	93.94	0.49	-1.97	93.88
		σ_{ξ_1}	1.00	1.00	0.49	96.59	0.99	-0.76	97.06	0.99	-0.78	93.94	1.01	1.16	93.88
		σ_{ξ_2}	1.00	1.00	-0.08	94.32	1.00	-0.01	100.00	1.00	-0.14	96.97	1.00	-0.24	93.88
		ν_{x_1}	0.00	0.00	-0.36	92.05	0.00	-0.34	89.71	-0.01	-0.82	98.48	0.00	-0.23	93.88
		ν_{x_2}	0.00	0.00	-0.34	95.45	-0.01	-0.84	91.18	-0.01	-1.41	100.00	-0.01	-0.70	91.84
		ν_{x_3}	0.00	0.00	-0.48	92.05	0.00	-0.32	85.29	0.00	-0.23	96.97	0.00	0.17	97.96
		ν_{x_4}	0.00	-0.01	-0.78	90.91	0.01	0.57	98.53	0.00	0.27	93.94	0.00	0.09	95.92

Table A.12: Results table for Study 3 simultaneous structural lag population model (D4) and simultaneous structural lag analysis model (A4) under $\phi_\zeta = 0.6$ *(continued)*

				W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^*		
ρ_η	N		θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%
		ν_{x_5}	0.00	0.00	-0.49	97.73	0.01	0.73	100.00	0.00	0.41	89.39	0.01	0.67	100.00
		ν_{x_6}	0.00	0.00	0.22	94.32	0.01	0.69	92.65	0.00	0.45	93.94	0.01	0.91	95.92
		ν_{y_2}	0.00	0.01	0.56	92.05	0.00	-0.31	97.06	0.01	0.65	95.45	0.01	0.60	97.96
		ν_{y_3}	0.00	0.00	-0.22	95.45	0.00	0.11	97.06	0.00	-0.01	95.45	-0.01	-0.98	93.88
		λ_{x_2}	1.00	1.00	0.35	97.73	1.00	-0.29	88.24	1.00	0.39	92.42	1.01	0.56	91.84
		λ_{x_3}	1.00	1.00	0.10	92.05	1.01	0.79	97.06	1.00	0.17	96.97	1.00	0.48	97.96
		λ_{x_4}	1.00	1.01	1.06	89.77	1.01	0.77	95.59	1.00	-0.48	90.91	1.01	1.07	91.84
		λ_{x_5}	1.00	1.01	0.87	94.32	1.00	0.48	98.53	1.00	-0.04	100.00	1.01	1.06	100.00
		λ_{y_2}	1.00	0.97	-2.57	95.45	0.98	-2.38	94.12	0.98	-1.50	95.45	0.99	-1.43	85.71
		λ_{y_3}	1.00	0.98	-2.45	92.05	0.99	-1.02	95.59	0.98	-2.20	95.45	0.99	-1.02	89.80
0.3															
		ρ_η	0.30	0.32	-11.19	94.25	0.30	-16.16	94.03	0.32	-12.84	98.44	0.32	-4.74	100.00
		ϕ_ζ	0.60	0.28	-53.38	81.61	0.29	-52.14	95.52	0.33	-45.15	100.00	0.49	-18.48	100.00
		α	0.00	0.01	0.54	94.25	-0.01	-0.61	98.51	-0.01	-0.56	95.31	0.00	-0.05	100.00
		γ_1	0.30	0.30	1.14	98.85	0.30	0.46	100.00	0.30	0.74	98.44	0.30	0.79	95.65
		γ_2	0.30	0.31	1.84	94.25	0.30	0.36	95.52	0.30	1.55	90.62	0.31	2.89	97.83
		γ_3	0.15	0.16	4.08	96.55	0.15	0.34	100.00	0.16	4.14	96.88	0.14	-3.93	95.65
		σ_{x_1}	0.50	0.50	0.41	98.85	0.49	-1.25	88.06	0.50	0.92	87.50	0.50	0.28	91.30
		σ_{x_2}	0.50	0.50	0.46	96.55	0.50	-0.20	98.51	0.50	0.34	98.44	0.49	-1.26	93.48
		σ_{x_3}	0.50	0.50	-0.32	96.55	0.50	-0.11	100.00	0.49	-2.29	93.75	0.49	-1.82	93.48
		σ_{x_4}	0.50	0.50	-0.35	93.10	0.50	-0.22	97.01	0.50	-0.64	96.88	0.51	1.11	93.48
		σ_{x_5}	0.50	0.50	-0.59	94.25	0.50	-0.68	94.03	0.51	1.01	98.44	0.49	-1.54	97.83
		σ_{x_6}	0.50	0.50	0.71	90.80	0.50	0.35	97.01	0.50	-0.39	98.44	0.50	0.42	95.65
		σ_{y_1}	0.50	0.49	-2.99	88.51	0.49	-1.16	88.06	0.49	-1.42	95.31	0.50	-1.00	86.96
		σ_{y_2}	0.50	0.50	-0.99	93.10	0.49	-1.69	91.04	0.49	-1.65	96.88	0.50	-0.15	95.65
		σ_{y_3}	0.50	0.50	-0.71	100.00	0.49	-1.07	97.01	0.50	-0.86	90.62	0.49	-1.38	91.30
		σ_{ξ_1}	1.00	1.00	-0.15	94.25	1.01	0.78	95.52	1.00	0.08	90.62	1.01	1.00	82.61
		σ_{ξ_2}	1.00	1.00	-0.40	93.10	1.02	1.68	91.04	1.01	0.72	90.62	1.00	-0.12	97.83
		ν_{x_1}	0.00	0.01	0.93	96.55	-0.01	-1.26	94.03	-0.02	-1.77	95.31	-0.02	-1.64	95.65
		ν_{x_2}	0.00	0.01	0.59	94.25	-0.01	-1.21	86.57	-0.02	-1.60	100.00	-0.01	-1.03	89.13
		ν_{x_3}	0.00	0.01	0.89	94.25	0.00	-0.16	91.04	-0.01	-1.47	98.44	-0.01	-0.71	91.30
		ν_{x_4}	0.00	-0.01	-0.75	97.70	-0.01	-1.14	94.03	0.00	0.21	92.19	0.00	-0.16	100.00
		ν_{x_5}	0.00	0.01	0.56	97.70	-0.01	-0.59	100.00	0.00	0.35	93.75	0.00	0.03	97.83
		ν_{x_6}	0.00	0.00	0.25	100.00	-0.01	-0.97	98.51	0.00	-0.24	93.75	0.00	0.01	100.00
		ν_{y_2}	0.00	-0.01	-0.72	97.70	-0.01	-0.79	91.04	0.01	0.79	90.62	0.00	0.14	95.65
		ν_{y_3}	0.00	0.00	-0.34	90.80	0.00	0.37	98.51	0.00	0.11	90.62	0.00	-0.18	97.83
		λ_{x_2}	1.00	1.01	0.87	97.70	1.01	0.82	98.51	1.01	1.21	93.75	1.01	0.57	93.48
		λ_{x_3}	1.00	1.00	0.42	91.95	1.01	1.16	100.00	1.01	0.95	92.19	1.00	0.39	93.48
		λ_{x_4}	1.00	1.01	0.91	93.10	1.00	-0.38	97.01	1.01	0.57	96.88	1.01	0.88	100.00
		λ_{x_5}	1.00	1.00	0.27	97.70	0.99	-0.54	95.52	1.00	0.17	98.44	1.01	0.76	97.83
		λ_{y_2}	1.00	0.98	-1.76	96.55	0.99	-0.64	97.01	0.98	-1.83	90.62	0.98	-1.70	93.48
		λ_{y_3}	1.00	0.99	-0.54	90.80	1.02	2.43	92.54	0.98	-2.07	92.19	0.99	-1.15	91.30
0.6															
		ρ_η	0.60	0.56	-6.05	96.47	0.56	-7.24	93.85	0.53	-11.27	95.24	0.34	-43.25	100.00
		ϕ_ζ	0.60	0.25	-59.09	57.65	0.26	-56.94	75.38	0.31	-48.54	100.00	0.49	-17.86	100.00
		α	0.00	0.00	0.13	85.88	0.00	-0.35	93.85	0.00	0.00	98.41	0.01	0.74	100.00
		γ_1	0.30	0.29	-4.24	100.00	0.30	1.02	100.00	0.31	1.93	92.06	0.31	1.83	95.00
		γ_2	0.30	0.31	2.56	97.65	0.31	2.49	98.46	0.30	1.51	88.89	0.30	1.03	95.00
		γ_3	0.15	0.14	-8.24	97.65	0.16	3.46	98.46	0.15	1.52	96.83	0.15	1.19	95.00
		σ_{x_1}	0.50	0.50	0.41	97.65	0.50	0.11	93.85	0.50	-0.28	96.83	0.50	-0.40	97.50
		σ_{x_2}	0.50	0.51	1.01	97.65	0.51	1.19	93.85	0.50	0.32	100.00	0.50	0.01	95.00
		σ_{x_3}	0.50	0.50	0.99	98.82	0.50	0.13	96.92	0.50	-0.52	98.41	0.50	0.28	97.50
		σ_{x_4}	0.50	0.49	-1.13	97.65	0.50	0.07	95.38	0.50	0.25	96.83	0.50	0.78	95.00
		σ_{x_5}	0.50	0.50	-0.14	88.24	0.50	0.59	89.23	0.50	-1.00	93.65	0.50	0.60	100.00
		σ_{x_6}	0.50	0.50	0.45	95.29	0.50	0.85	96.92	0.50	-0.39	100.00	0.50	-0.38	95.00

Table A.12: Results table for Study 3 simultaneous structural lag population model (D4) and simultaneous structural lag analysis model (A4) under $\phi_\zeta = 0.6$ *(continued)*

			W_C^{Low}			W_C^{Mid}			W_C^{High}			W_D^*		
ρ_η	N	θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%
	σ_{y_1}	0.50	0.50	-0.90	96.47	0.50	-0.74	89.23	0.50	-0.89	93.65	0.49	-2.15	87.50
	σ_{y_2}	0.50	0.49	-1.64	97.65	0.50	-0.96	93.85	0.49	-2.18	95.24	0.50	-0.50	100.00
	σ_{y_3}	0.50	0.50	-0.75	95.29	0.49	-1.22	98.46	0.50	0.20	93.65	0.49	-1.84	95.00
	σ_{ξ_1}	1.00	0.99	-1.43	91.76	1.00	-0.03	90.77	1.00	0.12	93.65	1.02	1.92	95.00
	σ_{ξ_2}	1.00	1.00	-0.06	92.94	0.99	-0.70	96.92	1.02	1.70	92.06	0.99	-0.61	90.00
	ν_{x_1}	0.00	0.00	0.49	97.65	0.01	0.56	95.38	-0.01	-0.60	98.41	0.01	0.77	97.50
	ν_{x_2}	0.00	0.00	0.37	97.65	0.00	-0.01	93.85	-0.01	-1.16	95.24	0.01	1.32	95.00
	ν_{x_3}	0.00	0.01	0.52	97.65	-0.01	-0.79	95.38	0.00	-0.28	95.24	0.00	0.34	95.00
	ν_{x_4}	0.00	-0.01	-0.69	92.94	-0.01	-0.78	93.85	0.00	0.22	98.41	0.01	0.71	87.50
	ν_{x_5}	0.00	0.00	0.01	95.29	-0.01	-0.82	93.85	0.00	0.48	90.48	0.01	0.84	90.00
	ν_{x_6}	0.00	0.00	-0.50	94.12	-0.01	-0.95	89.23	0.00	0.13	96.83	0.00	-0.01	87.50
	ν_{y_2}	0.00	0.00	0.22	90.59	-0.01	-0.67	93.85	0.00	-0.24	92.06	0.00	0.47	100.00
	ν_{y_3}	0.00	0.00	0.06	92.94	0.00	0.06	95.38	0.00	-0.29	93.65	-0.01	-0.55	100.00
	λ_{x_2}	1.00	1.00	0.39	95.29	1.01	0.61	95.38	1.01	0.70	95.24	1.01	0.61	97.50
	λ_{x_3}	1.00	0.99	-0.65	97.65	1.01	0.90	93.85	1.01	0.57	98.41	1.01	0.62	92.50
	λ_{x_4}	1.00	1.00	0.29	92.94	1.00	-0.14	96.92	1.00	0.06	92.06	1.00	0.13	100.00
	λ_{x_5}	1.00	1.00	0.20	95.29	1.00	0.07	92.31	1.00	-0.35	90.48	1.01	0.77	97.50
	λ_{y_2}	1.00	1.00	-0.34	95.29	0.98	-1.65	96.92	0.99	-1.35	90.48	0.97	-2.60	95.00
	λ_{y_3}	1.00	0.99	-1.26	94.12	1.00	0.17	92.31	0.97	-2.73	93.65	1.00	0.49	97.50

[1] W^* is the population and analysis W condition.

[2] N is the simulated sample size.

[3] θ is the simulated population value of each parameter.

[4] $\rho_\eta = ...$ is the simulated spatial autocorrelation of the endogenous lag.

[5] $\bar{\hat{\theta}}$ is the obtained average posterior mean across simulated iterations.

[6] $Bias(\bar{\hat{\theta}})\%$ is the average percent bias across simulation iterations when $\theta \neq 0$, otherwise $Bias(\bar{\hat{\theta}})\%$ provides absolute bias.

[7] Cover% is the percent of simulated trials in which the population value falls within the central 95% density of the posterior estimate.

[8] $\phi_\zeta = 0.6$ at the population level.

A.4 Study 4

Table A.13: Results table for measurement lag population model (D2) and endogenous structural lag analysis model (A3)

				$\rho_{y2} = 0$			$\rho_{y2} = 0.3$			$\rho_{y2} = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
W_C^*												
	49											
		ρ_η	0.00	0.21	21.90	0.00	0.49	48.93	0.00	0.57	57.05	0.00
		α	0.00	-0.01	-0.53	96.61	0.00	-0.08	96.87	0.00	-0.17	96.21
		γ_1	0.30	0.32	6.55	96.35	0.32	6.72	97.26	0.32	8.17	96.73
		γ_2	0.30	0.32	7.08	96.61	0.32	5.43	96.87	0.32	6.88	97.12
		γ_3	0.15	0.16	9.74	95.18	0.16	9.95	96.22	0.17	10.24	95.95
		σ_{x_1}	0.50	0.51	1.71	93.87	0.50	0.65	94.52	0.51	1.28	94.77
		σ_{x_2}	0.50	0.51	1.67	94.78	0.50	0.68	94.13	0.51	1.06	95.95
		σ_{x_3}	0.50	0.51	1.35	95.70	0.50	-0.08	94.26	0.50	0.60	94.64
		σ_{x_4}	0.50	0.52	3.01	93.87	0.51	1.99	94.65	0.51	1.72	95.95
		σ_{x_5}	0.50	0.49	-1.08	94.78	0.51	1.05	95.57	0.50	0.53	94.77
		σ_{x_6}	0.50	0.51	2.18	94.92	0.50	0.42	94.39	0.50	0.95	95.69
		σ_{y_1}	0.50	0.49	-1.20	95.57	0.49	-1.48	95.57	0.49	-1.38	93.99
		σ_{y_2}	0.50	0.50	-0.11	95.31	0.50	-0.75	95.18	0.51	1.69	95.56
		σ_{y_3}	0.50	0.50	0.57	95.05	0.51	1.16	94.39	0.51	2.74	93.33
		σ_{ξ_1}	1.00	1.01	1.25	94.00	1.01	1.05	94.26	1.01	0.57	96.34
		σ_{ξ_2}	1.00	1.01	1.09	92.57	1.00	0.27	97.13	1.01	1.17	93.59
		ν_{x_1}	0.00	-0.01	-0.92	96.09	0.00	0.15	95.18	-0.01	-0.59	95.29
		ν_{x_2}	0.00	-0.01	-1.12	95.44	-0.01	-0.66	93.61	-0.01	-0.65	94.77
		ν_{x_3}	0.00	-0.01	-0.68	95.96	0.00	0.21	95.05	0.00	-0.30	94.90
		ν_{x_4}	0.00	0.01	1.14	94.78	0.01	0.83	95.96	0.00	0.03	94.25
		ν_{x_5}	0.00	0.01	0.88	95.18	0.01	0.60	97.65	0.00	-0.21	95.69
		ν_{x_6}	0.00	0.01	0.89	93.35	0.01	1.24	96.09	0.00	0.34	96.60
		ν_{y_2}	0.00	0.00	0.16	94.39	0.00	0.15	95.57	0.00	0.07	95.95
		ν_{y_3}	0.00	0.00	0.14	94.39	0.01	0.60	88.53	0.00	0.00	86.14
		λ_{x_2}	1.00	1.01	0.98	94.92	1.02	2.22	95.96	1.03	3.03	95.82
		λ_{x_3}	1.00	1.02	1.63	95.31	1.02	2.47	95.70	1.04	3.78	94.38
		λ_{x_4}	1.00	1.03	2.57	94.39	1.03	2.86	96.87	1.02	2.13	95.82
		λ_{x_5}	1.00	1.02	1.99	96.87	1.03	2.77	95.83	1.02	1.86	94.90
		λ_{y_2}	1.00	0.92	-7.60	93.74	0.94	-6.44	93.87	0.91	-8.57	93.59
		λ_{y_3}	1.00	0.92	-8.36	92.57	0.93	-6.65	92.96	0.93	-7.39	93.86
	196											
		ρ_η	0.00	0.11	11.94	0.00	0.48	48.23	0.00	0.43	43.25	0.00
		α	0.00	0.00	0.06	94.31	0.00	-0.22	94.44	-0.01	-0.59	97.22
		γ_1	0.30	0.30	1.15	95.42	0.30	0.34	95.14	0.30	0.45	97.36
		γ_2	0.30	0.30	1.66	93.89	0.31	1.90	93.75	0.30	0.36	95.83
		γ_3	0.15	0.15	2.54	96.67	0.15	0.68	95.14	0.15	0.63	95.56
		σ_{x_1}	0.50	0.50	0.11	95.00	0.50	0.51	95.00	0.50	0.52	93.06
		σ_{x_2}	0.50	0.50	0.54	93.61	0.50	0.26	94.58	0.50	-0.21	94.31
		σ_{x_3}	0.50	0.50	-0.19	95.56	0.50	0.70	95.83	0.50	0.38	92.64
		σ_{x_4}	0.50	0.50	0.83	93.33	0.50	0.52	96.94	0.50	0.53	94.72
		σ_{x_5}	0.50	0.50	0.92	93.61	0.50	0.47	92.78	0.50	0.29	95.28
		σ_{x_6}	0.50	0.50	-0.68	95.00	0.50	0.53	95.56	0.50	0.81	93.89
		σ_{y_1}	0.50	0.50	-0.34	94.58	0.49	-1.11	95.97	0.49	-1.15	95.69
		σ_{y_2}	0.50	0.50	-0.29	96.39	0.50	0.08	96.39	0.50	-0.71	94.86
		σ_{y_3}	0.50	0.50	-0.99	95.42	0.50	0.14	94.58	0.51	2.87	88.61
		σ_{ξ_1}	1.00	1.00	-0.09	94.31	1.00	0.18	95.28	1.00	0.26	96.81
		σ_{ξ_2}	1.00	1.00	0.24	94.17	1.00	0.28	95.00	1.00	0.48	95.28
		ν_{x_1}	0.00	0.00	0.39	96.81	0.01	1.05	94.44	0.00	-0.42	95.28
		ν_{x_2}	0.00	0.00	0.47	95.97	0.01	0.70	91.94	0.00	-0.24	93.61
		ν_{x_3}	0.00	0.01	0.83	95.97	0.01	0.62	92.50	0.00	-0.46	95.28

Table A.13: Results table for Study 4 measurement lag population model (D2) and endogenous structural lag analysis model (A3) *(continued)*

				$\rho_{y2} = 0$			$\rho_{y2} = 0.3$			$\rho_{y2} = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
		ν_{x_4}	0.00	0.00	-0.45	95.42	-0.01	-0.87	95.00	0.00	0.13	95.42
		ν_{x_5}	0.00	-0.01	-0.58	95.42	-0.01	-0.65	93.61	0.00	-0.15	94.03
		ν_{x_6}	0.00	0.00	-0.25	95.69	-0.01	-0.82	96.39	0.00	0.20	93.89
		ν_{y_2}	0.00	0.00	-0.23	94.31	0.00	0.04	95.14	0.01	0.68	95.00
		ν_{y_3}	0.00	0.00	-0.13	93.33	0.00	0.22	92.08	0.00	0.46	88.75
		λ_{x_2}	1.00	1.01	0.62	94.44	1.00	0.14	95.42	1.01	0.57	96.67
		λ_{x_3}	1.00	1.01	0.59	95.00	1.00	0.32	94.86	1.00	0.35	95.42
		λ_{x_4}	1.00	1.01	0.76	94.17	1.00	0.38	95.83	1.01	0.62	94.31
		λ_{x_5}	1.00	1.01	0.95	95.83	1.00	0.37	94.31	1.01	0.92	95.00
		λ_{y_2}	1.00	0.99	-0.85	94.72	0.98	-1.81	95.83	0.99	-1.24	92.78
		λ_{y_3}	1.00	0.99	-0.90	94.17	0.99	-0.89	93.19	0.99	-0.81	95.00
	400											
		ρ_η	0.00	0.09	9.46	0.00	0.42	42.40	0.00	0.39	39.30	0.00
		α	0.00	0.00	-0.02	94.63	0.00	-0.03	93.97	0.00	0.00	95.92
		γ_1	0.30	0.30	-0.84	93.97	0.30	-0.08	94.50	0.30	-0.18	95.79
		γ_2	0.30	0.30	0.93	95.15	0.30	-0.52	95.41	0.30	0.19	94.74
		γ_3	0.15	0.15	-2.45	95.02	0.15	-1.98	94.36	0.15	-0.77	95.92
		σ_{x_1}	0.50	0.50	-0.02	95.02	0.50	0.03	95.67	0.50	0.33	93.95
		σ_{x_2}	0.50	0.50	0.72	96.33	0.50	0.07	95.02	0.50	0.08	95.26
		σ_{x_3}	0.50	0.50	0.09	95.15	0.50	0.48	94.63	0.50	0.56	95.00
		σ_{x_4}	0.50	0.50	-0.02	93.71	0.50	-0.14	95.41	0.50	0.11	95.00
		σ_{x_5}	0.50	0.50	-0.12	94.89	0.50	-0.03	94.50	0.50	0.08	94.87
		σ_{x_6}	0.50	0.50	0.11	93.18	0.50	0.53	95.41	0.50	-0.02	95.26
		σ_{y_1}	0.50	0.50	-0.55	95.54	0.50	-0.53	95.28	0.50	-0.93	93.16
		σ_{y_2}	0.50	0.50	-0.30	94.76	0.50	-0.73	93.97	0.50	-0.62	95.00
		σ_{y_3}	0.50	0.50	-0.43	95.02	0.50	-0.10	93.97	0.51	2.88	85.79
		σ_{ξ_1}	1.00	1.00	0.14	95.15	1.00	-0.03	94.76	1.00	-0.08	94.21
		σ_{ξ_2}	1.00	1.00	0.31	94.50	1.00	0.26	94.23	1.00	0.16	95.66
		ν_{x_1}	0.00	0.00	-0.04	95.94	0.00	0.06	96.85	0.01	0.51	95.39
		ν_{x_2}	0.00	0.00	-0.03	95.81	0.00	-0.08	96.85	0.00	0.42	95.26
		ν_{x_3}	0.00	0.00	-0.02	95.94	0.00	-0.04	95.41	0.00	0.33	95.39
		ν_{x_4}	0.00	0.00	-0.24	95.41	0.00	-0.01	96.20	0.00	0.03	95.13
		ν_{x_5}	0.00	0.00	-0.40	93.84	0.00	0.11	95.81	0.00	0.22	94.47
		ν_{x_6}	0.00	0.00	-0.34	93.71	0.00	0.01	96.20	0.00	0.31	93.82
		ν_{y_2}	0.00	0.00	0.06	96.20	0.00	0.00	94.50	0.00	0.16	94.08
		ν_{y_3}	0.00	0.00	-0.05	96.85	0.00	-0.03	91.35	0.00	0.07	88.03
		λ_{x_2}	1.00	1.00	-0.13	95.67	1.00	0.15	94.50	1.00	0.04	94.34
		λ_{x_3}	1.00	1.00	0.07	95.41	1.00	0.22	94.89	1.00	0.01	94.61
		λ_{x_4}	1.00	1.00	0.02	95.41	1.00	-0.01	95.81	1.00	-0.16	95.79
		λ_{x_5}	1.00	1.00	-0.33	94.10	1.00	-0.03	94.23	1.00	-0.04	94.74
		λ_{y_2}	1.00	0.99	-0.58	95.54	1.00	-0.47	93.84	0.99	-0.78	94.61
		λ_{y_3}	1.00	0.99	-0.82	94.50	1.00	0.47	92.79	1.00	0.03	92.63
W_D^*												
	49											
		ρ_η	0.00	0.50	50.09	0.00	0.50	50.15	0.00	0.50	50.31	0.00
		α	0.00	0.00	0.44	96.29	-0.01	-0.76	96.55	0.00	0.37	97.74
		γ_1	0.30	0.32	7.61	97.09	0.32	7.02	98.01	0.33	8.86	97.21
		γ_2	0.30	0.32	6.79	94.57	0.32	7.73	96.95	0.32	6.60	97.48
		γ_3	0.15	0.16	9.64	97.48	0.17	11.72	95.89	0.17	11.65	96.68
		σ_{x_1}	0.50	0.51	2.22	93.51	0.51	1.84	93.10	0.51	2.74	92.56
		σ_{x_2}	0.50	0.51	1.99	93.77	0.51	1.78	95.23	0.50	0.68	95.09
		σ_{x_3}	0.50	0.50	0.20	95.89	0.50	0.87	96.42	0.50	0.68	93.36
		σ_{x_4}	0.50	0.52	3.55	94.04	0.51	1.77	94.03	0.51	2.36	92.70
		σ_{x_5}	0.50	0.50	-0.38	94.70	0.50	-0.03	93.24	0.49	-1.57	93.89

Table A.13: Results table for Study 4 measurement lag population model (D2) and endogenous structural lag analysis model (A3) *(continued)*

			$\rho_{y2}=0$			$\rho_{y2}=0.3$			$\rho_{y2}=0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
		σ_{x_6} 0.50	0.51	1.45	94.97	0.50	-0.06	94.56	0.51	1.43	96.55
		σ_{y_1} 0.50	0.49	-1.68	95.36	0.49	-1.23	95.89	0.50	-0.71	95.09
		σ_{y_2} 0.50	0.51	1.15	94.70	0.50	0.60	94.03	0.50	0.25	96.02
		σ_{y_3} 0.50	0.50	0.83	92.72	0.50	-0.85	93.77	0.49	-1.34	95.75
		σ_{ξ_1} 1.00	1.00	-0.07	93.77	1.01	1.11	95.23	1.01	1.19	94.95
		σ_{ξ_2} 1.00	1.00	-0.21	93.25	1.01	0.95	95.09	1.00	0.45	96.15
		ν_{x_1} 0.00	0.01	0.67	94.57	0.00	0.29	96.29	0.00	0.18	96.02
		ν_{x_2} 0.00	0.00	0.30	95.89	0.00	0.12	96.42	0.00	-0.05	95.22
		ν_{x_3} 0.00	0.00	0.49	94.44	0.00	0.41	96.29	0.01	0.64	96.02
		ν_{x_4} 0.00	0.01	1.18	94.97	-0.01	-0.97	95.23	0.01	0.97	95.62
		ν_{x_5} 0.00	0.00	0.45	95.50	-0.01	-0.98	95.49	0.01	0.78	96.68
		ν_{x_6} 0.00	0.01	0.82	96.16	-0.01	-1.11	93.90	0.01	0.51	96.15
		ν_{y_2} 0.00	0.00	-0.09	94.57	0.00	0.19	94.03	0.00	-0.21	96.28
		ν_{y_3} 0.00	0.00	0.21	93.91	0.00	0.05	91.25	0.01	0.64	88.05
		λ_{x_2} 1.00	1.02	2.37	96.16	1.03	2.84	94.03	1.03	2.66	95.62
		λ_{x_3} 1.00	1.03	2.56	96.16	1.02	2.48	95.49	1.03	2.77	95.75
		λ_{x_4} 1.00	1.04	3.54	95.63	1.03	3.10	93.90	1.03	2.93	95.88
		λ_{x_5} 1.00	1.03	2.81	94.97	1.02	2.46	94.30	1.03	2.67	96.15
		λ_{y_2} 1.00	0.93	-6.78	92.72	0.92	-7.55	93.50	0.92	-7.65	94.56
		λ_{y_3} 1.00	0.92	-7.51	94.83	0.92	-8.13	93.37	0.90	-9.68	95.22
	196										
		ρ_η 0.00	0.50	50.01	0.00	0.50	50.04	0.00	0.50	50.11	0.00
		α 0.00	0.00	-0.28	95.50	0.00	0.06	95.22	0.00	-0.09	96.34
		γ_1 0.30	0.31	2.11	96.77	0.30	1.64	95.50	0.30	0.26	94.08
		γ_2 0.30	0.31	1.69	96.62	0.31	1.88	93.67	0.30	1.40	97.32
		γ_3 0.15	0.16	4.96	92.97	0.15	3.06	95.92	0.15	1.00	96.34
		σ_{x_1} 0.50	0.50	0.73	94.09	0.50	0.26	95.92	0.51	1.20	94.51
		σ_{x_2} 0.50	0.50	0.48	94.09	0.50	-0.16	95.36	0.50	0.48	91.83
		σ_{x_3} 0.50	0.50	-0.29	94.23	0.50	0.93	94.66	0.50	0.32	96.06
		σ_{x_4} 0.50	0.50	0.63	96.20	0.50	0.94	95.22	0.51	1.11	94.37
		σ_{x_5} 0.50	0.51	1.12	95.08	0.50	0.72	94.51	0.50	0.10	94.37
		σ_{x_6} 0.50	0.50	0.01	93.95	0.50	0.71	95.36	0.50	0.72	94.65
		σ_{y_1} 0.50	0.50	-0.89	95.08	0.50	-0.91	95.50	0.50	-0.58	94.37
		σ_{y_2} 0.50	0.50	-0.88	94.80	0.50	-0.28	95.64	0.50	-0.58	93.52
		σ_{y_3} 0.50	0.50	-0.66	95.78	0.50	-0.92	94.94	0.49	-1.07	95.21
		σ_{ξ_1} 1.00	1.00	0.13	95.64	1.00	0.35	94.80	1.00	0.19	94.37
		σ_{ξ_2} 1.00	1.00	-0.13	96.06	1.00	-0.45	93.67	1.00	-0.21	94.37
		ν_{x_1} 0.00	-0.01	-0.53	95.64	0.00	-0.06	95.22	0.00	-0.40	96.48
		ν_{x_2} 0.00	0.00	-0.40	94.94	0.00	0.01	94.23	0.00	-0.42	96.48
		ν_{x_3} 0.00	0.00	-0.48	94.94	0.00	0.16	96.06	-0.01	-0.54	95.49
		ν_{x_4} 0.00	0.00	-0.31	95.08	0.00	0.01	95.08	0.00	0.12	94.37
		ν_{x_5} 0.00	0.00	0.06	94.23	0.00	0.11	94.37	0.00	0.38	96.06
		ν_{x_6} 0.00	0.00	-0.21	92.97	0.00	-0.09	92.69	0.00	0.28	96.48
		ν_{y_2} 0.00	0.00	-0.10	95.78	0.00	0.06	95.64	0.00	-0.11	95.07
		ν_{y_3} 0.00	0.00	0.33	97.19	0.00	-0.17	89.73	0.00	-0.10	83.10
		λ_{x_2} 1.00	1.00	0.32	95.36	1.01	0.51	94.51	1.01	0.71	92.82
		λ_{x_3} 1.00	1.01	0.67	94.80	1.01	0.66	94.51	1.00	0.47	93.80
		λ_{x_4} 1.00	1.01	0.75	94.51	1.01	0.62	96.20	1.01	0.89	93.66
		λ_{x_5} 1.00	1.01	0.71	94.09	1.01	0.51	95.36	1.01	0.85	94.37
		λ_{y_2} 1.00	0.99	-1.39	96.91	0.99	-0.90	94.37	0.99	-0.83	94.08
		λ_{y_3} 1.00	0.97	-2.80	95.08	0.99	-0.93	94.37	0.99	-0.53	94.93
	400										
		ρ_η 0.00	0.50	50.03	0.00	0.50	50.05	0.00	0.50	50.10	0.00
		α 0.00	0.00	-0.15	96.52	0.00	-0.16	95.19	0.00	-0.02	95.44
		γ_1 0.30	0.30	-0.09	93.98	0.30	0.25	95.99	0.30	0.63	95.04

Table A.13: Results table for Study 4 measurement lag population model (D2) and endogenous structural lag analysis model (A3) *(continued)*

				$\rho_{y2}=0$			$\rho_{y2}=0.3$			$\rho_{y2}=0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
		γ_2	0.30	0.30	0.47	96.66	0.30	0.06	95.99	0.30	1.23	96.78
		γ_3	0.15	0.15	-0.17	97.06	0.15	-1.29	95.32	0.15	-0.01	95.04
		σ_{x_1}	0.50	0.50	0.40	92.78	0.50	-0.13	94.39	0.50	0.75	96.38
		σ_{x_2}	0.50	0.50	0.37	95.32	0.50	0.63	94.25	0.50	0.08	95.58
		σ_{x_3}	0.50	0.50	-0.06	95.45	0.50	-0.02	94.79	0.50	0.39	94.10
		σ_{x_4}	0.50	0.50	0.61	94.39	0.50	-0.19	93.58	0.50	-0.10	95.31
		σ_{x_5}	0.50	0.50	-0.10	93.85	0.50	0.18	95.45	0.50	0.32	96.11
		σ_{x_6}	0.50	0.50	-0.03	95.59	0.50	0.45	94.52	0.50	0.44	95.04
		σ_{y_1}	0.50	0.50	-0.66	95.59	0.50	-0.66	95.59	0.50	-0.65	95.04
		σ_{y_2}	0.50	0.50	-0.63	93.32	0.50	-0.55	95.99	0.50	-0.53	95.58
		σ_{y_3}	0.50	0.50	-0.63	95.32	0.50	-0.54	94.25	0.50	-0.79	95.44
		σ_{ξ_1}	1.00	1.00	0.05	96.26	1.00	0.33	94.39	1.00	0.08	95.44
		σ_{ξ_2}	1.00	1.00	0.23	95.45	1.00	0.27	93.72	1.00	0.05	95.98
		ν_{x_1}	0.00	0.00	-0.07	94.79	0.00	-0.07	95.59	0.00	0.11	95.98
		ν_{x_2}	0.00	0.00	0.25	95.72	0.00	-0.11	95.99	0.00	0.02	95.84
		ν_{x_3}	0.00	0.00	-0.04	95.05	0.00	-0.16	94.25	0.00	-0.01	95.58
		ν_{x_4}	0.00	0.00	0.15	95.19	0.00	-0.02	94.92	0.00	-0.26	95.17
		ν_{x_5}	0.00	0.00	0.10	95.19	0.00	-0.15	96.26	0.00	0.00	93.70
		ν_{x_6}	0.00	0.00	-0.01	96.39	0.00	0.07	95.32	0.00	-0.22	96.11
		ν_{y_2}	0.00	0.00	0.18	93.58	0.00	-0.23	93.45	0.00	-0.02	94.64
		ν_{y_3}	0.00	0.00	0.09	94.79	0.00	0.00	91.31	0.00	-0.08	85.66
		λ_{x_2}	1.00	1.00	0.27	94.79	1.00	0.08	93.85	1.00	0.40	94.50
		λ_{x_3}	1.00	1.00	0.23	95.32	1.00	0.27	93.98	1.00	0.43	95.44
		λ_{x_4}	1.00	1.00	0.34	92.51	1.00	-0.03	96.79	1.00	0.34	96.11
		λ_{x_5}	1.00	1.01	0.61	93.45	1.00	-0.03	95.05	1.00	0.21	95.71
		λ_{y_2}	1.00	0.99	-0.80	95.05	0.99	-1.17	93.32	0.99	-0.72	95.58
		λ_{y_3}	1.00	0.99	-0.63	96.12	0.99	-1.49	92.65	0.99	-1.14	91.69

[1] W^* is the population and analysis W condition.

[2] N is the simulated sample size.

[3] θ is the simulated population value of each parameter.

[4] $\rho_\eta = ...$ is the simulated spatial autocorrelation of the endogenous lag.

[5] $\bar{\hat{\theta}}$ is the obtained average posterior mean across simulated iterations.

[6] $Bias(\bar{\hat{\theta}})\%$ is the average percent bias across simulation iterations when $\theta \neq 0$, otherwise $Bias(\bar{\hat{\theta}})\%$ provides absolute bias.

[7] Cover% is the percent of simulated trials in which the population value falls within the central 95% density of the posterior estimate.

Table A.14: Results table for Study 4 measurement lag population model (D2) and Simultaneous structural lag analysis model (A4)

				$\rho_{y2} = 0$			$\rho_{y2} = 0.3$			$\rho_{y2} = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
W_C^*												
	49											
		ρ_η	0.00	0.18	18.17	0.00	0.19	19.18	89.10	0.20	19.94	17.14
		ϕ_ζ	0.00	0.37	37.28	0.00	0.38	38.22	0.00	0.39	38.62	0.00
		α	0.00	0.00	0.31	99.05	0.00	0.36	99.53	0.01	1.35	99.05
		γ_1	0.30	0.32	6.00	98.58	0.32	6.70	95.26	0.30	1.08	96.19
		γ_2	0.30	0.31	4.42	98.10	0.32	5.12	97.63	0.31	3.18	97.62
		γ_3	0.15	0.15	2.66	97.16	0.16	5.22	97.16	0.14	-3.65	98.10
		σ_{x_1}	0.50	0.50	-0.38	94.31	0.51	2.98	95.26	0.50	-0.70	94.76
		σ_{x_2}	0.50	0.51	1.49	94.79	0.51	2.30	94.31	0.50	0.41	93.81
		σ_{x_3}	0.50	0.52	3.45	93.84	0.50	0.60	95.26	0.52	3.86	95.24
		σ_{x_4}	0.50	0.51	1.49	93.84	0.50	-0.01	97.16	0.51	2.17	94.29
		σ_{x_5}	0.50	0.51	2.99	96.21	0.50	0.78	92.89	0.51	2.05	96.19
		σ_{x_6}	0.50	0.51	1.61	96.68	0.51	2.51	95.26	0.51	1.06	94.76
		σ_{y_1}	0.50	0.50	-0.46	91.94	0.50	-0.74	96.21	0.50	-0.01	94.29
		σ_{y_2}	0.50	0.51	2.85	95.26	0.51	1.53	94.79	0.51	1.33	98.57
		σ_{y_3}	0.50	0.51	1.49	94.79	0.50	-0.99	95.26	0.51	1.66	95.24
		σ_{ξ_1}	1.00	1.02	1.55	93.84	1.00	0.30	97.63	1.02	1.77	93.81
		σ_{ξ_2}	1.00	1.02	1.94	97.63	1.01	0.98	95.26	1.01	0.58	95.71
		ν_{x_1}	0.00	0.01	1.30	95.73	0.00	-0.25	95.26	-0.02	-1.83	97.14
		ν_{x_2}	0.00	0.02	1.57	96.21	0.00	-0.07	94.31	-0.01	-1.21	93.33
		ν_{x_3}	0.00	0.01	1.47	98.10	0.00	-0.21	95.26	0.00	-0.44	95.24
		ν_{x_4}	0.00	-0.01	-0.98	96.21	0.02	2.33	94.79	0.01	1.33	96.67
		ν_{x_5}	0.00	0.00	0.47	95.26	0.02	2.05	95.73	0.01	0.96	94.29
		ν_{x_6}	0.00	0.00	-0.23	95.73	0.02	2.02	95.26	0.01	0.56	94.29
		ν_{y_2}	0.00	0.00	0.47	95.26	0.00	0.34	95.26	-0.02	-2.14	94.29
		ν_{y_3}	0.00	0.02	1.78	91.47	-0.02	-2.41	91.47	-0.02	-1.51	88.57
		λ_{x_2}	1.00	1.03	3.09	94.79	1.02	1.70	97.63	1.02	2.19	96.67
		λ_{x_3}	1.00	1.01	1.30	94.31	1.02	2.43	97.16	1.01	0.56	98.57
		λ_{x_4}	1.00	1.00	0.35	95.73	1.02	2.28	93.36	1.03	2.95	96.19
		λ_{x_5}	1.00	1.00	0.44	96.68	1.01	1.46	94.79	1.03	3.20	94.29
		λ_{y_2}	1.00	0.88	-11.99	93.36	0.91	-9.29	93.84	0.93	-7.40	93.33
		λ_{y_3}	1.00	0.87	-12.51	91.47	0.92	-8.26	91.47	0.95	-5.22	95.71
	196											
		ρ_η	0.00	0.11	11.38	0.00	0.12	11.97	22.06	0.13	13.00	13.30
		ϕ_ζ	0.00	0.31	30.62	0.00	0.34	34.14	0.00	0.38	37.65	0.00
		α	0.00	0.00	-0.21	97.58	0.00	-0.30	98.04	0.00	0.03	98.03
		γ_1	0.30	0.30	-0.09	94.20	0.29	-3.09	95.59	0.29	-2.34	96.06
		γ_2	0.30	0.30	-1.32	92.27	0.30	-1.39	92.65	0.29	-3.46	99.51
		γ_3	0.15	0.15	0.67	92.75	0.14	-6.12	95.59	0.14	-4.13	97.54
		σ_{x_1}	0.50	0.50	0.46	97.58	0.50	0.87	90.20	0.50	0.05	96.06
		σ_{x_2}	0.50	0.50	0.18	93.72	0.50	-0.23	93.63	0.50	-0.47	94.09
		σ_{x_3}	0.50	0.51	1.45	95.65	0.50	0.10	93.63	0.50	0.56	92.12
		σ_{x_4}	0.50	0.50	0.81	91.30	0.50	0.93	95.10	0.50	-0.39	92.61
		σ_{x_5}	0.50	0.50	-0.21	98.07	0.50	-0.65	94.61	0.50	0.31	96.06
		σ_{x_6}	0.50	0.49	-1.36	93.72	0.50	0.43	93.14	0.50	0.96	98.03
		σ_{y_1}	0.50	0.50	-0.71	96.62	0.50	-0.99	93.14	0.50	-0.41	96.06
		σ_{y_2}	0.50	0.50	0.16	95.17	0.50	-0.85	93.63	0.50	0.19	97.04
		σ_{y_3}	0.50	0.50	-0.40	96.14	0.50	0.14	93.63	0.51	2.22	87.68
		σ_{ξ_1}	1.00	1.00	-0.03	95.17	1.00	0.01	97.06	1.01	0.75	96.06
		σ_{ξ_2}	1.00	1.00	0.17	96.14	1.01	1.40	95.59	1.00	-0.29	96.06
		ν_{x_1}	0.00	-0.01	-0.80	93.24	-0.01	-0.56	92.65	0.00	0.11	93.10
		ν_{x_2}	0.00	0.00	-0.34	95.17	0.00	-0.45	95.10	0.00	-0.28	92.61
		ν_{x_3}	0.00	0.00	-0.38	90.82	-0.01	-0.65	93.63	0.00	-0.22	93.60
		ν_{x_4}	0.00	0.01	0.65	94.69	0.01	0.68	93.63	-0.01	-0.80	95.07

Table A.14: Results table for Study 4 measurement lag population model (D2) and Simultaneous structural lag analysis model (A4) *(continued)*

			$\rho_{y2}=0$			$\rho_{y2}=0.3$			$\rho_{y2}=0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
	ν_{x_5}	0.00	0.01	0.75	95.17	0.00	0.25	93.63	-0.01	-0.57	94.09
	ν_{x_6}	0.00	0.00	0.40	96.62	0.00	0.44	91.67	0.00	-0.16	95.57
	ν_{y_2}	0.00	0.00	0.04	94.20	0.00	0.27	96.57	0.00	-0.40	97.04
	ν_{y_3}	0.00	0.00	-0.08	96.14	0.00	0.07	91.67	0.01	0.89	86.21
	λ_{x_2}	1.00	1.00	0.42	95.65	1.00	0.46	93.14	1.01	0.99	97.54
	λ_{x_3}	1.00	1.01	0.86	95.17	1.00	0.46	95.10	1.01	0.79	96.55
	λ_{x_4}	1.00	1.01	1.24	98.07	1.00	0.10	94.61	1.00	-0.04	95.07
	λ_{x_5}	1.00	1.01	0.98	92.75	1.00	0.07	94.61	1.00	0.27	94.09
	λ_{y_2}	1.00	0.97	-3.20	94.20	0.98	-1.93	95.59	0.98	-1.92	98.03
	λ_{y_3}	1.00	0.97	-3.03	91.79	0.99	-1.05	96.57	1.01	1.00	95.07
	400										
	ρ_η	0.00	0.06	6.28	0.00	0.07	6.68	0.83	0.08	7.69	0.00
	ϕ_ζ	0.00	0.31	30.60	0.00	0.37	37.10	0.00	0.42	42.45	0.00
	α	0.00	0.00	-0.22	94.35	0.00	0.40	98.33	0.00	0.44	98.33
	γ_1	0.30	0.31	2.25	95.16	0.30	1.26	97.50	0.30	-0.75	97.50
	γ_2	0.30	0.30	0.51	96.77	0.30	-0.02	95.00	0.29	-2.44	94.17
	γ_3	0.15	0.15	1.59	99.19	0.15	-0.87	98.33	0.15	-3.19	95.83
	σ_{x_1}	0.50	0.50	-0.17	97.58	0.50	0.07	95.00	0.51	1.02	95.83
	σ_{x_2}	0.50	0.50	-0.47	98.39	0.50	-0.10	100.00	0.50	-0.66	95.83
	σ_{x_3}	0.50	0.50	-0.56	91.13	0.50	0.35	95.00	0.50	-0.50	90.00
	σ_{x_4}	0.50	0.50	-0.19	96.77	0.50	0.43	95.00	0.50	-0.54	96.67
	σ_{x_5}	0.50	0.50	0.08	91.94	0.50	-0.60	93.33	0.50	0.26	90.83
	σ_{x_6}	0.50	0.50	-0.17	97.58	0.50	-0.76	95.00	0.50	-0.11	90.83
	σ_{y_1}	0.50	0.49	-1.34	92.74	0.49	-1.82	91.67	0.50	-0.29	96.67
	σ_{y_2}	0.50	0.50	-0.86	95.16	0.49	-1.48	85.83	0.49	-1.62	94.17
	σ_{y_3}	0.50	0.50	0.16	94.35	0.50	0.12	95.00	0.51	1.96	87.50
	σ_{ξ_1}	1.00	1.00	-0.03	97.58	1.00	0.07	95.83	1.00	-0.19	92.50
	σ_{ξ_2}	1.00	1.01	0.70	93.55	1.01	0.73	95.00	1.00	0.14	95.83
	ν_{x_1}	0.00	0.00	-0.49	97.58	0.01	1.12	94.17	0.01	1.13	95.83
	ν_{x_2}	0.00	-0.01	-0.58	94.35	0.01	1.09	99.17	0.00	0.44	97.50
	ν_{x_3}	0.00	-0.01	-1.03	95.16	0.01	1.31	95.00	0.01	0.63	97.50
	ν_{x_4}	0.00	0.00	-0.43	91.94	0.00	-0.01	95.83	0.01	0.85	91.67
	ν_{x_5}	0.00	0.00	-0.15	95.16	0.00	0.50	94.17	0.01	0.79	93.33
	ν_{x_6}	0.00	0.00	0.01	99.19	0.00	0.31	100.00	0.00	0.49	90.83
	ν_{y_2}	0.00	0.00	-0.09	96.77	-0.01	-0.69	95.00	0.00	-0.23	95.83
	ν_{y_3}	0.00	0.00	-0.25	95.16	0.00	0.11	85.83	0.00	0.05	82.50
	λ_{x_2}	1.00	1.00	0.18	91.13	1.00	-0.03	99.17	1.00	0.23	97.50
	λ_{x_3}	1.00	1.00	0.23	95.16	1.00	0.04	95.83	1.01	0.89	94.17
	λ_{x_4}	1.00	1.00	-0.32	92.74	1.00	-0.32	97.50	1.01	0.74	96.67
	λ_{x_5}	1.00	0.99	-0.94	93.55	1.00	-0.07	95.83	1.00	0.46	97.50
	λ_{y_2}	1.00	0.98	-2.28	88.71	0.98	-1.91	91.67	1.00	0.43	96.67
	λ_{y_3}	1.00	0.98	-2.20	93.55	1.00	0.27	92.50	1.02	2.08	95.00
W_D^*											
	49										
	ρ_η	0.00	0.33	32.57	0.00	0.33	33.40	100.00	0.33	33.21	95.57
	ϕ_ζ	0.00	0.47	46.82	0.00	0.46	46.48	0.00	0.48	47.52	0.00
	α	0.00	-0.01	-0.88	100.00	0.02	2.15	98.52	-0.01	-0.91	100.00
	γ_1	0.30	0.33	11.66	97.54	0.32	7.37	97.04	0.32	5.68	93.10
	γ_2	0.30	0.33	9.46	97.04	0.33	8.99	93.10	0.33	10.01	95.57
	γ_3	0.15	0.16	9.92	97.04	0.17	15.00	94.09	0.17	13.83	96.06
	σ_{x_1}	0.50	0.49	-1.11	92.61	0.52	3.86	95.07	0.51	1.04	94.58
	σ_{x_2}	0.50	0.50	0.42	96.06	0.51	2.88	94.58	0.50	0.85	93.60
	σ_{x_3}	0.50	0.51	1.80	94.09	0.51	2.55	96.06	0.50	0.85	96.55
	σ_{x_4}	0.50	0.52	4.43	96.06	0.51	2.32	96.06	0.51	2.89	94.09

Table A.14: Results table for Study 4 measurement lag population model (D2) and Simultaneous structural lag analysis model (A4) *(continued)*

			$\rho_{y2}=0$			$\rho_{y2}=0.3$			$\rho_{y2}=0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
	σ_{x_5}	0.50	0.50	-0.30	94.58	0.50	-0.66	97.04	0.50	-0.79	94.09
	σ_{x_6}	0.50	0.50	0.80	91.63	0.51	1.56	97.04	0.51	2.91	91.13
	σ_{y_1}	0.50	0.49	-2.47	93.60	0.50	-0.78	97.04	0.49	-1.55	95.57
	σ_{y_2}	0.50	0.50	0.37	95.57	0.51	1.78	96.06	0.49	-1.10	95.07
	σ_{y_3}	0.50	0.50	-0.21	92.61	0.49	-1.07	95.57	0.49	-1.03	94.58
	σ_{ξ_1}	1.00	1.02	2.22	95.07	1.02	2.17	89.66	1.02	2.39	94.09
	σ_{ξ_2}	1.00	1.01	0.62	92.61	1.01	1.31	92.12	0.99	-0.55	96.55
	ν_{x_1}	0.00	-0.03	-2.53	96.06	-0.02	-2.24	97.04	0.00	0.40	95.57
	ν_{x_2}	0.00	-0.01	-1.18	98.52	-0.02	-2.37	95.57	0.00	0.10	98.52
	ν_{x_3}	0.00	-0.01	-1.43	94.58	-0.02	-2.32	96.55	0.00	-0.39	99.01
	ν_{x_4}	0.00	-0.01	-0.61	97.54	-0.02	-1.63	93.10	0.01	0.90	94.58
	ν_{x_5}	0.00	-0.01	-0.68	93.60	-0.02	-1.95	98.52	0.00	-0.19	95.57
	ν_{x_6}	0.00	-0.01	-0.98	97.54	-0.01	-1.22	97.54	0.00	-0.09	94.58
	ν_{y_2}	0.00	0.00	-0.15	96.06	0.00	-0.33	99.01	-0.02	-1.65	96.55
	ν_{y_3}	0.00	0.00	-0.02	95.57	0.00	0.31	91.13	-0.01	-1.22	88.18
	λ_{x_2}	1.00	1.02	2.19	96.55	1.02	1.61	95.07	1.01	0.55	93.10
	λ_{x_3}	1.00	1.00	0.22	95.07	1.02	1.96	98.03	1.01	1.38	95.07
	λ_{x_4}	1.00	1.02	1.92	95.07	1.01	1.35	94.09	1.02	2.28	97.54
	λ_{x_5}	1.00	1.02	2.48	94.58	1.02	2.44	95.57	1.02	1.82	96.55
	λ_{y_2}	1.00	0.90	-10.20	94.09	0.90	-9.76	93.60	0.91	-8.72	91.13
	λ_{y_3}	1.00	0.90	-9.64	91.63	0.90	-9.75	90.64	0.91	-9.31	93.10
	196										
	ρ_η	0.00	0.32	31.62	0.00	0.30	30.20	95.98	0.30	30.27	94.90
	ϕ_ζ	0.00	0.46	46.05	0.00	0.47	47.37	0.00	0.48	47.62	0.00
	α	0.00	-0.01	-0.64	99.00	0.00	-0.01	100.00	-0.01	-0.65	98.98
	γ_1	0.30	0.31	2.80	97.51	0.30	-0.72	96.98	0.31	3.31	91.84
	γ_2	0.30	0.31	2.28	97.51	0.31	3.84	95.48	0.31	4.09	95.41
	γ_3	0.15	0.15	-2.40	98.01	0.15	-2.86	97.99	0.16	4.49	94.90
	σ_{x_1}	0.50	0.51	2.22	96.52	0.50	0.56	90.45	0.50	0.06	92.86
	σ_{x_2}	0.50	0.50	-0.33	95.02	0.50	0.31	93.97	0.50	0.31	96.94
	σ_{x_3}	0.50	0.50	0.14	97.01	0.50	0.95	92.96	0.50	-0.43	95.41
	σ_{x_4}	0.50	0.50	0.36	97.51	0.50	0.56	93.97	0.51	1.04	95.41
	σ_{x_5}	0.50	0.50	0.44	94.03	0.50	0.47	95.48	0.50	0.37	96.43
	σ_{x_6}	0.50	0.50	0.55	95.02	0.50	0.42	94.97	0.50	-0.35	95.92
	σ_{y_1}	0.50	0.49	-1.46	94.53	0.49	-1.09	95.98	0.50	-0.70	94.90
	σ_{y_2}	0.50	0.50	-0.24	96.02	0.50	-0.48	96.48	0.49	-1.43	94.90
	σ_{y_3}	0.50	0.50	-0.79	95.02	0.50	-0.57	95.98	0.49	-1.20	96.43
	σ_{ξ_1}	1.00	1.00	0.23	92.04	1.00	-0.18	95.48	1.00	0.27	96.43
	σ_{ξ_2}	1.00	1.00	-0.01	93.53	1.01	0.95	94.97	1.00	0.33	95.41
	ν_{x_1}	0.00	-0.01	-1.05	97.51	0.00	0.36	94.97	0.01	0.73	92.86
	ν_{x_2}	0.00	-0.01	-1.29	97.01	0.00	0.47	97.99	0.00	0.23	93.37
	ν_{x_3}	0.00	-0.01	-0.98	98.01	0.01	0.83	96.48	0.00	0.07	93.37
	ν_{x_4}	0.00	0.00	-0.28	97.51	0.01	0.76	94.97	0.00	0.32	95.41
	ν_{x_5}	0.00	0.00	-0.06	96.02	0.00	0.19	90.45	0.00	0.11	94.90
	ν_{x_6}	0.00	0.00	0.13	96.52	0.00	0.05	94.97	0.00	-0.08	97.45
	ν_{y_2}	0.00	-0.01	-0.91	91.54	-0.01	-0.70	92.96	0.00	-0.16	93.37
	ν_{y_3}	0.00	-0.01	-0.53	93.53	0.00	-0.37	90.95	-0.01	-1.09	81.63
	λ_{x_2}	1.00	1.01	0.84	95.52	1.00	-0.35	96.48	1.00	0.47	97.45
	λ_{x_3}	1.00	1.01	1.34	95.52	1.00	0.45	96.48	1.01	0.91	94.39
	λ_{x_4}	1.00	1.01	1.03	93.53	1.00	-0.27	95.48	1.01	0.52	95.41
	λ_{x_5}	1.00	1.00	-0.03	97.01	1.01	0.63	93.47	1.01	1.18	97.45
	λ_{y_2}	1.00	0.98	-1.83	95.02	0.98	-2.34	95.48	0.99	-0.78	96.43
	λ_{y_3}	1.00	0.97	-3.06	92.54	0.98	-2.19	94.97	0.99	-1.34	92.86
	400										
	ρ_η	0.00	0.30	30.49	14.04	0.31	31.28	100.00	0.31	31.10	100.00

Table A.14: Results table for Study 4 measurement lag population model (D2) and Simultaneous structural lag analysis model (A4) *(continued)*

			$\rho_{y2}=0$			$\rho_{y2}=0.3$			$\rho_{y2}=0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
	ϕ_ζ	0.00	0.50	49.62	0.00	0.50	49.94	0.00	0.50	49.90	0.00
	α	0.00	0.00	0.43	100.00	-0.02	-2.38	100.00	0.00	-0.47	100.00
	γ_1	0.30	0.31	1.80	94.74	0.29	-3.38	99.02	0.31	1.78	94.90
	γ_2	0.30	0.30	-1.12	97.37	0.31	3.36	90.20	0.30	1.39	94.90
	γ_3	0.15	0.15	-1.91	95.61	0.16	4.67	99.02	0.16	4.92	96.94
	σ_{x_1}	0.50	0.50	0.71	96.49	0.51	1.51	96.08	0.50	0.80	93.88
	σ_{x_2}	0.50	0.50	0.48	99.12	0.51	1.36	96.08	0.50	-0.14	95.92
	σ_{x_3}	0.50	0.50	-0.99	92.11	0.50	0.59	97.06	0.50	0.79	96.94
	σ_{x_4}	0.50	0.50	-0.07	100.00	0.50	0.87	91.18	0.50	0.08	89.80
	σ_{x_5}	0.50	0.50	0.45	94.74	0.50	0.85	90.20	0.50	-0.67	95.92
	σ_{x_6}	0.50	0.50	-0.06	98.25	0.50	-0.42	96.08	0.50	0.12	94.90
	σ_{y_1}	0.50	0.50	-0.81	92.98	0.50	0.04	91.18	0.49	-1.26	93.88
	σ_{y_2}	0.50	0.49	-1.18	89.47	0.50	-0.45	96.08	0.49	-1.23	90.82
	σ_{y_3}	0.50	0.50	-0.57	96.49	0.49	-1.49	90.20	0.49	-1.45	91.84
	σ_{ξ_1}	1.00	1.00	0.12	95.61	1.00	0.18	97.06	1.00	0.09	98.98
	σ_{ξ_2}	1.00	1.00	-0.11	90.35	0.99	-0.78	94.12	1.01	0.69	94.90
	ν_{x_1}	0.00	0.00	0.17	92.98	0.00	-0.25	96.08	-0.01	-0.56	89.80
	ν_{x_2}	0.00	0.00	0.18	91.23	0.00	-0.18	92.16	0.00	0.16	93.88
	ν_{x_3}	0.00	0.00	0.44	88.60	0.00	-0.14	87.25	-0.01	-0.52	90.82
	ν_{x_4}	0.00	0.01	0.58	91.23	0.00	-0.30	93.14	0.00	0.50	97.96
	ν_{x_5}	0.00	0.00	0.30	92.11	0.00	-0.27	98.04	0.00	0.03	100.00
	ν_{x_6}	0.00	0.00	-0.35	86.84	0.00	-0.44	96.08	-0.01	-0.82	98.98
	ν_{y_2}	0.00	0.00	0.14	93.86	0.00	0.08	90.20	0.00	-0.39	95.92
	ν_{y_3}	0.00	0.00	-0.43	96.49	-0.01	-0.68	85.29	0.00	-0.37	89.80
	λ_{x_2}	1.00	1.01	0.60	99.12	1.00	-0.21	96.08	1.01	0.77	91.84
	λ_{x_3}	1.00	1.01	0.71	91.23	1.00	-0.28	98.04	1.01	1.01	91.84
	λ_{x_4}	1.00	0.99	-0.56	92.98	1.01	0.98	98.04	1.00	-0.10	95.92
	λ_{x_5}	1.00	1.00	-0.21	94.74	1.01	0.92	94.12	1.01	0.88	97.96
	λ_{y_2}	1.00	0.99	-1.04	99.12	0.98	-1.82	99.02	0.98	-1.53	93.88
	λ_{y_3}	1.00	0.99	-0.79	98.25	0.97	-3.07	94.12	0.98	-1.59	95.92

[1] W^* is the population and analysis W condition.

[2] N is the simulated sample size.

[3] θ is the simulated population value of each parameter.

[4] $\rho_\eta = ...$ is the simulated spatial autocorrelation of the endogenous lag.

[5] $\bar{\hat{\theta}}$ is the obtained average posterior mean across simulated iterations.

[6] $Bias(\bar{\hat{\theta}})\%$ is the average percent bias across simulation iterations when $\theta \neq 0$, otherwise $Bias(\bar{\hat{\theta}})\%$ provides absolute bias.

[7] Cover% is the percent of simulated trials in which the population value falls within the central 95% density of the posterior estimate.

Table A.15: Results table for Study 4 endogenous structural lag population model (D3) and measurement lag analysis model (A2)

				$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
W_C^*												
	49											
		ρ_{y2}	0.00	0.44	44.30	0.00	0.45	44.56	100.00	0.45	45.23	100.00
		α	0.00	0.00	-0.15	95.20	0.00	0.28	90.80	0.00	-0.46	86.67
		γ_1	0.30	0.31	1.76	95.20	0.30	0.40	95.47	0.31	2.88	95.20
		γ_2	0.30	0.30	0.26	96.80	0.31	3.76	96.13	0.30	-1.38	96.13
		γ_3	0.15	0.14	-5.25	95.20	0.16	8.39	96.53	0.16	8.56	95.07
		σ_{x_1}	0.50	0.51	1.94	96.27	0.51	1.48	94.80	0.51	2.02	94.67
		σ_{x_2}	0.50	0.50	0.56	95.87	0.50	0.60	94.13	0.50	0.36	94.40
		σ_{x_3}	0.50	0.50	0.42	92.93	0.50	0.86	96.80	0.50	0.89	95.20
		σ_{x_4}	0.50	0.51	1.64	94.40	0.51	1.31	94.40	0.51	1.72	95.07
		σ_{x_5}	0.50	0.51	1.99	93.60	0.50	0.76	93.60	0.50	0.91	96.93
		σ_{x_6}	0.50	0.51	1.04	95.87	0.50	-0.24	96.27	0.50	0.80	94.67
		σ_{y_1}	0.50	0.50	-0.53	94.13	0.49	-1.26	95.73	0.50	0.24	95.47
		σ_{y_2}	0.50	0.51	2.51	95.60	0.51	1.62	93.60	0.50	-0.18	94.27
		σ_{y_3}	0.50	0.49	-1.05	93.47	0.39	-22.10	96.00	0.38	-23.85	94.00
		σ_{ξ_1}	1.00	1.00	0.49	96.13	1.01	1.38	95.73	1.01	0.58	96.13
		σ_{ξ_2}	1.00	1.00	0.00	96.53	1.00	0.41	95.60	1.00	-0.39	96.93
		ν_{x_1}	0.00	-0.01	-0.78	94.53	0.00	-0.27	95.73	-0.01	-0.85	96.40
		ν_{x_2}	0.00	-0.01	-0.66	94.53	0.00	-0.11	95.33	0.00	-0.05	97.07
		ν_{x_3}	0.00	-0.01	-0.81	92.53	0.00	-0.22	95.33	0.00	-0.12	96.00
		ν_{x_4}	0.00	-0.01	-1.18	95.47	0.00	-0.16	96.67	0.00	-0.27	96.13
		ν_{x_5}	0.00	-0.01	-1.07	96.00	0.00	0.16	95.33	0.00	-0.17	95.20
		ν_{x_6}	0.00	-0.01	-0.95	95.87	0.00	-0.46	96.13	0.00	0.28	95.47
		ν_{y_2}	0.00	0.00	-0.22	95.20	0.00	0.07	95.07	0.00	-0.34	93.73
		ν_{y_3}	0.00	0.00	-0.31	96.80	0.00	-0.06	98.27	0.00	-0.44	98.93
		λ_{x_2}	1.00	1.03	2.65	96.00	1.02	2.25	95.87	1.02	2.24	94.80
		λ_{x_3}	1.00	1.02	2.37	96.67	1.02	2.42	95.60	1.02	2.06	95.07
		λ_{x_4}	1.00	1.03	2.55	95.20	1.01	1.14	94.27	1.02	2.23	95.73
		λ_{x_5}	1.00	1.03	2.82	95.87	1.02	2.43	94.27	1.02	2.08	95.73
		λ_{y_2}	1.00	0.98	-1.64	93.87	0.98	-1.63	93.73	0.99	-0.81	94.93
		λ_{y_3}	1.00	0.99	-1.34	95.33	0.98	-2.41	94.27	0.98	-1.64	95.20
	196											
		ρ_{y2}	0.00	0.39	39.32	0.00	0.40	39.52	100.00	0.40	39.50	100.00
		α	0.00	0.00	-0.15	95.35	0.00	-0.01	91.81	0.00	0.28	83.05
		γ_1	0.30	0.29	-1.96	94.50	0.30	0.46	94.92	0.30	1.48	94.77
		γ_2	0.30	0.29	-1.88	96.47	0.30	-1.09	94.77	0.30	0.70	95.48
		γ_3	0.15	0.15	-1.00	94.08	0.15	2.74	95.06	0.15	-1.76	95.48
		σ_{x_1}	0.50	0.50	0.61	94.36	0.50	0.92	94.77	0.51	1.02	94.49
		σ_{x_2}	0.50	0.50	0.34	94.78	0.50	0.54	95.34	0.51	1.22	94.21
		σ_{x_3}	0.50	0.50	0.48	95.63	0.50	0.28	93.79	0.50	-0.10	93.36
		σ_{x_4}	0.50	0.50	0.68	95.35	0.50	0.45	95.62	0.50	0.55	95.34
		σ_{x_5}	0.50	0.50	0.25	93.37	0.50	0.47	96.75	0.50	-0.02	95.34
		σ_{x_6}	0.50	0.50	0.55	92.52	0.50	0.15	96.89	0.51	1.05	94.49
		σ_{y_1}	0.50	0.50	0.35	93.51	0.50	0.30	94.49	0.50	-0.07	95.62
		σ_{y_2}	0.50	0.50	0.23	93.94	0.50	0.89	95.90	0.50	0.54	93.64
		σ_{y_3}	0.50	0.51	1.08	89.00	0.40	-20.17	92.51	0.40	-20.31	88.70
		σ_{ξ_1}	1.00	1.00	0.10	94.92	1.00	0.10	95.48	1.00	0.09	94.77
		σ_{ξ_2}	1.00	1.01	0.69	95.77	1.00	0.13	93.36	1.00	0.01	95.48
		ν_{x_1}	0.00	0.00	0.36	96.47	0.00	0.08	95.48	0.00	0.47	95.06
		ν_{x_2}	0.00	0.00	0.20	95.06	0.00	-0.16	95.62	0.00	0.17	95.34
		ν_{x_3}	0.00	0.00	0.46	97.04	0.00	0.10	96.47	0.00	0.31	96.61
		ν_{x_4}	0.00	0.00	-0.39	95.49	0.00	-0.24	94.07	0.00	0.08	94.92
		ν_{x_5}	0.00	0.00	-0.33	95.63	0.00	-0.16	95.06	0.00	-0.24	95.20
		ν_{x_6}	0.00	0.00	-0.27	94.78	0.00	-0.33	95.20	0.00	0.12	93.36

Table A.15: Results table for Study 4 endogenous structural lag population model (D3) and measurement lag analysis model (A2) *(continued)*

				$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
		ν_{y_2}	0.00	0.00	-0.37	96.61	0.00	-0.04	97.60	0.00	0.01	95.48
		ν_{y_3}	0.00	0.00	0.11	96.76	0.00	-0.10	95.90	0.00	-0.07	95.90
		λ_{x_2}	1.00	1.01	0.74	96.61	1.01	1.07	93.64	1.01	0.52	94.49
		λ_{x_3}	1.00	1.00	0.25	93.51	1.01	0.63	94.35	1.01	1.01	95.90
		λ_{x_4}	1.00	1.00	0.47	96.19	1.00	0.47	95.48	1.01	1.02	94.35
		λ_{x_5}	1.00	1.01	0.62	94.08	1.00	0.37	96.05	1.01	0.76	94.49
		λ_{y_2}	1.00	1.00	-0.02	96.05	1.00	-0.15	95.34	1.00	-0.04	93.50
		λ_{y_3}	1.00	1.00	0.29	95.91	0.99	-0.54	95.48	1.00	-0.42	95.62
	400											
		ρ_{y2}	0.00	0.37	36.70	0.00	0.37	37.10	100.00	0.38	37.71	100.00
		α	0.00	0.00	0.24	94.93	0.00	-0.34	91.84	0.00	0.27	82.89
		γ_1	0.30	0.30	0.56	95.46	0.30	1.12	94.92	0.30	1.11	93.85
		γ_2	0.30	0.30	1.08	94.53	0.30	-1.12	94.79	0.30	-0.67	95.99
		γ_3	0.15	0.15	1.29	94.93	0.15	0.67	95.45	0.14	-3.63	94.52
		σ_{x_1}	0.50	0.50	0.41	93.72	0.50	0.22	95.59	0.50	0.03	92.91
		σ_{x_2}	0.50	0.50	-0.03	94.93	0.50	0.39	94.52	0.50	0.50	94.65
		σ_{x_3}	0.50	0.50	0.17	93.99	0.50	0.21	95.19	0.50	0.01	96.79
		σ_{x_4}	0.50	0.50	0.35	96.80	0.50	0.26	93.85	0.50	0.52	95.19
		σ_{x_5}	0.50	0.50	0.12	97.06	0.50	0.42	94.65	0.50	0.11	95.45
		σ_{x_6}	0.50	0.50	0.08	93.99	0.50	-0.06	94.12	0.50	-0.27	95.32
		σ_{y_1}	0.50	0.50	-0.30	96.13	0.50	0.27	95.05	0.50	0.04	94.12
		σ_{y_2}	0.50	0.50	0.61	96.53	0.50	0.22	94.52	0.50	0.31	96.26
		σ_{y_3}	0.50	0.49	-1.87	87.98	0.41	-17.96	85.56	0.41	-18.98	85.96
		σ_{ξ_1}	1.00	1.00	0.27	93.19	1.00	0.04	93.72	1.00	0.15	95.45
		σ_{ξ_2}	1.00	1.00	0.45	96.40	1.00	0.34	94.12	1.00	0.09	94.12
		ν_{x_1}	0.00	0.00	-0.18	94.79	0.00	-0.19	94.92	0.00	0.26	95.59
		ν_{x_2}	0.00	0.00	-0.23	94.53	0.00	-0.03	95.86	0.00	0.02	94.39
		ν_{x_3}	0.00	0.00	-0.07	94.53	0.00	-0.03	95.86	0.00	0.07	95.05
		ν_{x_4}	0.00	0.00	-0.16	94.13	0.00	0.26	94.52	0.00	0.04	94.52
		ν_{x_5}	0.00	0.00	-0.14	95.33	0.00	0.22	94.79	0.00	-0.04	92.65
		ν_{x_6}	0.00	0.00	-0.21	95.33	0.00	0.22	95.19	0.00	0.18	94.65
		ν_{y_2}	0.00	0.00	-0.12	95.46	0.00	0.28	94.92	0.00	-0.04	95.45
		ν_{y_3}	0.00	0.00	-0.01	95.59	0.01	0.54	96.79	0.00	-0.25	95.05
		λ_{x_2}	1.00	1.00	0.15	95.86	1.00	0.43	93.72	1.00	0.37	94.79
		λ_{x_3}	1.00	1.00	0.19	93.99	1.00	0.39	95.72	1.00	0.12	95.86
		λ_{x_4}	1.00	1.00	0.22	95.73	1.00	0.14	93.72	1.01	0.55	94.79
		λ_{x_5}	1.00	1.00	0.26	95.19	1.00	0.29	93.98	1.00	0.49	95.45
		λ_{y_2}	1.00	1.00	-0.18	94.66	1.00	-0.15	94.25	1.00	0.21	93.98
		λ_{y_3}	1.00	1.00	-0.13	93.59	1.00	-0.08	94.79	1.00	-0.32	93.72
W_D^*												
	49											
		ρ_{y2}	0.00	0.50	49.77	0.00	0.50	49.82	100.00	0.50	49.93	100.00
		α	0.00	0.00	-0.26	95.45	0.00	-0.15	91.97	0.01	0.92	82.20
		γ_1	0.30	0.29	-1.98	94.65	0.30	0.04	97.05	0.30	1.13	96.25
		γ_2	0.30	0.29	-2.29	95.85	0.29	-2.27	94.91	0.30	-1.28	95.18
		γ_3	0.15	0.14	-6.10	95.98	0.16	6.31	95.72	0.16	4.05	96.92
		σ_{x_1}	0.50	0.51	1.90	93.31	0.51	2.62	95.31	0.52	3.32	93.71
		σ_{x_2}	0.50	0.51	1.93	93.98	0.50	0.21	92.50	0.50	0.66	94.65
		σ_{x_3}	0.50	0.50	0.83	95.18	0.51	1.39	94.11	0.51	1.42	94.65
		σ_{x_4}	0.50	0.51	2.01	93.71	0.51	2.41	95.45	0.51	1.15	95.58
		σ_{x_5}	0.50	0.50	-0.99	93.84	0.51	2.00	94.38	0.51	1.45	94.51
		σ_{x_6}	0.50	0.51	2.33	94.78	0.51	1.29	92.24	0.51	2.23	93.17
		σ_{y_1}	0.50	0.48	-3.20	94.38	0.49	-1.96	93.98	0.49	-1.35	94.24
		σ_{y_2}	0.50	0.51	2.12	95.45	0.50	-0.02	94.91	0.50	0.84	95.31

Table A.15: Results table for Study 4 endogenous structural lag population model (D3) and measurement lag analysis model (A2) *(continued)*

				$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
		σ_{y_3}	0.50	0.49	-1.40	93.98	0.35	-30.27	94.24	0.35	-29.91	94.91
		σ_{ξ_1}	1.00	1.01	0.59	94.91	1.01	1.04	94.65	1.01	0.74	93.31
		σ_{ξ_2}	1.00	1.01	1.32	93.04	1.02	2.42	94.65	1.01	1.12	95.05
		ν_{x_1}	0.00	0.00	0.00	95.18	0.00	-0.11	94.65	-0.01	-0.90	95.72
		ν_{x_2}	0.00	0.00	-0.29	94.78	0.00	-0.42	95.85	-0.01	-0.78	96.12
		ν_{x_3}	0.00	0.00	-0.32	95.72	0.00	-0.39	95.18	-0.01	-0.93	95.85
		ν_{x_4}	0.00	0.00	0.32	95.85	0.00	0.22	95.18	-0.01	-0.71	96.25
		ν_{x_5}	0.00	0.00	-0.45	95.18	-0.01	-0.51	96.25	0.00	-0.22	97.05
		ν_{x_6}	0.00	0.00	0.26	94.91	0.00	0.35	95.58	0.00	0.44	95.05
		ν_{y_2}	0.00	0.01	0.65	96.12	0.00	-0.12	94.38	0.00	0.47	95.18
		ν_{y_3}	0.00	0.00	0.03	99.20	0.00	-0.21	97.99	0.01	1.01	97.59
		λ_{x_2}	1.00	1.02	2.03	95.72	1.03	2.99	96.79	1.03	2.90	96.39
		λ_{x_3}	1.00	1.02	2.31	96.79	1.02	2.24	95.98	1.03	2.68	94.78
		λ_{x_4}	1.00	1.03	2.79	96.52	1.02	1.97	96.25	1.02	2.27	95.45
		λ_{x_5}	1.00	1.01	1.48	95.72	1.01	1.11	96.39	1.02	2.24	95.31
		λ_{y_2}	1.00	0.98	-1.81	95.18	0.99	-0.91	96.39	0.98	-1.58	94.38
		λ_{y_3}	1.00	0.98	-2.10	96.12	0.99	-1.37	94.78	0.98	-2.00	93.98
	196											
		ρ_{y2}	0.00	0.49	49.29	0.00	0.49	49.44	100.00	0.50	49.71	100.00
		α	0.00	0.00	0.03	96.59	0.00	-0.31	89.49	-0.01	-0.95	86.08
		γ_1	0.30	0.31	2.17	95.31	0.30	1.62	96.16	0.29	-2.18	94.89
		γ_2	0.30	0.30	-0.15	95.03	0.30	0.04	94.46	0.30	-0.52	94.32
		γ_3	0.15	0.15	-0.49	96.31	0.16	6.84	94.32	0.15	-1.03	95.45
		σ_{x_1}	0.50	0.51	1.19	94.74	0.50	0.66	95.17	0.50	0.45	95.17
		σ_{x_2}	0.50	0.50	-0.24	95.88	0.50	-0.06	94.89	0.50	0.26	94.89
		σ_{x_3}	0.50	0.50	-0.28	94.03	0.50	0.02	94.89	0.50	0.46	94.74
		σ_{x_4}	0.50	0.50	-0.01	93.32	0.51	1.10	93.32	0.51	1.27	94.89
		σ_{x_5}	0.50	0.50	0.43	94.60	0.50	0.57	94.46	0.50	0.63	94.60
		σ_{x_6}	0.50	0.50	0.74	94.46	0.50	0.46	96.02	0.50	0.54	93.18
		σ_{y_1}	0.50	0.50	0.28	94.03	0.50	0.23	94.74	0.50	0.18	94.18
		σ_{y_2}	0.50	0.50	0.49	92.90	0.50	-0.41	96.02	0.50	0.29	94.46
		σ_{y_3}	0.50	0.50	0.45	85.23	0.34	-32.23	86.22	0.34	-32.68	87.07
		σ_{ξ_1}	1.00	1.00	0.30	95.60	1.01	0.65	92.90	1.00	-0.17	95.17
		σ_{ξ_2}	1.00	1.00	0.14	95.17	1.00	0.42	95.17	1.00	-0.28	95.03
		ν_{x_1}	0.00	0.00	-0.11	96.73	0.00	-0.24	93.61	0.00	-0.32	94.89
		ν_{x_2}	0.00	0.00	0.13	94.32	-0.01	-0.56	94.60	-0.01	-0.56	94.46
		ν_{x_3}	0.00	0.00	0.01	94.60	0.00	-0.26	95.45	0.00	-0.20	95.88
		ν_{x_4}	0.00	0.00	-0.08	92.90	0.00	0.02	95.60	0.00	-0.23	96.31
		ν_{x_5}	0.00	0.00	0.08	94.60	0.00	-0.01	94.18	0.00	-0.49	94.60
		ν_{x_6}	0.00	0.00	-0.07	94.32	0.00	-0.47	95.17	0.00	-0.40	95.31
		ν_{y_2}	0.00	0.01	0.70	95.03	0.00	0.13	96.59	0.00	0.08	95.17
		ν_{y_3}	0.00	0.01	0.63	97.87	0.00	0.06	98.15	0.00	0.34	97.73
		λ_{x_2}	1.00	1.01	0.62	94.89	1.01	0.73	95.03	1.00	0.45	94.18
		λ_{x_3}	1.00	1.01	0.70	94.89	1.01	0.74	93.89	1.01	1.01	95.60
		λ_{x_4}	1.00	1.01	0.62	95.74	1.00	0.02	95.17	1.01	0.66	93.32
		λ_{x_5}	1.00	1.00	0.21	93.89	1.00	0.27	95.31	1.01	0.80	93.18
		λ_{y_2}	1.00	1.00	0.00	95.45	1.00	-0.06	95.74	1.00	-0.14	93.61
		λ_{y_3}	1.00	1.00	-0.12	96.02	1.00	-0.36	95.17	1.00	-0.16	94.74
	400											
		ρ_{y2}	0.00	0.49	49.02	0.00	0.49	49.21	100.00	0.49	49.11	100.00
		α	0.00	0.00	-0.04	94.35	0.00	0.10	87.23	0.00	0.01	85.35
		γ_1	0.30	0.30	-0.24	94.49	0.30	-0.37	95.16	0.31	2.60	95.03
		γ_2	0.30	0.30	0.22	96.64	0.30	0.91	94.49	0.30	-0.56	95.03
		γ_3	0.15	0.16	5.43	92.88	0.15	-0.85	95.97	0.15	1.58	94.35
		σ_{x_1}	0.50	0.50	0.18	95.83	0.50	0.43	93.95	0.50	-0.23	93.68

Table A.15: Results table for Study 4 endogenous structural lag population model (D3) and measurement lag analysis model (A2) *(continued)*

			$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
	σ_{x_2}	0.50	0.50	0.17	96.24	0.50	-0.04	95.16	0.50	0.24	95.03
	σ_{x_3}	0.50	0.50	-0.23	94.22	0.50	-0.06	96.24	0.50	0.21	93.55
	σ_{x_4}	0.50	0.50	0.52	93.28	0.50	0.45	96.77	0.50	0.53	94.22
	σ_{x_5}	0.50	0.50	0.11	94.35	0.50	0.51	95.97	0.50	0.52	95.03
	σ_{x_6}	0.50	0.50	0.23	96.24	0.50	0.14	94.62	0.50	-0.52	93.95
	σ_{y_1}	0.50	0.50	0.24	95.56	0.50	-0.06	95.43	0.50	-0.22	95.16
	σ_{y_2}	0.50	0.50	0.47	95.56	0.50	0.33	94.62	0.50	0.61	94.76
	σ_{y_3}	0.50	0.50	-0.49	77.42	0.34	-32.39	75.13	0.34	-32.98	75.13
	σ_{ξ_1}	1.00	1.00	0.24	92.34	1.00	0.00	93.95	1.00	0.13	95.16
	σ_{ξ_2}	1.00	1.00	0.19	96.10	1.00	0.12	95.43	1.00	-0.13	96.64
	ν_{x_1}	0.00	0.00	0.04	93.95	0.00	0.02	95.56	0.00	0.05	94.76
	ν_{x_2}	0.00	0.00	0.02	97.04	0.00	0.04	95.83	0.00	0.17	95.43
	ν_{x_3}	0.00	0.00	-0.04	95.70	0.00	-0.01	96.10	0.00	0.05	96.24
	ν_{x_4}	0.00	0.00	0.14	94.09	0.00	0.32	95.03	0.00	-0.27	91.26
	ν_{x_5}	0.00	0.00	0.19	95.83	0.00	0.18	95.03	0.00	-0.21	93.28
	ν_{x_6}	0.00	0.00	0.22	93.41	0.00	0.34	94.89	0.00	-0.15	95.16
	ν_{y_2}	0.00	0.00	-0.15	93.95	0.00	-0.05	94.76	0.00	0.02	95.16
	ν_{y_3}	0.00	0.00	0.04	96.77	0.00	-0.10	97.04	0.00	0.08	96.77
	λ_{x_2}	1.00	1.00	0.16	94.76	1.00	0.45	95.30	1.00	0.19	94.22
	λ_{x_3}	1.00	1.00	0.33	94.09	1.00	0.30	93.55	1.00	0.16	95.56
	λ_{x_4}	1.00	1.00	0.27	96.51	1.00	-0.18	94.35	1.00	0.34	95.16
	λ_{x_5}	1.00	1.00	0.17	95.70	1.00	-0.01	94.62	1.00	0.21	95.97
	λ_{y_2}	1.00	1.00	0.00	94.49	1.00	-0.39	94.76	0.99	-0.61	93.68
	λ_{y_3}	1.00	1.00	0.00	93.41	1.00	-0.12	93.95	0.99	-0.51	94.89

[1] W^* is the population and analysis W condition.

[2] N is the simulated sample size.

[3] θ is the simulated population value of each parameter.

[4] $\rho_\eta = ...$ is the simulated spatial autocorrelation of the endogenous lag.

[5] $\bar{\hat{\theta}}$ is the obtained average posterior mean across simulated iterations.

[6] $Bias(\bar{\hat{\theta}})\%$ is the average percent bias across simulation iterations when $\theta \neq 0$, otherwise $Bias(\bar{\hat{\theta}})\%$ provides absolute bias.

[7] Cover% is the percent of simulated trials in which the population value falls within the central 95% density of the posterior estimate.

Table A.16: Results table for Study 4 endogenous structural lag population model (D3) and simultaneous structural lag analysis model (A4)

				$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
W_C^*												
	49											
		ρ_η	-	0.15	15.06	0.00	0.25	-17.35	99.46	0.41	-31.28	86.02
		ϕ_ζ	0.00	0.36	36.25	0.00	0.36	35.99	0.00	0.36	36.07	0.00
		α	0.00	0.00	0.23	99.46	0.01	0.99	98.92	0.00	0.09	95.70
		γ_1	0.30	0.32	5.42	97.85	0.31	3.84	96.24	0.32	6.82	97.85
		γ_2	0.30	0.33	9.83	97.31	0.32	6.51	95.70	0.33	8.97	96.77
		γ_3	0.15	0.17	10.51	95.16	0.16	8.71	98.39	0.16	9.37	96.77
		σ_{x_1}	0.50	0.50	0.29	94.09	0.50	0.64	92.47	0.50	0.33	96.24
		σ_{x_2}	0.50	0.49	-1.16	94.62	0.51	2.98	98.39	0.51	2.92	97.31
		σ_{x_3}	0.50	0.51	1.34	93.55	0.50	0.10	93.55	0.51	1.12	93.55
		σ_{x_4}	0.50	0.50	-0.86	93.55	0.51	2.91	95.70	0.50	-0.85	92.47
		σ_{x_5}	0.50	0.51	1.94	93.55	0.50	0.05	95.70	0.50	0.29	94.09
		σ_{x_6}	0.50	0.51	1.36	91.94	0.51	1.84	92.47	0.51	1.35	95.70
		σ_{y_1}	0.50	0.50	-0.32	95.70	0.49	-1.71	94.62	0.50	0.34	96.77
		σ_{y_2}	0.50	0.50	-0.74	94.09	0.52	3.08	96.24	0.50	-0.33	96.24
		σ_{y_3}	0.50	0.51	1.54	96.77	0.50	0.91	99.46	0.50	0.46	96.24
		σ_{ξ_1}	1.00	1.01	1.27	93.01	1.02	1.85	94.62	0.99	-0.60	96.24
		σ_{ξ_2}	1.00	1.00	0.26	92.47	1.00	0.39	91.94	1.00	0.16	90.32
		ν_{x_1}	0.00	0.01	1.16	94.09	0.00	0.30	96.24	0.00	-0.42	95.70
		ν_{x_2}	0.00	0.01	1.29	95.16	0.01	1.01	97.31	0.00	0.12	94.62
		ν_{x_3}	0.00	0.01	0.92	97.85	0.01	1.47	98.92	0.00	0.26	91.40
		ν_{x_4}	0.00	-0.01	-1.46	94.62	0.01	1.06	93.01	-0.01	-0.54	91.40
		ν_{x_5}	0.00	-0.01	-1.34	95.70	0.01	0.51	94.09	-0.01	-0.62	95.70
		ν_{x_6}	0.00	-0.01	-1.08	96.24	0.01	1.22	96.24	0.00	0.11	94.09
		ν_{y_2}	0.00	0.00	-0.31	94.62	-0.01	-0.59	96.77	0.01	0.69	95.70
		ν_{y_3}	0.00	0.00	-0.32	95.16	0.00	-0.41	95.70	0.00	0.05	95.70
		λ_{x_2}	1.00	1.02	1.66	95.16	1.03	3.24	94.62	1.01	1.29	93.01
		λ_{x_3}	1.00	1.01	0.79	95.70	1.02	2.44	96.24	1.02	1.81	96.77
		λ_{x_4}	1.00	1.02	1.95	95.16	1.02	1.76	95.16	1.02	1.98	94.62
		λ_{x_5}	1.00	1.03	2.60	95.70	1.02	2.02	97.31	1.02	1.66	94.62
		λ_{y_2}	1.00	0.90	-9.75	88.71	0.94	-6.39	91.40	0.93	-7.44	93.01
		λ_{y_3}	1.00	0.90	-9.63	95.70	0.91	-9.48	92.47	0.96	-4.17	93.55
	196											
		ρ_η	-	0.10	10.03	0.00	0.25	-15.58	91.94	0.54	-10.08	94.02
		ϕ_ζ	0.00	0.31	31.19	0.00	0.29	29.20	0.00	0.27	26.50	0.00
		α	0.00	0.00	-0.10	95.16	0.00	0.35	91.94	0.00	-0.08	94.02
		γ_1	0.30	0.30	0.17	96.24	0.30	0.35	95.70	0.30	1.59	98.37
		γ_2	0.30	0.30	1.20	94.62	0.30	-0.35	95.16	0.31	2.50	95.11
		γ_3	0.15	0.15	-1.60	91.94	0.15	0.34	94.62	0.15	0.75	98.37
		σ_{x_1}	0.50	0.51	1.04	95.16	0.50	0.45	92.47	0.50	0.24	94.02
		σ_{x_2}	0.50	0.50	0.25	97.31	0.50	-0.17	96.24	0.50	0.47	96.20
		σ_{x_3}	0.50	0.51	1.19	94.09	0.50	0.56	97.31	0.50	0.15	94.57
		σ_{x_4}	0.50	0.50	0.37	97.85	0.50	-0.27	98.39	0.50	-0.49	92.93
		σ_{x_5}	0.50	0.50	0.56	96.77	0.50	0.97	94.09	0.50	-0.45	95.65
		σ_{x_6}	0.50	0.50	-0.05	93.55	0.50	0.25	93.01	0.50	0.74	91.30
		σ_{y_1}	0.50	0.49	-1.31	95.16	0.50	-0.64	94.09	0.50	-0.75	94.02
		σ_{y_2}	0.50	0.50	-0.45	95.70	0.50	-0.73	94.62	0.50	-0.97	91.30
		σ_{y_3}	0.50	0.50	0.18	95.16	0.50	0.20	94.09	0.49	-1.10	92.93
		σ_{ξ_1}	1.00	1.00	-0.31	91.94	1.00	0.15	93.01	1.00	-0.09	91.30
		σ_{ξ_2}	1.00	1.00	0.38	96.24	1.00	0.36	96.24	1.00	-0.19	96.20
		ν_{x_1}	0.00	0.00	0.46	96.24	0.00	-0.20	95.16	0.00	0.32	95.65
		ν_{x_2}	0.00	0.01	0.60	92.47	0.00	-0.23	96.24	0.00	-0.01	94.57
		ν_{x_3}	0.00	0.01	0.54	93.55	0.00	-0.33	94.62	0.00	-0.11	95.65
		ν_{x_4}	0.00	0.00	-0.13	94.62	0.00	0.39	96.77	0.00	0.15	91.85

Table A.16: Results table for Study 4 endogenous structural lag population model (D3) and simultaneous structural lag analysis model (A4) *(continued)*

			$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
		ν_{x_5} 0.00	0.00	-0.36	94.09	0.01	0.66	96.24	0.00	0.05	94.02
		ν_{x_6} 0.00	0.00	-0.32	95.70	0.01	0.62	96.77	0.01	0.56	92.39
		ν_{y_2} 0.00	0.00	0.29	97.31	0.00	-0.26	92.47	0.00	-0.11	92.39
		ν_{y_3} 0.00	0.01	0.74	95.70	0.00	0.22	94.62	-0.01	-0.64	93.48
		λ_{x_2} 1.00	1.01	0.79	93.01	1.00	0.28	97.31	1.01	0.53	97.28
		λ_{x_3} 1.00	1.00	0.06	95.70	1.01	1.14	96.77	1.00	0.31	92.39
		λ_{x_4} 1.00	1.00	0.38	94.09	1.00	0.28	96.24	1.01	0.91	92.93
		λ_{x_5} 1.00	1.01	0.73	95.70	1.00	0.14	95.16	1.01	1.11	95.11
		λ_{y_2} 1.00	0.97	-3.28	94.47	0.98	-1.51	97.85	0.97	-2.84	93.48
		λ_{y_3} 1.00	0.98	-2.40	93.55	0.98	-2.10	96.77	0.97	-2.63	90.76
	400										
		ρ_η -	0.14	14.81	0.00	0.32	-9.70	97.12	0.57	-5.60	92.08
		ϕ_ζ 0.00	0.30	29.52	0.00	0.27	27.34	16.35	0.25	24.56	16.83
		α 0.00	0.01	0.54	94.44	0.00	0.18	92.31	0.00	0.46	95.05
		γ_1 0.30	0.30	1.08	94.44	0.30	0.86	97.12	0.31	3.16	91.09
		γ_2 0.30	0.30	1.51	92.59	0.30	0.11	98.08	0.30	-0.81	93.07
		γ_3 0.15	0.15	1.42	99.07	0.14	-3.94	93.27	0.14	-4.43	92.08
		σ_{x_1} 0.50	0.50	0.68	97.22	0.50	-0.66	100.00	0.50	-0.33	98.02
		σ_{x_2} 0.50	0.50	-0.03	94.44	0.50	-0.54	93.27	0.50	0.69	97.03
		σ_{x_3} 0.50	0.50	-0.54	96.30	0.50	0.92	94.23	0.50	-0.78	97.03
		σ_{x_4} 0.50	0.50	-0.19	93.52	0.50	0.14	95.19	0.51	1.28	96.04
		σ_{x_5} 0.50	0.50	-0.09	95.37	0.50	0.15	97.12	0.50	-0.18	96.04
		σ_{x_6} 0.50	0.51	1.03	94.44	0.50	0.98	96.15	0.50	0.71	96.04
		σ_{y_1} 0.50	0.50	-0.59	94.44	0.49	-1.81	97.12	0.49	-1.29	94.06
		σ_{y_2} 0.50	0.49	-2.02	90.74	0.49	-1.39	89.42	0.49	-1.56	95.05
		σ_{y_3} 0.50	0.49	-1.06	95.37	0.50	-0.55	100.00	0.50	-0.60	95.05
		σ_{ξ_1} 1.00	1.00	0.21	96.30	1.01	0.70	96.15	1.00	0.43	98.02
		σ_{ξ_2} 1.00	1.00	0.08	95.37	1.00	0.47	96.15	0.99	-0.90	95.05
		ν_{x_1} 0.00	0.01	0.56	95.37	0.00	-0.03	95.19	0.00	-0.02	95.05
		ν_{x_2} 0.00	0.01	0.66	93.52	0.00	0.01	92.31	0.00	-0.27	99.01
		ν_{x_3} 0.00	0.01	1.05	95.37	0.00	0.06	92.31	0.00	0.39	99.01
		ν_{x_4} 0.00	0.00	-0.46	94.44	0.01	0.99	94.23	0.00	0.26	97.03
		ν_{x_5} 0.00	0.00	0.22	97.22	0.01	0.71	95.19	0.01	0.68	97.03
		ν_{x_6} 0.00	0.00	0.17	95.37	0.02	1.73	93.27	0.01	0.79	95.05
		ν_{y_2} 0.00	-0.01	-0.64	98.15	0.00	0.23	89.42	0.00	-0.01	99.01
		ν_{y_3} 0.00	0.00	-0.32	95.37	0.00	0.09	92.31	0.00	0.17	97.03
		λ_{x_2} 1.00	1.01	0.61	98.15	1.00	0.17	97.12	1.00	0.13	95.05
		λ_{x_3} 1.00	1.01	0.86	94.44	1.00	-0.22	96.15	1.00	0.00	94.06
		λ_{x_4} 1.00	1.00	0.19	94.44	1.00	-0.19	97.12	1.01	0.68	94.06
		λ_{x_5} 1.00	1.00	0.02	92.59	0.99	-0.52	93.27	1.00	0.42	96.04
		λ_{y_2} 1.00	1.01	0.55	97.22	0.99	-0.52	95.19	0.99	-1.42	90.10
		λ_{y_3} 1.00	0.99	-1.32	92.59	0.99	-0.83	93.27	0.98	-2.16	92.08
W_D^*											
	49										
		ρ_η -	0.33	33.42	0.00	0.33	9.90	100.00	0.34	-42.80	100.00
		ϕ_ζ 0.00	0.47	47.49	0.00	0.48	47.76	0.00	0.48	47.97	0.00
		α 0.00	-0.01	-0.52	100.00	0.01	0.92	100.00	0.00	0.40	100.00
		γ_1 0.30	0.32	8.28	93.41	0.33	8.72	95.81	0.33	8.69	97.60
		γ_2 0.30	0.33	9.26	96.41	0.33	9.64	97.01	0.32	7.35	95.21
		γ_3 0.15	0.16	4.24	97.01	0.16	8.33	98.80	0.16	7.66	97.01
		σ_{x_1} 0.50	0.51	1.94	92.22	0.50	-0.04	94.61	0.51	2.24	98.20
		σ_{x_2} 0.50	0.51	2.57	93.41	0.51	2.54	98.80	0.49	-1.88	92.81
		σ_{x_3} 0.50	0.50	0.48	96.41	0.51	1.70	97.60	0.52	3.53	94.61
		σ_{x_4} 0.50	0.52	3.01	94.61	0.50	-0.78	94.61	0.51	2.26	95.81

Table A.16: Results table for Study 4 endogenous structural lag population model (D3) and simultaneous structural lag analysis model (A4) *(continued)*

			$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
	σ_{x_5}	0.50	0.52	3.17	95.81	0.50	0.26	94.61	0.52	3.17	92.81
	σ_{x_6}	0.50	0.51	2.16	94.61	0.51	2.26	94.61	0.50	-0.69	97.60
	σ_{y_1}	0.50	0.50	-0.83	93.41	0.50	-0.80	95.21	0.49	-2.14	94.61
	σ_{y_2}	0.50	0.51	1.91	94.01	0.50	-0.73	98.20	0.50	0.08	95.21
	σ_{y_3}	0.50	0.50	0.94	92.22	0.50	0.50	98.80	0.51	1.55	95.81
	σ_{ξ_1}	1.00	1.02	1.91	95.81	1.02	2.47	95.81	1.01	1.43	96.41
	σ_{ξ_2}	1.00	1.00	-0.03	97.60	1.03	3.00	98.20	1.03	2.65	97.01
	ν_{x_1}	0.00	-0.01	-0.74	92.22	0.01	0.94	98.20	0.00	0.23	95.81
	ν_{x_2}	0.00	-0.01	-0.85	95.81	0.00	0.07	95.81	0.00	0.43	96.41
	ν_{x_3}	0.00	-0.01	-1.14	93.41	0.01	0.60	97.60	0.00	0.42	99.40
	ν_{x_4}	0.00	0.00	0.20	98.20	-0.02	-1.94	97.60	0.03	2.59	95.81
	ν_{x_5}	0.00	0.01	0.51	94.61	-0.02	-1.77	98.20	0.01	0.60	97.01
	ν_{x_6}	0.00	0.00	0.27	97.01	-0.02	-1.75	96.41	0.02	1.77	98.20
	ν_{y_2}	0.00	0.00	-0.43	94.01	-0.02	-1.94	96.41	-0.01	-0.84	95.21
	ν_{y_3}	0.00	0.01	0.73	96.41	-0.01	-1.06	94.01	0.00	-0.42	96.41
	λ_{x_2}	1.00	1.02	2.49	92.81	1.01	0.98	98.20	1.02	2.03	98.80
	λ_{x_3}	1.00	1.02	2.47	95.81	1.01	1.39	95.21	1.01	0.77	98.20
	λ_{x_4}	1.00	1.02	1.71	97.60	1.01	1.05	96.41	1.01	1.10	95.81
	λ_{x_5}	1.00	1.02	2.12	97.01	1.02	1.91	95.21	1.02	1.59	96.41
	λ_{y_2}	1.00	0.89	-10.60	95.41	0.94	-6.33	92.81	0.91	-8.91	90.42
	λ_{y_3}	1.00	0.91	-8.95	94.61	0.91	-9.23	95.81	0.92	-8.47	91.62
	196										
	ρ_η	-	0.30	29.62	0.00	0.32	6.70	98.80	0.36	-39.18	98.20
	ϕ_ζ	0.00	0.48	47.89	0.00	0.48	48.06	0.00	0.48	48.49	0.00
	α	0.00	0.01	0.72	99.40	0.00	0.32	100.00	-0.01	-0.73	100.00
	γ_1	0.30	0.30	1.61	94.61	0.31	2.17	97.01	0.30	1.44	95.21
	γ_2	0.30	0.31	3.97	92.22	0.31	2.63	95.21	0.30	-0.54	97.01
	γ_3	0.15	0.16	4.91	94.61	0.15	2.07	97.01	0.16	4.02	95.81
	σ_{x_1}	0.50	0.50	0.77	96.41	0.50	0.67	97.01	0.50	0.18	92.22
	σ_{x_2}	0.50	0.50	0.40	95.21	0.51	2.12	89.82	0.51	1.11	97.01
	σ_{x_3}	0.50	0.50	-0.52	94.01	0.50	0.38	91.62	0.50	0.08	95.81
	σ_{x_4}	0.50	0.50	0.67	94.01	0.50	0.26	96.41	0.50	0.76	97.01
	σ_{x_5}	0.50	0.51	1.43	95.21	0.50	0.76	94.01	0.51	1.26	95.81
	σ_{x_6}	0.50	0.50	0.58	94.61	0.50	0.83	96.41	0.51	1.20	94.61
	σ_{y_1}	0.50	0.50	-0.90	95.21	0.50	-0.99	93.41	0.49	-2.54	91.02
	σ_{y_2}	0.50	0.49	-1.39	93.41	0.49	-1.23	95.81	0.50	-0.26	97.60
	σ_{y_3}	0.50	0.49	-1.46	97.01	0.50	-0.61	94.61	0.50	0.19	96.41
	σ_{ξ_1}	1.00	1.00	-0.28	97.60	1.00	0.31	96.41	1.01	0.65	93.41
	σ_{ξ_2}	1.00	1.00	-0.31	97.01	1.01	0.78	95.21	1.01	0.74	94.61
	ν_{x_1}	0.00	0.00	-0.24	93.41	0.01	0.89	90.42	0.00	-0.21	94.01
	ν_{x_2}	0.00	0.00	-0.28	98.80	0.00	0.26	92.81	0.00	-0.13	95.21
	ν_{x_3}	0.00	0.00	0.33	97.60	0.01	0.54	90.42	-0.01	-0.64	92.81
	ν_{x_4}	0.00	-0.01	-0.95	97.01	0.00	-0.20	95.81	-0.01	-0.98	94.61
	ν_{x_5}	0.00	0.00	-0.42	97.60	-0.01	-0.66	97.60	-0.01	-1.03	94.61
	ν_{x_6}	0.00	-0.01	-0.58	97.01	-0.01	-1.02	94.61	-0.01	-0.80	97.01
	ν_{y_2}	0.00	0.00	-0.12	94.01	0.01	0.53	95.81	0.01	0.68	91.62
	ν_{y_3}	0.00	0.00	0.48	90.42	0.00	-0.23	95.81	0.01	0.63	96.41
	λ_{x_2}	1.00	1.01	1.18	97.01	1.00	0.23	96.41	0.99	-0.50	97.01
	λ_{x_3}	1.00	1.01	0.82	97.60	1.00	0.02	94.01	1.01	0.54	93.41
	λ_{x_4}	1.00	1.00	0.45	94.61	1.00	-0.21	95.81	1.00	0.05	97.01
	λ_{x_5}	1.00	1.01	0.67	97.60	1.00	-0.20	96.41	1.00	-0.35	94.01
	λ_{y_2}	1.00	0.99	-1.39	94.22	0.98	-2.46	94.61	0.97	-3.00	94.01
	λ_{y_3}	1.00	0.97	-2.60	96.41	0.96	-3.93	95.21	0.96	-4.42	97.01
	400										
	ρ_η	-	0.30	30.17	19.64	0.31	-7.94	100.00	0.33	-44.31	100.00

Table A.16: Results table for Study 4 endogenous structural lag population model (D3) and simultaneous structural lag analysis model (A4) *(continued)*

			$\rho_\eta = 0$			$\rho_\eta = 0.3$			$\rho_\eta = 0.6$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\theta)\%$	Cover%
	ϕ_ζ	0.00	0.49	49.40	19.64	0.49	49.28	20.37	0.49	49.48	21.15
	α	0.00	0.00	-0.20	100.00	0.00	0.25	100.00	0.01	0.58	100.00
	γ_1	0.30	0.31	1.88	92.86	0.31	2.21	92.59	0.31	2.20	98.08
	γ_2	0.30	0.31	1.88	94.64	0.31	3.45	98.15	0.32	5.20	94.23
	γ_3	0.15	0.15	-2.24	92.86	0.15	0.89	90.74	0.15	-0.93	100.00
	σ_{x_1}	0.50	0.50	0.23	98.21	0.50	-0.60	98.15	0.50	-0.40	96.15
	σ_{x_2}	0.50	0.50	0.93	96.43	0.49	-1.97	88.89	0.50	0.41	98.08
	σ_{x_3}	0.50	0.51	1.16	98.21	0.51	1.28	94.44	0.50	-0.12	94.23
	σ_{x_4}	0.50	0.49	-1.23	92.86	0.50	-0.14	92.59	0.50	-0.34	98.08
	σ_{x_5}	0.50	0.50	0.85	100.00	0.50	-0.59	100.00	0.50	-0.05	100.00
	σ_{x_6}	0.50	0.50	0.96	100.00	0.50	-0.72	98.15	0.50	-0.69	88.46
	σ_{y_1}	0.50	0.49	-2.62	89.29	0.49	-2.35	87.04	0.49	-2.61	90.38
	σ_{y_2}	0.50	0.50	-0.11	91.07	0.50	-0.93	88.89	0.50	-0.33	94.23
	σ_{y_3}	0.50	0.49	-1.76	94.64	0.50	-0.83	88.89	0.49	-2.05	92.31
	σ_{ξ_1}	1.00	1.01	1.05	92.86	1.01	0.90	92.59	1.02	1.87	90.38
	σ_{ξ_2}	1.00	1.00	-0.01	94.64	1.00	-0.46	94.44	1.00	-0.46	98.08
	ν_{x_1}	0.00	0.01	0.88	94.64	0.01	1.26	98.15	0.01	0.99	98.08
	ν_{x_2}	0.00	0.00	0.44	98.21	0.02	1.76	92.59	0.02	1.71	92.31
	ν_{x_3}	0.00	0.01	0.75	98.21	0.02	1.80	100.00	0.01	0.89	98.08
	ν_{x_4}	0.00	0.01	1.16	98.21	0.01	1.25	100.00	0.00	0.18	94.23
	ν_{x_5}	0.00	0.02	1.83	100.00	0.01	0.89	100.00	0.00	0.05	94.23
	ν_{x_6}	0.00	0.01	1.16	98.21	0.01	1.45	98.15	0.00	-0.23	92.31
	ν_{y_2}	0.00	0.00	0.10	96.43	0.01	0.92	100.00	0.01	0.70	100.00
	ν_{y_3}	0.00	-0.01	-0.67	98.21	0.00	0.43	98.15	-0.01	-0.63	98.08
	λ_{x_2}	1.00	1.01	0.90	100.00	1.02	1.55	100.00	1.00	0.36	94.23
	λ_{x_3}	1.00	1.01	1.02	96.43	1.00	0.08	94.44	0.99	-0.66	98.08
	λ_{x_4}	1.00	1.01	0.60	92.86	1.00	0.44	98.15	1.01	1.38	96.15
	λ_{x_5}	1.00	1.01	0.63	100.00	1.01	0.63	98.15	1.02	1.99	96.15
	λ_{y_2}	1.00	0.98	-2.16	91.07	0.96	-3.77	92.59	0.96	-4.47	92.31
	λ_{y_3}	1.00	0.98	-2.09	93.86	0.98	-1.73	96.30	0.98	-1.79	94.23

[1] W^* is the population and analysis W condition.

[2] N is the simulated sample size.

[3] θ is the simulated population value of each parameter.

[4] $\rho_\eta = ...$ is the simulated spatial autocorrelation of the endogenous lag.

[5] $\bar{\hat{\theta}}$ is the obtained average posterior mean across simulated iterations.

[6] $Bias(\bar{\hat{\theta}})\%$ is the average percent bias across simulation iterations when $\theta \neq 0$, otherwise $Bias(\bar{\hat{\theta}})\%$ provides absolute bias.

[7] Cover% is the percent of simulated trials in which the population value falls within the central 95% density of the posterior estimate.

Table A.17: Results table for Study 4 simultaneous structural lag population model (D4) and measurement lag analysis model (A2)

				$\rho_\eta = 0.3$						$\rho_\eta = 0.6$					
				$\phi_\zeta = 0.3$			$\phi_\zeta = 0.6$			$\phi_\zeta = 0.3$			$\phi_\zeta = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%
W_C^*															
	49														
		ρ_{y2}	0.00	0.45	44.63	100.00	0.45	44.67	99.75	0.46	46.29	100.00	0.46	46.37	100.00
		α	0.00	0.00	0.46	94.03	-0.01	-1.50	94.01	0.02	1.71	80.35	0.00	0.16	79.40
		γ_1	0.30	0.32	7.41	97.01	0.32	5.77	97.51	0.34	14.34	92.54	0.35	15.59	91.96
		γ_2	0.30	0.32	7.81	98.01	0.33	8.51	96.51	0.35	16.42	91.54	0.34	13.53	92.71
		γ_3	0.15	0.16	8.04	96.77	0.17	13.20	97.26	0.18	19.87	93.78	0.18	18.66	94.72
		σ_{x_1}	0.50	0.51	1.71	94.78	0.51	1.47	95.76	0.50	0.83	94.03	0.50	0.38	94.22
		σ_{x_2}	0.50	0.51	1.05	92.04	0.51	1.07	95.51	0.51	1.71	94.53	0.51	2.62	94.22
		σ_{x_3}	0.50	0.51	1.48	94.53	0.49	-1.05	92.77	0.51	1.59	94.28	0.50	0.64	94.47
		σ_{x_4}	0.50	0.51	1.86	93.03	0.51	2.78	92.52	0.51	2.41	94.03	0.51	2.42	95.98
		σ_{x_5}	0.50	0.50	0.23	96.77	0.51	1.48	95.76	0.51	1.40	96.02	0.50	0.88	94.47
		σ_{x_6}	0.50	0.51	1.32	93.03	0.50	0.10	96.26	0.50	0.66	94.53	0.51	1.67	95.98
		σ_{y_1}	0.50	0.50	-0.93	95.02	0.50	-0.46	95.76	0.51	1.76	95.52	0.50	-0.53	94.47
		σ_{y_2}	0.50	0.50	-0.35	94.53	0.50	-0.10	96.26	0.51	1.33	95.02	0.50	0.72	94.97
		σ_{y_3}	0.50	0.37	-26.32	90.30	0.37	-26.40	89.78	0.36	-27.40	89.30	0.37	-26.96	89.20
		σ_{ξ_1}	1.00	1.01	0.78	92.79	1.00	-0.08	95.51	1.01	1.30	95.27	1.02	2.10	94.47
		σ_{ξ_2}	1.00	1.01	1.07	95.77	1.00	0.36	95.51	1.01	1.09	93.78	1.01	0.74	94.97
		ν_{x_1}	0.00	0.00	-0.33	97.51	0.00	-0.40	96.01	0.03	2.91	96.27	0.01	0.50	95.98
		ν_{x_2}	0.00	0.00	-0.16	94.03	-0.01	-1.24	96.76	0.02	1.56	96.77	0.00	0.19	96.73
		ν_{x_3}	0.00	0.00	-0.49	98.01	-0.01	-0.88	96.76	0.03	2.65	95.77	0.00	0.07	96.48
		ν_{x_4}	0.00	0.00	-0.17	95.52	-0.01	-1.18	95.76	0.00	-0.44	95.77	-0.01	-0.83	95.23
		ν_{x_5}	0.00	0.00	0.44	96.52	-0.01	-1.16	96.51	0.00	0.07	96.52	0.00	-0.31	94.72
		ν_{x_6}	0.00	0.00	-0.23	97.26	-0.01	-1.29	97.01	-0.01	-0.75	95.52	0.00	-0.31	94.22
		ν_{y_2}	0.00	-0.01	-0.73	95.77	0.01	0.71	94.51	0.00	0.43	95.27	-0.01	-0.72	95.73
		ν_{y_3}	0.00	-0.01	-0.76	96.77	0.01	0.89	96.76	0.01	0.77	97.51	0.00	-0.09	96.98
		λ_{x_2}	1.00	1.02	2.04	95.52	1.03	2.70	96.01	1.02	2.27	95.77	1.01	1.20	95.73
		λ_{x_3}	1.00	1.02	2.04	96.52	1.03	3.37	95.51	1.02	2.46	95.27	1.02	2.02	96.48
		λ_{x_4}	1.00	1.02	2.35	95.27	1.03	3.09	95.76	1.02	1.60	95.27	1.03	2.57	96.73
		λ_{x_5}	1.00	1.02	1.74	96.52	1.03	3.03	95.26	1.02	1.62	95.27	1.03	2.88	95.23
		λ_{y_2}	1.00	0.93	-6.99	95.52	0.94	-5.71	95.26	0.93	-7.23	95.27	0.96	-4.29	94.22
		λ_{y_3}	1.00	0.90	-9.96	94.03	0.90	-10.29	92.77	0.90	-10.49	94.78	0.92	-8.15	90.70
	196														
		ρ_{y2}	0.00	0.41	41.48	100.00	0.41	41.15	100.00	0.44	43.98	100.00	0.44	44.06	100.00
		α	0.00	0.00	0.16	89.45	0.00	0.28	92.42	0.01	0.52	76.07	0.00	-0.25	77.24
		γ_1	0.30	0.31	4.26	93.97	0.31	4.54	95.96	0.33	10.15	88.92	0.33	10.69	85.17
		γ_2	0.30	0.31	3.60	93.97	0.31	3.22	93.43	0.33	9.80	85.39	0.33	10.63	83.38
		γ_3	0.15	0.16	3.59	95.48	0.15	3.00	95.20	0.17	11.71	92.95	0.17	11.24	91.05
		σ_{x_1}	0.50	0.50	0.87	94.97	0.50	0.54	94.70	0.50	0.97	94.71	0.50	-0.20	94.12
		σ_{x_2}	0.50	0.50	0.70	95.48	0.50	0.18	93.69	0.50	0.31	94.46	0.50	0.32	92.58
		σ_{x_3}	0.50	0.50	-0.23	92.96	0.50	0.03	92.93	0.50	0.33	94.21	0.50	0.47	94.88
		σ_{x_4}	0.50	0.50	0.11	93.72	0.50	0.51	92.17	0.50	0.50	94.71	0.50	0.61	97.19
		σ_{x_5}	0.50	0.50	1.00	94.97	0.50	0.34	94.70	0.50	0.09	95.97	0.50	0.22	95.65
		σ_{x_6}	0.50	0.50	-0.14	95.23	0.50	-0.13	94.70	0.50	0.75	96.22	0.50	0.31	95.91
		σ_{y_1}	0.50	0.50	-0.34	94.47	0.50	-0.72	94.19	0.50	-0.40	96.22	0.50	0.59	95.14
		σ_{y_2}	0.50	0.50	-0.06	96.73	0.50	-0.49	94.19	0.50	0.71	95.21	0.50	-0.09	95.65
		σ_{y_3}	0.50	0.39	-22.54	80.15	0.39	-22.58	80.81	0.37	-25.93	79.85	0.38	-24.99	81.33
		σ_{ξ_1}	1.00	1.00	-0.43	94.47	1.00	0.13	93.43	1.01	0.67	94.46	1.01	0.83	94.12
		σ_{ξ_2}	1.00	1.00	-0.14	96.23	1.00	0.22	95.71	1.01	0.85	96.47	1.00	0.27	93.86
		ν_{x_1}	0.00	0.00	0.09	92.71	0.00	0.38	94.95	0.00	0.07	96.22	-0.01	-0.73	95.40
		ν_{x_2}	0.00	0.00	0.00	94.47	0.01	0.54	93.94	0.00	-0.03	95.47	0.00	-0.46	94.63
		ν_{x_3}	0.00	0.00	0.17	94.72	0.00	0.42	94.95	0.00	0.13	94.71	0.00	-0.41	93.86
		ν_{x_4}	0.00	0.00	0.23	94.47	0.00	-0.26	93.18	0.01	0.97	95.72	0.00	-0.13	96.16

Table A.17: Results table for Study 4 simultaneous structural lag population model (D4) and measurement lag analysis model (A2) *(continued)*

				$\rho_\eta = 0.3$						$\rho_\eta = 0.6$					
				$\phi_\zeta = 0.3$			$\phi_\zeta = 0.6$			$\phi_\zeta = 0.3$			$\phi_\zeta = 0.3$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%
		ν_{x_5}	0.00	0.00	0.18	95.23	0.00	-0.09	94.19	0.01	0.75	95.21	0.00	0.24	93.86
		ν_{x_6}	0.00	0.00	0.25	94.47	0.00	-0.21	92.68	0.00	0.33	95.21	0.00	0.18	95.65
		ν_{y_2}	0.00	0.00	-0.08	94.47	0.00	0.10	96.72	0.00	-0.25	95.47	0.00	0.03	97.44
		ν_{y_3}	0.00	0.00	-0.03	94.47	0.00	-0.06	95.71	0.00	-0.26	96.47	0.00	0.04	96.68
		λ_{x_2}	1.00	1.00	0.44	95.73	1.00	0.33	94.95	1.00	0.11	92.44	1.00	0.46	95.40
		λ_{x_3}	1.00	1.01	0.85	96.23	1.01	0.78	95.20	1.00	-0.16	95.97	1.00	0.09	96.16
		λ_{x_4}	1.00	1.00	0.04	91.21	1.00	0.28	95.71	1.00	0.07	96.47	1.01	0.80	96.42
		λ_{x_5}	1.00	1.01	0.55	94.22	1.01	0.69	95.96	1.00	0.04	92.95	1.01	0.54	95.40
		λ_{y_2}	1.00	0.99	-1.39	95.48	0.98	-2.30	95.20	0.98	-1.90	94.71	0.99	-0.65	93.61
		λ_{y_3}	1.00	0.97	-2.89	94.72	0.98	-2.44	92.93	0.96	-3.60	93.20	0.97	-3.00	92.33
	400														
		ρ_{y2}	0.00	0.42	41.74	100.00	0.42	42.24	100.00	0.45	44.51	100.00	0.45	44.82	100.00
		α	0.00	0.00	-0.22	91.47	0.00	-0.15	88.98	0.00	-0.08	71.32	0.00	0.02	76.68
		γ_1	0.30	0.30	1.63	95.74	0.31	2.02	95.28	0.32	7.79	80.23	0.32	7.82	83.40
		γ_2	0.30	0.31	2.35	94.19	0.31	2.70	93.31	0.32	6.05	82.56	0.32	7.55	83.00
		γ_3	0.15	0.15	0.56	94.19	0.15	2.73	96.85	0.16	7.57	91.09	0.16	8.33	91.30
		σ_{x_1}	0.50	0.50	0.12	94.57	0.50	0.69	94.88	0.50	0.32	95.35	0.50	-0.19	93.68
		σ_{x_2}	0.50	0.50	-0.06	94.96	0.50	0.29	93.70	0.50	0.08	94.96	0.50	-0.24	97.23
		σ_{x_3}	0.50	0.50	0.07	92.64	0.50	-0.24	96.06	0.50	0.23	93.80	0.50	0.46	95.26
		σ_{x_4}	0.50	0.50	0.00	96.90	0.50	0.06	94.09	0.51	1.01	98.45	0.50	0.51	96.84
		σ_{x_5}	0.50	0.50	0.08	94.96	0.50	-0.18	95.67	0.50	0.33	93.80	0.50	0.51	92.09
		σ_{x_6}	0.50	0.50	0.77	93.41	0.50	0.30	96.85	0.50	-0.29	94.19	0.50	-0.44	96.05
		σ_{y_1}	0.50	0.49	-1.02	95.74	0.50	-0.74	94.09	0.50	0.13	96.12	0.50	0.42	94.86
		σ_{y_2}	0.50	0.50	0.05	94.19	0.50	-0.41	93.70	0.50	-0.52	97.29	0.50	0.26	98.02
		σ_{y_3}	0.50	0.38	-24.60	67.83	0.38	-23.60	69.29	0.37	-26.60	61.24	0.37	-26.36	62.45
		σ_{ξ_1}	1.00	1.00	0.41	95.35	1.00	0.27	93.31	1.00	0.31	93.41	0.99	-0.60	90.91
		σ_{ξ_2}	1.00	1.00	-0.40	96.51	1.00	0.10	94.09	1.01	0.89	96.51	0.99	-0.61	93.28
		ν_{x_1}	0.00	0.00	0.36	95.35	0.00	-0.35	92.13	0.00	-0.24	91.86	0.00	-0.18	96.05
		ν_{x_2}	0.00	0.00	0.10	93.41	0.00	-0.22	93.70	0.00	-0.48	90.70	0.00	0.29	94.07
		ν_{x_3}	0.00	0.00	0.04	94.19	-0.01	-0.59	92.52	0.00	-0.28	93.02	0.00	0.06	94.86
		ν_{x_4}	0.00	0.00	-0.32	95.74	0.00	-0.12	94.88	0.00	0.07	94.57	0.01	0.74	94.86
		ν_{x_5}	0.00	0.00	-0.22	95.74	0.00	-0.46	96.46	0.00	-0.13	94.57	0.00	0.30	93.68
		ν_{x_6}	0.00	0.00	-0.41	94.96	0.00	-0.21	95.67	0.00	-0.12	94.57	0.00	0.34	95.26
		ν_{y_2}	0.00	0.00	0.38	93.41	0.00	-0.37	93.70	0.00	0.01	93.41	0.00	0.34	96.44
		ν_{y_3}	0.00	0.00	-0.05	94.96	0.00	-0.02	95.28	0.00	0.00	94.96	0.00	0.36	94.47
		λ_{x_2}	1.00	1.00	0.27	96.12	1.00	0.14	98.03	1.00	0.46	94.19	1.01	0.79	97.23
		λ_{x_3}	1.00	1.00	0.28	96.90	1.00	0.29	96.06	1.00	0.28	96.12	1.00	0.22	95.65
		λ_{x_4}	1.00	1.00	0.44	97.29	1.01	0.62	92.91	1.00	-0.13	96.51	1.01	0.63	94.07
		λ_{x_5}	1.00	1.01	0.54	93.41	1.00	0.32	95.67	1.00	0.19	94.96	1.00	0.26	96.05
		λ_{y_2}	1.00	0.99	-0.58	96.12	0.98	-1.50	94.49	1.00	0.21	93.41	0.99	-1.17	92.49
		λ_{y_3}	1.00	0.99	-0.74	95.74	0.98	-1.75	94.88	0.99	-1.22	93.41	0.98	-2.25	93.68
W_D^*															
	49														
		ρ_{y2}	0.00	0.49	49.13	100.00	0.49	48.83	100.00	0.49	49.02	100.00	0.49	49.07	100.00
		α	0.00	0.01	0.96	97.01	0.01	0.87	92.22	0.00	-0.40	82.63	0.02	2.14	76.05
		γ_1	0.30	0.31	4.46	97.01	0.32	7.32	94.01	0.32	5.83	96.41	0.32	5.32	97.60
		γ_2	0.30	0.32	6.78	95.81	0.33	9.33	97.01	0.33	10.37	94.61	0.32	7.07	95.21
		γ_3	0.15	0.16	7.14	95.21	0.16	6.67	96.41	0.17	11.57	95.81	0.16	4.93	97.60
		σ_{x_1}	0.50	0.51	1.74	97.01	0.50	0.20	92.81	0.52	4.77	94.01	0.51	1.48	96.41
		σ_{x_2}	0.50	0.51	2.42	98.20	0.51	1.83	95.21	0.50	-0.40	92.22	0.50	-0.20	91.02
		σ_{x_3}	0.50	0.50	-0.68	95.21	0.52	3.39	94.61	0.50	-0.51	95.81	0.53	5.03	95.21
		σ_{x_4}	0.50	0.50	0.65	96.41	0.52	4.15	96.41	0.50	-0.43	94.01	0.51	1.13	91.02
		σ_{x_5}	0.50	0.51	1.19	92.81	0.50	-0.73	96.41	0.50	0.54	91.62	0.51	1.02	97.01

Table A.17: Results table for Study 4 simultaneous structural lag population model (D4) and measurement lag analysis model (A2) *(continued)*

				$\rho_\eta = 0.3$						$\rho_\eta = 0.6$					
				$\phi_\zeta = 0.3$			$\phi_\zeta = 0.6$			$\phi_\zeta = 0.3$			$\phi_\zeta = 0.3$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%
		σ_{x_6}	0.50	0.50	-0.76	96.41	0.51	2.94	92.81	0.52	3.00	94.01	0.50	0.56	94.61
		σ_{y_1}	0.50	0.49	-2.12	94.61	0.50	-0.16	94.01	0.50	-0.87	97.60	0.50	-0.10	98.20
		σ_{y_2}	0.50	0.50	0.79	95.21	0.50	-0.13	95.21	0.50	0.22	92.81	0.51	2.76	94.01
		σ_{y_3}	0.50	0.34	-31.91	83.23	0.34	-31.12	88.02	0.35	-30.89	91.62	0.34	-32.30	88.02
		σ_{ξ_1}	1.00	0.99	-0.82	92.22	1.01	1.25	96.41	1.00	-0.33	95.81	1.00	0.35	95.21
		σ_{ξ_2}	1.00	1.00	0.01	94.61	0.98	-1.93	96.41	1.01	1.43	94.01	1.01	0.67	95.21
		ν_{x_1}	0.00	0.02	1.97	98.20	0.02	2.02	97.01	0.01	1.46	95.21	0.02	1.52	98.20
		ν_{x_2}	0.00	0.02	2.06	95.81	0.02	2.37	95.21	0.02	1.55	97.01	0.01	0.58	93.41
		ν_{x_3}	0.00	0.01	1.44	97.01	0.02	1.97	98.80	0.01	0.96	97.01	0.01	1.08	96.41
		ν_{x_4}	0.00	-0.02	-1.64	98.20	0.00	0.44	93.41	-0.02	-1.89	95.21	0.02	1.65	96.41
		ν_{x_5}	0.00	-0.02	-1.74	98.80	0.01	1.21	95.21	-0.02	-1.88	93.41	0.02	2.46	94.01
		ν_{x_6}	0.00	-0.02	-1.89	96.41	0.01	1.06	97.01	-0.03	-2.51	95.21	0.01	1.18	94.61
		ν_{y_2}	0.00	0.00	-0.40	92.81	0.00	0.20	95.21	0.00	-0.33	94.61	-0.01	-1.15	96.41
		ν_{y_3}	0.00	-0.01	-0.59	98.80	0.01	1.25	99.40	0.00	-0.15	97.60	0.00	-0.23	97.01
		λ_{x_2}	1.00	1.02	1.62	98.80	1.03	2.65	95.81	1.04	3.96	96.41	1.04	3.58	91.62
		λ_{x_3}	1.00	1.03	2.72	98.80	1.02	2.46	95.81	1.05	4.92	95.81	1.03	3.32	95.81
		λ_{x_4}	1.00	1.02	1.86	95.81	1.04	4.22	95.81	1.01	1.09	97.01	1.02	2.02	97.60
		λ_{x_5}	1.00	1.02	2.25	94.01	1.02	2.41	96.41	1.01	0.73	96.41	1.01	1.00	94.61
		λ_{y_2}	1.00	0.95	-5.32	92.22	0.93	-6.84	94.01	0.89	-10.52	84.43	0.93	-7.38	92.81
		λ_{y_3}	1.00	0.95	-5.00	88.62	0.90	-9.82	92.81	0.86	-13.62	89.22	0.92	-8.08	92.22
	196														
		ρ_{y2}	0.00	0.48	47.87	100.00	0.48	47.71	100.00	0.48	48.04	100.00	0.48	47.80	100.00
		α	0.00	0.00	0.11	97.55	-0.01	-0.51	93.12	0.01	0.93	68.52	0.01	1.16	70.89
		γ_1	0.30	0.30	1.33	97.55	0.31	2.22	95.00	0.30	0.52	93.21	0.31	2.96	94.94
		γ_2	0.30	0.31	2.89	96.32	0.30	-0.89	94.38	0.30	-1.04	93.21	0.31	1.79	94.94
		γ_3	0.15	0.15	-1.51	95.09	0.15	1.37	97.50	0.15	0.14	93.83	0.16	4.63	94.94
		σ_{x_1}	0.50	0.50	0.50	93.25	0.50	0.92	96.88	0.51	1.41	95.68	0.50	0.21	94.94
		σ_{x_2}	0.50	0.50	0.03	93.87	0.50	0.52	96.88	0.50	-0.01	97.53	0.50	0.64	96.20
		σ_{x_3}	0.50	0.50	0.72	95.71	0.50	-0.40	96.25	0.50	0.56	95.06	0.50	0.10	95.57
		σ_{x_4}	0.50	0.50	-0.42	97.55	0.50	0.89	93.12	0.50	0.20	93.83	0.51	1.32	92.41
		σ_{x_5}	0.50	0.50	0.58	91.41	0.51	1.13	91.88	0.50	0.54	93.83	0.50	-0.29	98.10
		σ_{x_6}	0.50	0.50	0.80	95.09	0.50	-0.01	96.88	0.50	-0.25	95.06	0.50	0.82	97.47
		σ_{y_1}	0.50	0.50	-0.21	95.09	0.50	-0.83	94.38	0.49	-1.03	95.06	0.50	-0.50	93.67
		σ_{y_2}	0.50	0.50	-0.92	95.09	0.49	-1.12	90.62	0.50	-0.55	96.30	0.50	0.14	94.94
		σ_{y_3}	0.50	0.34	-32.95	68.71	0.35	-30.99	77.50	0.33	-34.04	68.52	0.34	-31.96	79.75
		σ_{ξ_1}	1.00	0.99	-0.68	96.32	0.99	-0.52	95.00	1.00	0.21	95.06	1.01	0.72	94.30
		σ_{ξ_2}	1.00	1.01	0.68	98.16	1.00	0.20	94.38	1.01	0.58	97.53	1.01	0.81	93.04
		ν_{x_1}	0.00	0.00	0.36	95.09	-0.01	-0.89	95.62	-0.01	-0.63	94.44	0.00	-0.12	96.20
		ν_{x_2}	0.00	0.00	-0.35	97.55	0.00	-0.30	92.50	0.00	-0.28	93.83	0.00	0.11	92.41
		ν_{x_3}	0.00	0.00	0.43	98.77	-0.01	-1.27	91.25	0.00	-0.32	95.68	0.00	0.37	94.30
		ν_{x_4}	0.00	0.00	0.49	95.71	0.00	-0.18	96.25	0.01	1.14	93.21	0.00	0.36	91.77
		ν_{x_5}	0.00	0.01	0.88	95.09	0.00	0.25	95.62	0.01	1.21	97.53	0.00	0.20	94.94
		ν_{x_6}	0.00	0.00	0.44	95.09	0.00	0.05	95.00	0.01	0.87	93.83	0.00	0.44	93.04
		ν_{y_2}	0.00	0.01	0.56	97.55	0.00	0.48	95.00	0.00	0.22	97.53	0.00	-0.50	93.04
		ν_{y_3}	0.00	0.00	0.32	97.55	0.00	0.01	98.12	0.01	0.53	96.30	-0.01	-1.20	98.73
		λ_{x_2}	1.00	1.00	0.17	95.09	1.01	0.88	95.00	1.00	0.37	96.91	1.01	0.81	94.30
		λ_{x_3}	1.00	1.01	1.02	96.32	1.01	0.91	91.88	1.01	0.96	95.68	1.01	1.41	94.30
		λ_{x_4}	1.00	1.00	0.39	94.48	1.00	0.29	97.50	1.00	-0.02	95.06	1.01	0.94	96.84
		λ_{x_5}	1.00	1.01	0.53	95.09	1.01	1.04	95.62	1.00	0.03	94.44	1.00	0.27	96.84
		λ_{y_2}	1.00	0.95	-4.57	90.18	0.99	-1.24	93.75	0.99	-0.52	94.44	0.97	-2.77	91.77
		λ_{y_3}	1.00	0.98	-2.00	96.93	1.00	-0.15	96.88	1.00	-0.35	93.83	0.98	-1.51	93.67
	400														
		ρ_{y2}	0.00	0.50	49.67	100.00	0.49	48.84	100.00	0.49	48.83	100.00	0.50	49.83	100.00
		α	0.00	0.01	0.67	87.88	0.00	-0.13	84.38	0.00	-0.44	75.76	0.01	0.72	75.00

Table A.17: Results table for Study 4 simultaneous structural lag population model (D4) and measurement lag analysis model (A2) *(continued)*

				$\rho_\eta = 0.3$						$\rho_\eta = 0.6$					
				$\phi_\zeta = 0.3$			$\phi_\zeta = 0.6$			$\phi_\zeta = 0.3$			$\phi_\zeta = 0.3$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%
		γ_1	0.30	0.31	3.15	84.85	0.30	0.29	93.75	0.31	2.68	100.00	0.30	-1.58	92.86
		γ_2	0.30	0.30	-0.87	93.94	0.31	3.25	90.62	0.30	1.56	93.94	0.30	-0.58	89.29
		γ_3	0.15	0.16	6.67	96.97	0.15	-0.55	93.75	0.15	0.78	96.97	0.15	2.36	100.00
		σ_{x_1}	0.50	0.50	0.26	96.97	0.50	-0.56	96.88	0.51	1.08	93.94	0.50	-0.37	96.43
		σ_{x_2}	0.50	0.50	-0.06	96.97	0.49	-1.18	93.75	0.50	-0.34	93.94	0.50	0.13	96.43
		σ_{x_3}	0.50	0.50	-0.03	100.00	0.49	-2.37	100.00	0.50	-0.07	100.00	0.51	1.78	96.43
		σ_{x_4}	0.50	0.50	-0.95	90.91	0.50	0.72	90.62	0.50	0.92	90.91	0.50	0.72	96.43
		σ_{x_5}	0.50	0.50	0.31	87.88	0.51	1.14	93.75	0.51	1.33	87.88	0.51	1.12	96.43
		σ_{x_6}	0.50	0.50	-0.56	96.97	0.50	-0.37	96.88	0.51	2.04	87.88	0.50	-0.84	92.86
		σ_{y_1}	0.50	0.50	-0.37	96.97	0.50	-0.23	90.62	0.50	-0.56	96.97	0.49	-1.10	96.43
		σ_{y_2}	0.50	0.50	0.09	90.91	0.50	-0.28	96.88	0.50	-0.52	96.97	0.49	-1.42	100.00
		σ_{y_3}	0.50	0.33	-34.43	60.61	0.33	-33.08	78.12	0.32	-35.08	63.64	0.33	-34.00	60.71
		σ_{ξ_1}	1.00	1.00	0.22	100.00	1.01	0.87	96.88	0.99	-0.82	96.97	0.99	-0.62	96.43
		σ_{ξ_2}	1.00	1.01	0.66	93.94	1.00	0.39	90.62	0.99	-0.81	90.91	1.01	0.90	89.29
		ν_{x_1}	0.00	0.00	0.14	100.00	0.00	-0.36	93.75	0.00	-0.24	90.91	0.02	1.89	100.00
		ν_{x_2}	0.00	0.00	0.12	93.94	0.00	-0.07	96.88	-0.01	-0.57	90.91	0.02	2.11	92.86
		ν_{x_3}	0.00	0.00	0.44	96.97	0.00	0.11	90.62	-0.01	-1.14	90.91	0.01	1.24	92.86
		ν_{x_4}	0.00	0.02	1.65	90.91	0.00	-0.46	100.00	0.01	1.06	96.97	-0.01	-0.73	85.71
		ν_{x_5}	0.00	0.01	0.52	96.97	0.01	0.60	93.75	0.00	0.44	96.97	-0.02	-1.71	92.86
		ν_{x_6}	0.00	0.02	2.06	93.94	0.00	0.10	96.88	0.00	-0.38	100.00	-0.02	-1.63	96.43
		ν_{y_2}	0.00	0.00	0.49	96.97	0.01	0.55	93.75	0.00	0.24	87.88	0.00	-0.28	100.00
		ν_{y_3}	0.00	0.00	0.01	100.00	0.01	0.60	96.88	-0.01	-0.57	100.00	0.00	0.49	100.00
		λ_{x_2}	1.00	1.01	0.73	96.97	1.00	-0.27	96.88	1.00	0.31	96.97	1.00	0.30	96.43
		λ_{x_3}	1.00	1.00	-0.07	93.94	1.00	0.04	96.88	1.01	0.66	96.97	1.00	0.25	89.29
		λ_{x_4}	1.00	1.00	-0.41	96.97	1.00	0.33	100.00	1.01	0.53	90.91	0.99	-1.19	96.43
		λ_{x_5}	1.00	1.01	0.62	87.88	1.00	0.19	90.62	1.00	0.44	96.97	1.00	-0.02	92.86
		λ_{y_2}	1.00	1.01	0.90	96.97	0.99	-1.26	93.75	0.98	-2.13	100.00	1.01	1.44	100.00
		λ_{y_3}	1.00	0.98	-1.94	96.97	1.00	-0.25	87.50	0.99	-0.67	96.97	1.01	0.85	96.43

[1] W^* is the population and analysis W condition.

[2] N is the simulated sample size.

[3] θ is the simulated population value of each parameter.

[4] $\rho_{y2} = \ldots$ is the simulated spatial autocorrelation of the endogenous lag.

[5] $\bar{\hat{\theta}}$ is the obtained average posterior mean across simulated iterations.

[6] $Bias(\bar{\hat{\theta}})\%$ is the average percent bias across simulation iterations when $\theta \neq 0$, otherwise $Bias(\bar{\hat{\theta}})\%$ provides absolute bias.

[7] Cover% is the percent of simulated trials in which the population value falls within the central 95% density of the posterior estimate.

Table A.18: Results table for Study 4 simultaneous structural lag population model (D4) and endogenous structural lag analysis model (A3)

				$\rho_\eta = 0.3$						$\rho_\eta = 0.6$					
				$\phi_\zeta = 0.3$			$\phi_\zeta = 0.6$			$\phi_\zeta = 0.3$			$\phi_\zeta = 0.6$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%
W_C^*															
	49														
		ρ_η	-	0.50	65.94	100.00	0.49	64.89	100.00	0.54	-9.81	100.00	0.53	-11.89	100.00
		α	0.00	0.01	1.21	94.44	-0.01	-1.26	92.37	0.01	1.30	79.80	0.00	-0.28	80.15
		γ_1	0.30	0.31	4.24	95.96	0.32	7.27	96.18	0.33	9.67	95.71	0.32	5.82	95.88
		γ_2	0.30	0.32	6.26	97.73	0.32	7.85	95.17	0.33	9.26	94.70	0.32	7.43	97.16
		γ_3	0.15	0.16	8.05	98.23	0.17	10.11	95.67	0.17	10.59	95.71	0.16	9.82	96.65
		σ_{x_1}	0.50	0.50	0.06	95.96	0.51	1.24	97.20	0.50	-0.43	94.19	0.50	0.99	95.36
		σ_{x_2}	0.50	0.51	2.77	93.69	0.51	1.07	96.95	0.52	3.17	92.68	0.51	1.64	94.59
		σ_{x_3}	0.50	0.50	-0.33	97.73	0.50	0.87	93.13	0.51	2.49	93.43	0.51	1.73	95.10
		σ_{x_4}	0.50	0.50	0.90	94.95	0.51	1.71	93.89	0.51	1.90	94.19	0.50	0.78	96.39
		σ_{x_5}	0.50	0.50	0.81	94.44	0.50	-0.27	96.95	0.51	2.04	94.44	0.51	1.70	95.62
		σ_{x_6}	0.50	0.51	1.63	91.67	0.51	1.99	94.91	0.50	0.85	93.69	0.51	2.35	95.62
		σ_{y_1}	0.50	0.50	-0.17	95.71	0.50	-0.88	96.44	0.50	-0.79	94.70	0.49	-1.43	94.85
		σ_{y_2}	0.50	0.50	0.36	96.46	0.50	-0.56	95.93	0.50	0.47	95.20	0.51	1.17	95.36
		σ_{y_3}	0.50	0.50	-0.01	93.43	0.51	1.28	96.18	0.50	0.25	95.20	0.51	1.17	96.91
		σ_{ξ_1}	1.00	1.02	1.61	94.95	1.00	-0.22	95.42	1.02	1.54	93.94	1.03	2.54	96.13
		σ_{ξ_2}	1.00	1.01	1.21	93.69	0.99	-0.93	95.42	1.02	1.83	95.20	1.01	1.04	94.33
		ν_{x_1}	0.00	0.00	0.08	99.24	-0.01	-1.10	96.18	0.00	-0.22	94.95	0.01	0.63	95.36
		ν_{x_2}	0.00	0.01	0.71	96.72	-0.01	-1.44	95.93	0.00	-0.05	96.97	0.01	0.53	95.88
		ν_{x_3}	0.00	0.00	0.49	98.23	-0.01	-0.94	95.42	0.00	-0.21	95.45	0.00	0.10	95.88
		ν_{x_4}	0.00	0.00	-0.01	94.95	-0.01	-0.65	95.93	0.01	1.23	96.21	-0.02	-1.68	96.91
		ν_{x_5}	0.00	0.01	0.51	95.45	0.00	0.33	95.93	0.02	2.12	95.45	-0.01	-0.78	96.39
		ν_{x_6}	0.00	0.00	0.18	94.44	-0.01	-1.43	95.93	0.01	0.94	97.47	-0.01	-0.91	96.39
		ν_{y_2}	0.00	-0.01	-1.38	96.72	0.01	0.65	94.40	0.00	-0.28	92.68	0.00	-0.37	95.88
		ν_{y_3}	0.00	-0.01	-1.07	95.45	0.01	1.23	95.42	0.00	0.09	94.70	0.00	0.01	96.91
		λ_{x_2}	1.00	1.01	0.95	95.96	1.02	2.12	98.73	1.01	1.02	96.97	1.01	0.51	97.16
		λ_{x_3}	1.00	1.01	1.23	94.44	1.03	2.78	95.67	1.01	1.01	95.96	1.01	0.92	96.91
		λ_{x_4}	1.00	1.01	1.45	94.19	1.03	3.26	93.89	1.02	1.72	95.45	1.03	2.60	96.65
		λ_{x_5}	1.00	1.02	1.98	95.96	1.02	1.64	94.66	1.01	1.36	94.19	1.02	2.16	97.16
		λ_{y_2}	1.00	0.94	-6.43	92.93	0.93	-6.80	94.40	0.92	-8.45	92.42	0.93	-6.79	91.49
		λ_{y_3}	1.00	0.93	-6.92	93.94	0.90	-9.83	93.89	0.92	-8.22	94.19	0.94	-5.86	94.59
	196														
		ρ_η	-	0.51	69.08	100.00	0.50	66.49	100.00	0.63	4.55	97.47	0.62	3.65	98.45
		α	0.00	0.00	0.35	92.93	0.00	-0.20	93.13	0.00	-0.09	77.53	0.00	-0.50	74.74
		γ_1	0.30	0.31	3.30	94.19	0.31	3.15	96.44	0.31	3.64	95.71	0.31	3.75	95.10
		γ_2	0.30	0.31	2.74	95.45	0.31	2.30	93.64	0.31	3.29	94.44	0.31	2.18	96.13
		γ_3	0.15	0.15	2.74	96.97	0.15	0.04	94.40	0.16	4.94	93.69	0.15	3.28	96.91
		σ_{x_1}	0.50	0.50	0.35	94.95	0.50	0.68	96.69	0.50	0.61	95.71	0.50	-0.27	94.33
		σ_{x_2}	0.50	0.50	0.00	94.70	0.50	-0.16	92.37	0.50	-0.26	95.45	0.50	0.51	95.88
		σ_{x_3}	0.50	0.50	-0.07	92.68	0.50	0.49	94.15	0.51	1.21	95.71	0.50	0.42	92.53
		σ_{x_4}	0.50	0.50	0.52	94.19	0.50	-0.25	92.88	0.51	1.26	95.71	0.50	0.20	96.65
		σ_{x_5}	0.50	0.50	0.82	94.95	0.51	1.47	93.38	0.50	0.07	94.95	0.51	1.10	95.10
		σ_{x_6}	0.50	0.50	0.38	95.71	0.50	-0.62	93.64	0.50	0.50	97.47	0.50	0.62	95.36
		σ_{y_1}	0.50	0.50	-0.93	95.96	0.50	-0.30	95.93	0.49	-1.35	95.96	0.50	-0.34	94.85
		σ_{y_2}	0.50	0.50	-0.49	96.46	0.50	-0.28	96.69	0.50	-0.13	94.70	0.50	-0.36	96.65
		σ_{y_3}	0.50	0.50	0.07	94.95	0.50	-0.04	96.69	0.50	-0.48	95.45	0.50	-0.31	96.65
		σ_{ξ_1}	1.00	1.00	-0.04	92.93	1.01	0.50	96.69	1.01	0.62	94.70	1.00	0.47	94.85
		σ_{ξ_2}	1.00	1.00	0.20	94.95	1.01	0.55	96.18	1.00	0.26	96.72	1.01	0.89	94.07
		ν_{x_1}	0.00	0.00	-0.50	95.45	0.00	-0.43	96.44	0.00	-0.29	95.20	0.00	-0.29	97.68
		ν_{x_2}	0.00	0.00	-0.31	95.96	0.00	-0.47	94.91	0.00	0.32	94.70	0.00	-0.35	95.36
		ν_{x_3}	0.00	0.00	-0.49	95.96	-0.01	-0.55	97.46	0.00	-0.06	93.94	0.00	-0.46	96.13
		ν_{x_4}	0.00	0.01	1.07	94.44	0.00	-0.38	94.91	0.00	0.07	96.72	-0.01	-0.59	96.91

Table A.18: Results table for Study 4 simultaneous structural lag population model (D4) and endogenous structural lag analysis model (A3) *(continued)*

				$\rho_\eta = 0.3$						$\rho_\eta = 0.6$					
				$\phi_\zeta = 0.3$			$\phi_\zeta = 0.6$			$\phi_\zeta = 0.3$			$\phi_\zeta = 0.3$		
W^*	N		θ	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%	$\bar{\bar{\theta}}$	$Bias(\bar{\bar{\theta}})\%$	Cover%
		ν_{x_5}	0.00	0.01	0.83	93.43	0.00	-0.42	94.66	0.00	0.28	97.22	0.00	-0.19	93.56
		ν_{x_6}	0.00	0.01	1.02	92.42	0.00	-0.43	93.64	0.00	-0.14	95.20	0.00	-0.14	95.10
		ν_{y_2}	0.00	0.00	0.15	95.71	0.00	0.05	98.47	0.00	0.13	95.45	0.00	0.10	93.81
		ν_{y_3}	0.00	0.00	0.07	94.70	0.00	-0.28	95.17	0.00	0.21	94.44	0.00	0.07	95.36
		λ_{x_2}	1.00	1.00	0.31	96.21	1.00	0.06	93.89	1.00	0.32	94.44	1.01	0.87	96.65
		λ_{x_3}	1.00	1.01	0.99	94.70	1.00	0.24	95.67	1.00	0.12	97.22	1.01	0.91	95.10
		λ_{x_4}	1.00	1.00	0.37	92.93	1.00	0.13	96.44	1.01	0.90	93.69	1.01	0.81	96.65
		λ_{x_5}	1.00	1.00	0.25	94.44	1.00	0.18	96.18	1.01	0.92	94.70	1.00	0.08	97.42
		λ_{y_2}	1.00	0.98	-2.28	92.93	0.98	-1.92	96.44	0.98	-1.54	94.95	0.99	-0.89	95.88
		λ_{y_3}	1.00	0.97	-2.98	93.94	0.98	-2.08	91.35	0.98	-1.65	93.43	0.98	-1.64	93.56
	400														
		ρ_η	-	0.52	73.63	100.00	0.52	72.87	100.00	0.68	13.25	82.07	0.68	12.71	83.94
		α	0.00	0.00	0.13	91.16	0.00	0.22	89.20	0.00	-0.17	73.48	0.00	0.37	77.72
		γ_1	0.30	0.31	1.78	95.20	0.30	-0.13	95.63	0.31	2.15	92.42	0.31	2.05	94.56
		γ_2	0.30	0.31	2.38	94.70	0.31	1.79	94.60	0.30	0.66	96.21	0.31	1.89	95.08
		γ_3	0.15	0.15	0.55	95.96	0.15	-1.82	95.63	0.15	0.81	93.94	0.15	2.27	95.34
		σ_{x_1}	0.50	0.50	0.03	95.20	0.50	0.96	93.83	0.50	0.02	95.71	0.50	0.13	95.60
		σ_{x_2}	0.50	0.50	-0.14	93.69	0.50	0.54	93.83	0.50	-0.05	95.71	0.50	-0.07	96.37
		σ_{x_3}	0.50	0.50	-0.08	93.94	0.50	0.24	96.92	0.50	0.35	93.43	0.50	0.10	96.89
		σ_{x_4}	0.50	0.50	-0.12	94.95	0.50	-0.21	94.34	0.50	0.56	97.98	0.50	0.36	95.34
		σ_{x_5}	0.50	0.50	0.61	95.20	0.50	-0.62	95.37	0.50	0.49	93.43	0.50	0.30	93.78
		σ_{x_6}	0.50	0.50	0.41	94.44	0.50	0.78	96.40	0.50	-0.27	96.21	0.50	-0.16	95.60
		σ_{y_1}	0.50	0.50	-0.67	94.70	0.50	-0.60	96.40	0.50	-0.76	93.94	0.50	-0.36	94.82
		σ_{y_2}	0.50	0.50	-0.24	94.44	0.50	-0.69	94.34	0.50	-0.84	95.45	0.50	-0.50	95.60
		σ_{y_3}	0.50	0.50	-0.63	96.97	0.50	-0.09	97.43	0.49	-1.10	91.16	0.49	-1.06	93.78
		σ_{ξ_1}	1.00	1.00	0.10	94.95	1.00	0.21	91.52	1.01	0.56	92.93	1.00	0.21	92.75
		σ_{ξ_2}	1.00	1.00	0.19	94.95	1.00	0.40	95.12	1.01	0.81	96.97	1.00	-0.45	94.30
		ν_{x_1}	0.00	0.01	0.68	95.71	0.00	-0.17	93.57	-0.01	-0.68	93.18	0.00	0.38	96.63
		ν_{x_2}	0.00	0.01	0.85	94.19	0.00	0.05	95.37	-0.01	-0.58	94.95	0.00	0.37	94.56
		ν_{x_3}	0.00	0.00	0.26	94.70	0.00	-0.09	93.83	-0.01	-0.61	94.70	0.01	0.62	95.08
		ν_{x_4}	0.00	0.00	0.18	95.96	0.00	0.43	93.06	0.00	-0.12	94.70	0.01	0.90	96.37
		ν_{x_5}	0.00	0.00	-0.09	96.97	0.00	0.16	95.37	0.00	-0.28	95.20	0.00	0.27	92.23
		ν_{x_6}	0.00	0.00	-0.21	95.45	0.00	0.48	93.06	0.00	-0.29	94.70	0.00	0.45	96.89
		ν_{y_2}	0.00	0.00	0.06	93.43	0.00	-0.36	92.54	0.00	-0.15	93.43	0.00	0.34	95.60
		ν_{y_3}	0.00	0.00	-0.02	93.94	0.00	-0.17	92.54	0.00	-0.18	91.92	0.00	0.28	95.60
		λ_{x_2}	1.00	1.00	0.23	94.44	1.00	-0.16	96.92	1.00	0.25	93.94	1.00	0.41	96.37
		λ_{x_3}	1.00	1.00	0.34	95.45	1.00	0.04	96.40	1.00	0.23	95.96	1.00	0.24	96.37
		λ_{x_4}	1.00	1.00	0.16	95.96	1.01	0.51	94.09	1.00	-0.18	95.71	1.00	0.29	94.04
		λ_{x_5}	1.00	1.00	0.40	94.44	1.00	0.29	95.63	1.00	0.01	95.20	1.00	0.23	94.30
		λ_{y_2}	1.00	0.99	-0.66	95.71	0.99	-0.65	95.89	1.00	0.49	94.44	0.99	-0.70	94.56
		λ_{y_3}	1.00	0.99	-0.91	97.22	0.98	-1.54	94.60	1.00	-0.08	93.69	1.00	0.23	94.30
W_D^*															
	49														
		ρ_η	-	0.50	65.94	100.00	0.49	64.89	100.00	0.54	-9.81	100.00	0.53	-11.89	100.00
		α	0.00	0.01	1.20	94.41	0.01	1.26	93.37	0.01	1.30	79.80	0.00	-0.28	80.15
		γ_1	0.30	0.31	4.24	95.96	0.32	7.27	96.18	0.33	9.67	95.71	0.32	5.82	95.88
		γ_2	0.30	0.32	6.26	97.73	0.32	7.85	95.17	0.33	9.26	94.70	0.32	7.43	97.16
		γ_3	0.15	0.16	8.07	98.23	0.17	10.11	95.67	0.17	10.59	95.71	0.16	9.82	96.65
		σ_{x_1}	0.50	0.50	0.06	95.96	0.51	1.24	97.20	0.50	-0.43	94.19	0.50	0.99	95.36
		σ_{x_2}	0.50	0.51	2.77	93.29	0.51	1.07	96.95	0.52	3.17	92.68	0.51	1.64	94.59
		σ_{x_3}	0.50	0.50	0.33	97.73	0.50	0.87	93.13	0.51	2.49	93.43	0.51	1.73	95.10
		σ_{x_4}	0.50	0.50	0.90	94.95	0.51	1.71	93.89	0.51	1.90	94.19	0.50	0.78	96.39
		σ_{x_5}	0.50	0.50	0.81	94.44	0.50	-0.17	96.95	0.51	2.04	94.44	0.51	1.70	95.62

Table A.18: Results table for Study 4 simultaneous structural lag population model (D4) and endogenous structural lag analysis model (A3) *(continued)*

				$\rho_\eta = 0.3$						$\rho_\eta = 0.6$					
				$\phi_\zeta = 0.3$			$\phi_\zeta = 0.6$			$\phi_\zeta = 0.3$			$\phi_\zeta = 0.3$		
W^*	N		θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%
		σ_{x_6}	0.50	0.51	1.63	91.67	0.51	1.99	94.91	0.50	0.85	93.69	0.51	2.35	95.62
		σ_{y_1}	0.50	0.50	0.07	95.71	0.50	0.18	96.44	0.50	0.49	94.70	0.49	-1.43	93.85
		σ_{y_2}	0.50	0.50	0.36	96.46	0.50	-0.56	95.93	0.50	0.47	95.20	0.51	1.17	95.36
		σ_{y_3}	0.50	0.50	0.01	93.43	0.51	1.28	96.18	0.50	0.25	95.20	0.51	1.17	96.91
		σ_{ξ_1}	1.00	1.02	1.61	94.95	1.00	-0.22	95.42	1.02	1.54	93.94	1.03	2.54	96.13
		σ_{ξ_2}	1.00	1.01	1.21	93.69	0.99	-0.93	95.42	1.02	1.83	95.20	1.01	1.04	94.33
		ν_{x_1}	0.00	0.00	0.08	99.25	-0.01	-1.10	96.18	0.00	-0.22	94.95	0.01	0.63	95.36
		ν_{x_2}	0.00	0.01	0.71	96.72	-0.01	-1.44	95.93	0.00	-0.05	96.97	0.01	0.53	95.18
		ν_{x_3}	0.00	0.00	0.49	98.23	-0.01	-0.94	95.42	0.00	-0.22	95.45	0.00	0.10	95.38
		ν_{x_4}	0.00	0.00	-0.01	94.95	-0.01	-0.65	95.93	0.01	1.23	96.21	-0.02	-1.68	96.91
		ν_{x_5}	0.00	0.01	0.51	95.45	0.00	0.33	95.93	0.02	2.12	95.45	-0.01	-0.78	96.39
		ν_{x_6}	0.00	0.00	0.18	94.44	-0.01	-1.43	95.93	0.01	0.94	97.47	-0.01	-0.91	96.39
		ν_{y_2}	0.00	0.01	1.33	96.72	0.01	0.65	94.40	0.00	-0.28	92.68	0.00	-0.37	95.64
		ν_{y_3}	0.00	0.01	1.04	95.45	0.01	1.23	95.42	0.00	0.09	94.70	0.00	0.01	96.78
		λ_{x_2}	1.00	1.01	0.95	95.96	1.02	2.12	98.73	1.01	1.02	96.97	1.01	0.51	97.16
		λ_{x_3}	1.00	1.01	1.23	94.44	1.03	2.78	95.67	1.01	1.01	95.96	1.01	0.92	96.91
		λ_{x_4}	1.00	1.01	1.45	94.19	1.03	3.26	93.89	1.02	1.72	95.45	1.03	2.60	96.65
		λ_{x_5}	1.00	1.02	1.98	95.96	1.02	1.64	94.66	1.01	1.36	94.19	1.02	2.16	97.06
		λ_{y_2}	1.00	0.94	-6.43	92.93	0.93	-6.80	94.40	0.92	-8.45	92.42	0.93	-6.79	91.49
		λ_{y_3}	1.00	0.93	-6.92	93.94	0.90	-9.83	93.89	0.92	-8.22	94.19	0.94	-5.86	94.39
	196														
		ρ_η	-	0.49	51.08	100.00	0.50	66.49	100.00	0.63	4.55	97.47	0.62	3.65	98.45
		α	0.00	0.00	0.35	92.93	0.00	-0.20	93.13	0.00	-0.09	77.53	0.00	-0.50	74.62
		γ_1	0.30	0.31	3.30	94.19	0.31	3.15	96.44	0.31	3.64	95.71	0.31	3.75	95.09
		γ_2	0.30	0.31	2.74	95.45	0.31	2.30	93.64	0.31	3.29	94.44	0.31	2.18	96.11
		γ_3	0.15	0.15	2.74	96.97	0.15	0.04	94.40	0.16	4.94	93.69	0.15	3.28	96.94
		σ_{x_1}	0.50	0.50	0.35	94.95	0.50	0.68	96.69	0.50	0.61	95.71	0.50	-0.27	94.33
		σ_{x_2}	0.50	0.50	0.00	94.70	0.50	-0.16	92.37	0.50	-0.26	95.45	0.50	0.51	95.88
		σ_{x_3}	0.50	0.50	0.11	92.68	0.50	0.49	94.15	0.51	1.21	95.71	0.50	0.42	92.53
		σ_{x_4}	0.50	0.50	0.52	94.19	0.50	0.22	92.88	0.51	1.26	95.71	0.50	0.20	96.64
		σ_{x_5}	0.50	0.50	0.82	94.95	0.51	1.47	93.38	0.50	0.07	94.95	0.51	1.10	95.10
		σ_{x_6}	0.50	0.50	0.38	95.71	0.50	-0.62	93.64	0.50	0.50	97.47	0.50	0.62	95.36
		σ_{y_1}	0.50	0.50	-0.93	95.96	0.50	-0.30	95.93	0.49	-1.55	95.96	0.50	-0.34	94.85
		σ_{y_2}	0.50	0.50	-0.49	96.46	0.50	-0.28	96.69	0.50	-0.13	94.70	0.50	-0.36	96.65
		σ_{y_3}	0.50	0.50	0.07	94.95	0.50	-0.04	96.69	0.50	-0.48	95.45	0.50	-0.31	96.65
		σ_{ξ_1}	1.00	1.00	-0.04	92.93	1.01	0.50	96.69	1.01	0.62	94.70	1.00	0.47	94.85
		σ_{ξ_2}	1.00	1.00	0.20	94.95	1.00	0.09	96.18	1.00	0.26	96.72	1.01	0.89	94.07
		ν_{x_1}	0.00	0.00	-0.50	95.45	0.00	-0.43	96.44	0.00	-0.29	95.20	0.00	-0.29	97.68
		ν_{x_2}	0.00	0.00	-0.31	95.96	0.00	-0.47	94.91	0.00	0.32	94.70	0.00	-0.35	95.36
		ν_{x_3}	0.00	0.00	-0.49	95.96	-0.01	-0.55	97.46	0.00	-0.06	93.94	0.00	-0.46	96.13
		ν_{x_4}	0.00	0.01	1.07	94.44	0.00	0.38	94.91	0.00	0.07	96.72	-0.01	-0.59	96.91
		ν_{x_5}	0.00	0.01	0.83	93.43	0.00	-0.42	94.66	0.00	0.28	97.22	0.00	-0.19	93.56
		ν_{x_6}	0.00	0.01	1.02	92.42	0.00	0.43	93.64	0.00	0.14	95.20	0.00	-0.14	95.10
		ν_{y_2}	0.00	0.00	0.15	95.71	0.00	0.05	98.47	0.00	0.13	95.45	0.00	0.10	93.81
		ν_{y_3}	0.00	0.00	0.07	94.70	0.00	-0.28	95.17	0.00	0.21	94.44	0.00	0.07	95.36
		λ_{x_2}	1.00	1.00	0.31	96.21	1.00	0.06	93.89	1.00	0.32	94.44	1.01	0.87	96.65
		λ_{x_3}	1.00	1.01	0.99	94.70	1.00	0.24	95.67	1.00	0.12	97.22	1.01	0.91	95.10
		λ_{x_4}	1.00	1.00	0.37	92.93	1.00	0.13	96.44	1.01	0.90	93.69	1.01	0.81	96.65
		λ_{x_5}	1.00	1.00	0.25	94.44	1.00	0.18	96.18	1.01	0.92	94.70	1.00	0.08	97.42
		λ_{y_2}	1.00	0.98	-2.28	92.93	0.98	-1.92	96.44	0.98	-1.54	94.95	0.99	-0.89	93.88
		λ_{y_3}	1.00	0.97	-2.98	93.94	0.98	-2.08	91.35	0.98	-1.65	93.43	0.98	-1.64	93.56
	400														
		ρ_η	-	0.52	73.63	100.00	0.52	72.87	100.00	0.68	13.25	82.07	0.68	12.71	83.94
		α	0.00	0.00	0.13	91.16	0.00	0.22	89.20	0.00	-0.17	73.48	0.00	0.37	77.72

Table A.18: Results table for Study 4 simultaneous structural lag population model (D4) and endogenous structural lag analysis model (A3) *(continued)*

			$\rho_\eta = 0.3$						$\rho_\eta = 0.6$					
			$\phi_\zeta = 0.3$			$\phi_\zeta = 0.6$			$\phi_\zeta = 0.3$			$\phi_\zeta = 0.3$		
W^*	N	θ	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%	$\bar{\hat{\theta}}$	$Bias(\bar{\hat{\theta}})\%$	Cover%
	γ_1	0.30	0.31	1.78	95.20	0.30	-0.13	95.63	0.31	2.15	92.42	0.31	2.05	94.56
	γ_2	0.30	0.31	2.38	94.70	0.31	1.79	94.60	0.30	0.66	96.21	0.31	1.89	95.08
	γ_3	0.15	0.15	0.55	95.96	0.15	-1.82	95.63	0.15	0.81	93.94	0.15	2.27	95.34
	σ_{x_1}	0.50	0.50	0.03	95.20	0.50	0.96	93.83	0.50	0.02	95.71	0.50	0.13	95.60
	σ_{x_2}	0.50	0.50	-0.14	93.69	0.50	0.54	93.83	0.50	-0.05	95.71	0.50	-0.07	96.37
	σ_{x_3}	0.50	0.50	0.08	93.94	0.50	0.24	96.92	0.50	0.35	93.43	0.50	0.10	96.89
	σ_{x_4}	0.50	0.50	0.12	94.95	0.50	-0.21	94.34	0.50	0.56	97.98	0.50	0.36	95.34
	σ_{x_5}	0.50	0.50	-0.65	95.20	0.50	-0.62	95.37	0.50	0.49	93.43	0.50	0.30	93.78
	σ_{x_6}	0.50	0.50	0.41	94.44	0.50	0.78	96.40	0.50	-0.27	96.21	0.50	0.16	95.60
	σ_{y_1}	0.50	0.50	0.67	94.70	0.50	-0.60	96.40	0.50	-0.76	93.94	0.50	0.36	94.82
	σ_{y_2}	0.50	0.50	0.24	94.44	0.50	-0.69	94.34	0.50	-0.84	95.45	0.50	0.20	95.60
	σ_{y_3}	0.50	0.50	0.63	96.97	0.50	-0.09	97.43	0.49	-1.10	91.16	0.49	-1.06	93.78
	σ_{ξ_1}	1.00	1.00	0.10	94.95	1.00	0.21	91.52	1.01	0.56	92.93	1.00	0.21	92.75
	σ_{ξ_2}	1.00	1.00	0.19	94.95	1.00	0.40	95.12	1.01	0.81	96.97	1.00	-0.45	94.30
	ν_{x_1}	0.00	0.01	0.68	95.71	0.00	-0.17	93.57	-0.01	-0.68	93.18	0.00	0.38	96.63
	ν_{x_2}	0.00	0.01	0.85	94.19	0.00	0.05	95.37	-0.01	-0.58	94.95	0.00	0.37	94.56
	ν_{x_3}	0.00	0.00	0.26	94.70	0.00	-0.09	93.83	-0.01	-0.61	94.70	0.01	0.62	95.08
	ν_{x_4}	0.00	0.00	-0.18	95.96	0.00	0.43	93.06	0.00	-0.12	94.70	0.01	0.90	96.37
	ν_{x_5}	0.00	0.00	0.09	96.97	0.00	0.16	95.37	0.00	-0.28	95.20	0.00	0.27	92.23
	ν_{x_6}	0.00	0.00	0.21	95.45	0.00	0.48	93.06	0.00	-0.29	94.70	0.00	0.45	96.89
	ν_{y_2}	0.00	0.00	0.06	93.43	0.00	-0.36	92.54	0.00	-0.15	93.43	0.00	0.34	96.60
	ν_{y_3}	0.00	0.00	-0.02	93.94	0.00	-0.17	92.54	0.00	-0.18	91.92	0.00	0.28	95.60
	λ_{x_2}	1.00	1.00	0.23	94.44	1.00	-0.16	96.92	1.00	0.25	93.94	1.00	0.41	96.33
	λ_{x_3}	1.00	1.00	0.34	95.45	1.00	0.04	96.40	1.00	0.23	95.96	1.00	0.24	96.33
	λ_{x_4}	1.00	1.00	0.16	95.96	1.01	0.51	94.09	1.00	-0.18	95.71	1.00	0.29	94.14
	λ_{x_5}	1.00	1.00	0.40	94.44	1.00	0.29	95.63	1.00	0.01	95.20	1.00	0.23	94.32
	λ_{y_2}	1.00	0.99	-0.66	95.71	0.99	-0.65	95.89	1.00	0.49	94.44	0.99	-0.70	93.56
	λ_{y_3}	1.00	0.99	-0.91	97.22	0.98	-1.54	94.60	1.00	-0.08	93.69	1.00	0.23	93.80

[1] W^* is the population and analysis W condition.

[2] N is the simulated sample size.

[3] θ is the simulated population value of each parameter.

[4] $\rho_\eta = ...$ is the simulated spatial autocorrelation of the endogenous lag.

[5] $\bar{\hat{\theta}}$ is the obtained average posterior mean across simulated iterations.

[6] $Bias(\bar{\hat{\theta}})\%$ is the average percent bias across simulation iterations when $\theta \neq 0$, otherwise $Bias(\bar{\hat{\theta}})\%$ provides absolute bias.

[7] Cover% is the percent of simulated trials in which the population value falls within the central 95% density of the posterior estimate.

A.5 Empirical Example

Table A.19: Correlation matrix of the extended US homicide data.

	Rape Rate	Assault Rate	Burglary Rate	Murder Rate	Larceny Rate	Motor Vehicle Theft Rate	Robbery Rate	Gini Coefficient	Average Income
Rape Rate									
Assault Rate	0.27								
Burglary Rate	0.17	0.46							
Murder Rate	0.26	0.68	0.55						
Larceny Rate	0.26	0.42	0.62	0.46					
Motor Vehicle Theft Rate	0.14	0.62	0.52	0.65	0.63				
Robbery Rate	-0.08	0.54	0.51	0.82	0.47	0.73			
Gini Coefficient	0.21	0.28	0.35	0.35	0.03	0.23	0.18		
Average Income	-0.38	0.08	-0.36	0.11	-0.04	0.27	0.39	-0.49	
Unemployment Rate	0.41	0.21	0.02	0.22	-0.09	0.18	0.39	0.13	0.28